Matlab Based Design of Adaptive Filters Using Least Pth Norm: FIR vs IIR

Srishtee Chaudhary
GPCG, ECE Dept, Patiala, Punjab, India
Email: srishtee_chaudhary@yahoo.co.in

Abstract—**Adaptive filters are considered nonlinear systems; therefore their behavior analysis is more complicated than for fixed filters. As adaptive filters are self designing filters, their design can be considered less involved than in the case of digital filters with fixed coefficients. The paper discusses adaptive filters, adaptive filtering with various approaches, optimization methods, algorithms for a filter, IIR and FIR filter designs, in order to improve a prescribed performance criterion. Further least Pth norm approach is compared for both FIR and IIR filter. Since no specifications are available, the adaptive algorithm that determines the updating of the filter coefficients requires extra information that is usually given in the form of a signal. This signal is in general called a desired or reference signal, whose choice depends on the application.**

Index Terms—**adaptive filters, FIR, IIR, least pth**

I. Introduction

Adaptive filter is a filter that self-adjusts its transfer function according to an optimization algorithm driven by an error signal. Because of the complexity of the optimization algorithms, most adaptive filters are digital filters. An adaptive filter is required when either the fixed specifications are unknown or the specifications cannot be satisfied by time-invariant filters [1]. Adaptive filtering techniques are used in wide range of applications, including echo cancellation, adaptive equalization, and adaptive noise cancellation. In many application of noise cancellation, the changes in signal characteristics could be quite fast. This requires utilization of adaptive algorithms, which converge rapidly. An adaptive filter is a nonlinear filter since its characteristics are dependent on the input signal. However, if we freeze the filter parameters at a given instant of time, most adaptive filters considered in this text are linear in the sense that their output signals are linear functions of their input signals [2]. The adaptive filters are time-varying since their parameters are continually changing in order to meet a performance requirement. Practically when the environment is not well defined the procedure could be costly and difficult to implement on-line. The solution to this problem is to employ an adaptive filter that performs on-line updating of its parameters through a rather simple algorithm, using only the information available in the environment. In other words, the adaptive filter performs a data-driven approximation step. Adaptive filters self learn. Adaptive filters require two inputs: the signal and a noise or reference input. As the signal into the filter continues, the adaptive filter coefficients adjust themselves to achieve the desired result, such as identifying an unknown filter or canceling noise in the input signal. New coefficients are sent to the filter from coefficient generator. The coefficient generator is an adaptive algorithm that modifies the coefficients in response to an incoming signal. In most applications the goal of the coefficient generator is to match the filter coefficient to the noise so the adaptive filter can subtract the noise out from the signal. Since, the noise signal changes the coefficients must vary to match it, hence the name adaptive filters. Designing the filter does not require any other frequency response information or specification. To define the self-learning process, select the adaptive algorithm used to reduce the error between the output signal $y(k)$ and the desired signal $d(k)$ [3], [4].

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II. Adaptive Filtering Algorithm

Adaptive filters are dynamic filters which iteratively alter their characteristics in order to achieve an optimal desired output. An adaptive filter algorithmically alters its parameters in order to minimize a function of the difference between the desired output and its actual output. This function is known as the cost function of the adaptive algorithm. Adaptive filtering can be classified into three categories: adaptive filter structures, adaptive algorithms, and applications. The adaptive filters can be implemented in a number of different structures or realizations. The choice of the structure can influence the computational complexity (amount of arithmetic operations per iteration) of the process and also the necessary number of iterations to achieve a desired performance level. Basically, there are two major classes of adaptive digital filter realizations, distinguished by the form of impulse response, namely the finite-duration impulse response (FIR) filter and infinite-duration impulse response (IIR) filters. An adaptive digital filter can be built up using an IIR (Infinite impulse response) or FIR (Finite impulse response) filter. The most widely used adaptive FIR filter structure is the transversal filter, also called tapped delay line, that implements an all-zero transfer function with a canonic direct form realization without any feedback. The adaptive FIR filter structure output is a linear combination of the adaptive filter

