generated a three dimensional trajectory and variation of this trajectory all over the limit cycle reflected the ECG. Different waves like P, Q, R, S, T of ECG are defined by specific conditions and these conditions have fixed angles with unit circle, when trajectory approaches these angles, it is moved up or down from the limit cycle as shown in Figure 2.

The trajectory is generated by three coupled single degree differential equations of motion as given below

$$\dot{x} = \alpha x - \omega y$$
$$\dot{y} = \alpha y + \omega x$$
$$\dot{z} = \sum_{i \in \{P,Q,R,S,T\}} \mathrm{a_i} \Delta\theta_i \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) - (z - z_0) \qquad (1)$$

where $\alpha = 1 - \sqrt{x^2 + y^2}$, $\Delta\theta_i = (\theta - \theta_i)\mathrm{mod}2\pi$, $\theta = atan2(y, x)$,and ω is angular velocity of the trajectory given by $\omega(t) = \frac{2\pi}{T(t)}$, $T(t)$ denotes the time series originated by RR-process with power spectrum $s(f)$ which is given in equation (3)[4]. The low frequency artifact, baseline wander can be introduced by blending the baseline value z_0 in equation (1) to frequency f_2 corresponding to respiratory action is given by

$$z_0(t) = \mathrm{A}\sin(2\pi f_2 \mathrm{t}) \qquad (2)$$

where $A = 0.15mv$.

This model also incorporated the effects of both parasympathetic & sympathetic activities. Parasympathetic activity is indicated using spectral analysis of RR tachogram, the periodic pulsation in RR tachograms ascribed to parasympathetic activity is called respiratory sinus arrhythmia (RSA) which is in rhythm with respiratory cycle and effect of Mayer waves due to fluctuations in the blood pressure rhythms. These two effects are reflected by bi-modal spectrum which is the combination of duo Gaussian functions [4].

$$s(f) = \frac{\sigma_1^2}{\sqrt{2\pi c_1^2}} exp\left(\frac{(f-f_1)^2}{1c_2^2}\right) + \frac{\sigma_2^2}{\sqrt{2\pi c_2^2}} exp\left(\frac{(f-f_2)^2}{2c_2^2}\right) \quad (3)$$

wheref_1, f_2 are means, c_1, c_2 are corresponding standard deviations. In spectral analysis of RR tachogram, two frequency bands referred as low band (0.04-0.15 Hz) and high band (0.15-0.4) respectively are considered. The σ_1^2,σ_2^2 are powers in low and high band respectively. In this synthetic ECG generation (ECGSYN) function [5], RR intervals with different mean and variance are used to represent the transitions between physiological states like sleep states [6]. Extension of this model also introduced a model based filtering, classification and compression by fitting the parameter values which are specified by ECGSYN function [7], [8]. MATLAB is very efficient tool for signal analysis and processing. Further Ackora-Prah generated synthetic ECG signals by using several inbuilt MATLAB functions(Synecg) to analyse normal &abnormal ECG waveforms and also applied *sgolay filt* function which is used towards smooth out a noisy signal whose frequency is high [9]. Kovas introduced a piecewise polynomial approximation method to model ECG curve that was based on 15 control points denoting these points by $x_1 \dots\dots\dots.\dots. x_{15}$,to find the spline S, the following equation is used

$$S^{(i)}(x_k) = f^{(i)}(x_k) \qquad (4)$$

where $k = 1, 2, \dots\dots..15$; (i=0,1,2)

These points depend on geometrical parameters (tangents and curvature) and diagnostic parameters (intervals, amplitude and positions). Selection of base points depends on diagnostic attributes, because they dictate the diagnostic intervals and amplitude of ECG wave [10]. In this series for understanding HRV generation, Zeeman has proposed mathematical formulation of heart beat using Van der pol-Lienard equation [11]. This equation basically described relaxation oscillators in electronic circuits, and has been used customarily in mathematical formulation of the cardiac rhythm. The usual form of this equation is:

$$\ddot{x} + \mathrm{a}(1 - \mathrm{b}x^2)\dot{x} + \mathrm{c}x = r(t) \qquad (5)$$

where system parameters are denoted by a, b and c and $r(t)$ signifies an external input. The distinctive characteristics of an isolated VdP oscillator resemble closely that of heart actuation potential [12]. But this Zeeman model has not considered, parasympathetic and sympathetic effects, due to this modified Zeeman model was presented including these effects. In addition, a novel neural network technique was also introduced for ECG signal modeling and error identification. ECG waves are combination of repeating sin waves and triangular waves Fourier series is another approach to ECG simulation by using MATLAB software. In this idea each of the first elements are approximated in mathematical function then these functions were applied in models [13]. In this series Fourier model and Gaussian model were also presented to morphology based modeling of single lead ECG. In Fourier model, single lead ECG signal was considered as a periodic signal and three segmented zones (PR, QRS, ST) individually are represented by harmonic component of Fourier given by

$$x(t) = c_{0+\sum_{n=1}^{n} C_n \ \sin(n\omega_0 t + \theta_n)} \qquad (6)$$
$$C_{n = \sqrt{A_n^2 + B_n^2}}$$
$$\theta_n = tan^{-1}\left(\frac{B_n}{A_n}\right)$$

where $x(t)$, c_0 are instantaneous value and average value of zone potential respectively, ω_0 is angular frequency and n is total count of data points. For modeling ω_0, c_0 and θ_n are considered as model parameters and the curve fitting tool in MATLAB has been utilized for getting the model coefficients. Next the Gaussian model of ECG zones (PR, QRS, and ST) is given by

$$\mathrm{GC}(x) = \sum_{i=1}^{N} a_i \, e^{-\left(\frac{x-\mu_i}{2\sigma_i}\right)^2} \quad (7)$$