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coefficients. The performance surface of the objective cost function is quadratic which yields a single optimal point. Alternative adaptive FIR filter structures improve performance in terms of convergence speed [5]. For simple implementation and easy analysis; most adaptive IIR filter structures use the canonic direct form realization. Some other realizations are also presented to overcome some drawbacks of canonic direct form realization, like slow convergence rate and the need for stable monitoring. An algorithm is a procedure used to adjust adaptive filter coefficients in order to minimize the cost function. The choice of algorithm is highly dependent on the signals of interest and the operating environment, as well as the convergence time required and computation power available. The algorithm determines several important features of the whole adaptive procedure, such as computational complexity, convergence to suboptimal solutions, biased solutions, objective cost function and error signal. The algorithm is determined by defining search method (minimization algorithm), the objective function, and the error signal nature. Adaptive filters utilize different training techniques for updating the filter weights in dynamic environments. Although many adaptive training approaches were introduced for real-time filtering applications, but the LMS adaptive algorithm is practically used due to its simplicity and demonstrated efficient performance. An adaptive filter is required when either fixed specifications are unknown or the specifications cannot be satisfied by time-invariant filters. The algorithm used in equalization is LMS and is known for its simplification, low complexity and better performance in different running environments [6]. Further symmetric approach can be employed to reduce the complexity with partial serial MAC based approach to optimize speed and area [7]. Fractionally spaced equalizer (FSE) can be used to compensate for channel distortion before aliasing effects occur due to symbol rate sampling. FSE is used to reduce computational requirements and to improve convergence [8]. The LMS algorithm with varying step size results change in MSE.

When designing systems, it is important to have a systematic approach so that the design can be done timely and efficiently, which ultimately leads to lower cost. Among different algorithms for updating coefficients of an adaptive filter, LMS algorithm is used more because of its low computational processing tasks and high robustness. This algorithm is a member of stochastic gradient algorithm. It uses Mean Square Error (MSE) as a criterion. LMS uses a step size parameter, input signal and the difference of desired signal and filter output signal to frequently calculate the update of the filter coefficients set. The convergence time in case of LMS depends upon the step size parameter. If step size is small it will take long convergence time and smaller MSE. On the other hand large step size results faster convergence but large MSE. But if it is too large it will never converge. Thus the choice of step size determines the performance characteristics of adaptive algorithm in terms of convergence rate and amount of steady-state mean square error (MSE). The performance of LMS is a tradeoff between step size and filter order. The performance is also a tradeoff between convergence rate and MSE. To eliminate the tradeoff between convergence rate and MSE, one would use a variable step-size [9]-[11]. RLS and LMS algorithms can be compared in terms of complexity, convergence, performance and multiplications see Table I.

TABLE I. COMPARISON OF RLS AND LMS

Description	RLS	LMS
Complex	More	Less
Convergence	Faster	Slow
Performance	Superior	Less
Multiplication	3M(3+M)/2, more	2M+1, less

Adaptive filters are implemented using different techniques. Fast Block Least Mean Square (FBLMS) is one of the fastest and computationally efficient adaptive algorithms. Distributed Arithmetic further enhances the throughput of FBLMS algorithm with reduced delay, minimum area requirement and reduced hardware multipliers. Distributed arithmetic (DA) is a bit level rearrangement of a multiply accumulate to hide the multiplications [12]. But the reduced hardware complexity of high order filters was at the expense of increased memory and adder requirement. And technique is suitable for higher order filters. It is powerful technique for reducing the size of a parallel hardware multiply-accumulate that is well suited to FPGA designs. DA is one of the efficient techniques, in which, by means of a bit level rearrangement of a multiply accumulate terms; FFT can be implemented without multiplier. Since the main hardware complexity of the system is due to hardware multipliers and introduction of DA eliminates the need of that multipliers and resulting system have high throughput and also low power dissipation. The unconstrained optimization problem of non-recursive filter to minimize the difference between actual and desired response of magnitude is solved using least squares design method for L2p norm [13]. Least square error design method for optimal design of FIR filter showed that as the order of the filter is increased the ripple content in the stop band diminishes. Also the design using least pth norm showed that the ripple content disappears and smoothen the response and give a constant response in stop band. The Parks-McClellan algorithm are efficient tools for mini-max design of FIR filters but these are applied to only linear class of FIR filters. Least pth are commonly used for mini-max design of IIR filters.

An efficient filter structures with optimized code is there to create a system-on-chip (SOC) solution for various adaptive filtering problems specially unknown system identification. Based on the error signal the filter's coefficients are updated and becomes almost exactly as the unknown system coefficients. Several different adaptive algorithms have been coded in MATLAB as well as in VHDL. The design is evaluated in terms of speed, hardware resources, and power consumption.

Adaptive filters implementation can also be compared and evaluated in terms of hardware and software implementation respectively. The comparison can be in terms of current usage (both idle and active), area usage (for hardware-assisted implementation) and latency and CPU utilization. Hardware implementation is generally more power efficient although increased idle power usage may negate the savings if the task is not executed properly.

III. FIR VS IIR

FIR is inherently stable because its structure involves forward paths only, no feedback exists. The presence of feed back to the input may lead the filter to be unstable and oscillation may occur. On the other hand, IIR filters are dependent on both input and output, but FIR is dependent upon input only. IIR filters are difficult to control and have no particular phase, whereas FIR filters make a linear phase always possible. IIR filters make poly-phase implementation possible, whereas FIR can always be made casual. FIR filters are helpful to achieve fractional constant delays. MAD (stands for a number of multiplications and additions), and is used as a criterion for an IIR and FIR filter comparison. IIR filters require more MAD when compared to FIR, because FIR is of a higher order comparison to IIR, which is of lower order, and uses poly-phase structures. FIR filters are dependent upon linear-phase characteristics, whereas IIR filters are used for applications which are not linear. FIR's delay characteristics are much better, but they require more memory. IIR filters consist of zeros and poles, and require less memory than FIR filters, whereas FIR only consists of zeros. IIR filters can become difficult to implement, and also delay and distort adjustments can alter the poles & zeroes, which make the filters unstable, whereas FIR filters, remain stable. FIR filters are used for tapping of a higher-order, and IIR filters are better for tapping of lower-orders, since IIR filters may become unstable with tapping higher-orders. FIR filters have only numerators when compared to IIR filters, which have both numerators and denominators. Where the system response is infinite, we use IIR filters, and where the system response is zero, we use FIR filters. FIR filters are also preferred over IIR filters because they have a linear phase response and are non recursive, whereas IIR filters are recursive, and feedback is also involved. The high computational efficiency of IIR filters, with short delays, often makes the IIR popular as an alternative.

IV. LEAST PTH NORM

The most commonly used algorithm is that LMS provides low complexity and stability. Minimax algorithms are essentially sequential algorithm that involves a series of unconstrained optimization. A representative algorithm of this class is so called least Pth algorithm. Further the need of filter to minimize the difference between actual and desired response of magnitude is solved using least Pth design method. But for FIR filters to a target frequency response one can apply a rectangular window to the impulse response. However, the resulting ringing is usually not acceptable and is not an optimal choice. For matching non-noisy target frequency responses, Least Pth is considered. The Pth optimization as a design tool is not new. It was used quite successfully for the minimax design of IIR filters. The method does not need to update the weighting function, and it is an unconstrained convex minimization approach. [13], [14] More important, the algorithm enjoys global convergence to the mini-max design regardless of initial design used. This property is an immediate consequence of the fact that for each even power p, the weighted Lp objective function is convex in the entire parameter space. The approach has advantages as filter quality, mathematical verification of the properties such as causality, stability, etc using the pole zero and magnitude plots. The Least Pth norm algorithm has a larger gradient driving it to converge faster when away from the optimum. However, the LMS will have more desirable characteristics in the neighborhood of the optimum. The Least Pth norm algorithm is defined by the following cost function:

$$Jn = E[en^p] \tag{1}$$

where the error

$$en = dn + wn - \mathbf{c}Tn\mathbf{x}n \tag{2}$$

dn is the desired value, $\mathbf{c}n$ is the filter coefficient of the adaptive filter (with $\mathbf{c}opt$ is its optimal value), $\mathbf{x}n$ is the input vector and wn is the additive noise.

V. RESULTS

The optimal design of FIR and IIR filter using least Pth norm is implemented under MATLAB and is compared. The filters vary in terms of desired filter characteristics and consequently in the number of coefficients depending upon the order of the filter. Simulation results are presented for the case of ten coefficient filter.