In above equation (7) mean μ and standard deviation σ are the parameters that identify the shape of Gaussian. Both Fourier and Gaussian model parameters are calculated with the help of curve fitting toolbox in MATLAB with 4th order Fourier and 2nd order Gaussian model respectively [14]. Now many researchers also working on neural network to generate large volume synthetic beats for automatic machine learning medical systems further these aspects can account to modeled ECG with number of abnormalities [15].

III. ECG Simulation

The aim of this work to generate artificially ECG signal that provide efficient tool to study normal and abnormal ECG waveforms without using heavy medical equipment. In First step, analysis of occurrences and values of segments, intervals and waves of real ECG signal is necessary. If adequate knowledge of different part of ECG waves is available then these parts are composed by different mathematical functions and mostly these functions are implemented using MATLAB. This paper presents a run-through of easy MATLAB functions that reflect all the important features of real ECG wave. A normal heart beat depends on age and mental activity level. 60 - 100 beats per minute is normal heart rate for adults under resting condition. An abnormal heart rhythm has too fast, slow or irregular heartbeats which is also called arrhythmia [16]. The most common type abnormal heart rhythms are Tachycardia (> 100 bpm in adults) and Bradycardia (< 60 bpm in adults) [17]. Fig. 3, Fig. 4, and Fig. 5 show simulated artificial ECG waves with normal (80 bpm) and abnormal (40 bpm and 160 bpm) heart rate and Table II represents the values of segments and intervals.

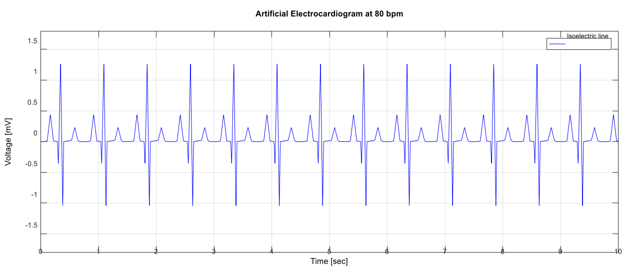

Fig. 3. Simulated synthetic ECG wave with 80 bpm heart rate.

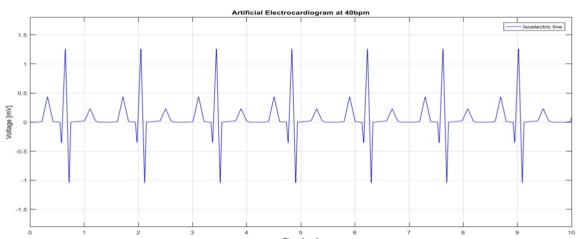

Fig. 4. Simulated synthetic ECG wave with 40 bpm heart rate.

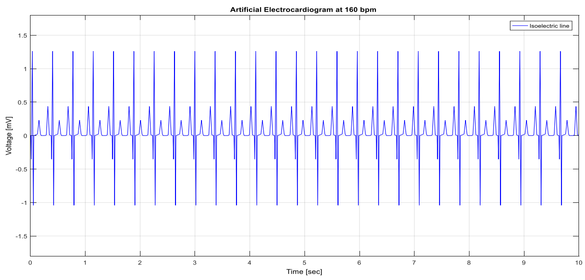

Fig. 5. Simulated synthetic ECG wave with 160 bpm heart rate.

TABLE I: CLINICAL SIGNATURES OF AN ARTIFICIALLY GENERATED ECG SIGNAL

Fig.	Synthetic ECG in bpm	Type of heart rhythm	Peak value in (mv)	RR interval(secs)	QT interval (secs)	PR interval (secs)	ST segment (secs)	PR segment (secs)
3	80	Normal	1.3	0.75	0.36	0.19	0.14	0.07
4	40	Bradycardia	1.3	1.42	0.62	0.35	0.25	0.13
5	160	Tachycardia	1.3	0.36	0.17	0.07	0.06	0.032

IV. Conclusions and Discussion

ECG modeling may be used in various aspects like filtering, segmentation, and classification. For these applications, all the variables of the model should be well defined that can replicate all the important features of ECG signals. In this paper, a brief summary of methods involved in synthetic ECG generation has been presented. The simulated waveforms shown have been realized using ECGSYN & SYNECG functions in MATLAB which consist of several other related functions. Form literature review, it was observed that there were many extensions of dynamical model for synthetic ECG generation by McSharry, however and still many clinical conditions are yet to be explored. The morphology based Fourier and Gaussian modeling is another approach for ECG synthesis by using curve fitting toolbox of MATLAB software. The simulation result of any synthetic ECG generation method further can be enhanced by well-defined mathematical model function that will help to reproduce the morphology of Ectopic beats and several other medical conditions.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Alka Mishra conducted the research and execute the code for the same; Surekha Bhusnur analyzed the data and wrote the paper.

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