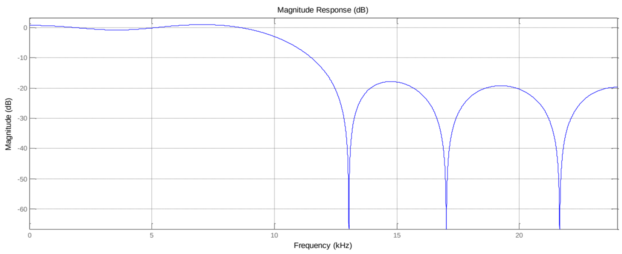

Figure 1. Magnitude response of FIR filter

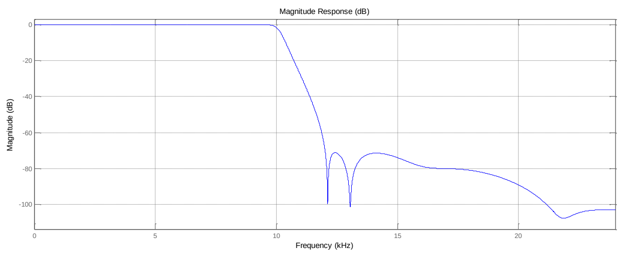

Figure 2. Magnitude response of IIR filter

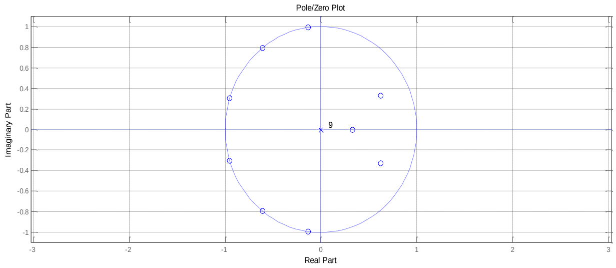

Figure 3. Pole and zero plot for FIR filter

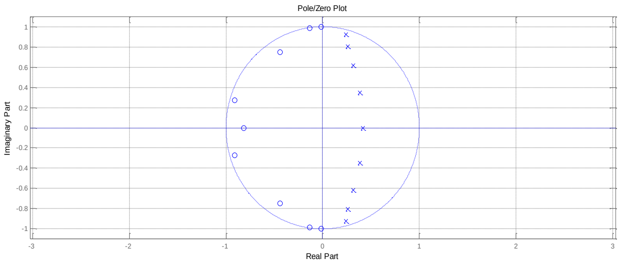

Figure 4. Pole and zero plot for IIR filter

TABLE II. COMPARISON OF FIR AND IIR

Description	**FIR**	**IIR**
Multipliers	10	19
Adders	9	18
Gain	0.0052015 dB	1 dB

The Fig. 1 and Fig. 2 show the magnitude responses of FIR and IIR filters and Fig. 3 and Fig. 4 show the pole and zero plot for the same. Both FIR and IIR filters are stable. The implementation cost for IIR filters is more comparing to that of FIR filters with 19 multipliers and 18 adders to that of 10 multipliers and 9 adders in case of FIR filters. But IIR filter provides the better gain (1dB) than that of FIR filter (0.0052015dB) see Table II.

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Srishtee Chaudhary is Master of Engineering (Electronics & Communication Engineering) from National Institute of Technical Teachers' Training and Research, Chandigarh, India. She has completed B.Tech degree in Electronics and Communication from Chitkara Institute of Engg. And Tech, Rajpura, Punjab, in 2007. Miss Srishtee Chaudhary has six years of teaching experience and has authored papers as "Speech Analysis and synthesis in MATLAB", International Conference on Advances in engineering and Technology (ICAET-2011), "Isolated Word Recognition Using Dynamic time Warping", International Conference on Signal, Image and Video Processing (ICSIVP-2012), Jan 2012, "Adaptive Filters: Analytical Comparison Using Different Techniques", International Conference on Communication and Electronics (ICCE) Proceedings, Oct 2012, "Adaptive Filters Design and Analysis using Least Square and Least Pth Norm" , International Journal of Advances in Engineering and Technology (IJAET), Vol. 6, Issue 2, May 2013, "FPGA Based Adaptive Filters Design using Least Pth Norm Technique", International Journal of Soft Computing and Engineering (IJSCE), Vol. 3, Issue 2, May 2013.

