Alpha Stable Filter and Distance for Multifocus Image Fusion

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***Abstract*—Image fusion, an important branch of data fusion, is the process of combining relevant information from two or more images into a single image where the resulting image will be more informative than any of the input images. The result image should be more suitable for visual perception and machine perception or computer processing. The goal of image fusion is to reduce uncertainty and minimize redundancy in the output as well maximize relevant information particular to an application or task. Mutlifocus image can be fused using Laplacian Pyramid (LP). LP is computed using two basic operation: reduce and expand that involve low-pas filter. The filter used in LP is Gaussian. In this paper, we propose a multifocus image fusion substituting in LP the Gaussian filter by Alpha Stable filters and using an adapted distance as integration rule in LP. We apply this method to multifocus images where the blurred part is generated using Gaussian filter and we compare with some news methods. The proposed method give the better results.**

***Index Terms*—Laplacian Pyramid, gaussian filter, alpha stable filter, multi focus image fusion**

I. Introduction

Due to the limited depth-of-focus of optical lenses, it is often difficult to capture an image that contains all relevant objects in focus. Only the objects within the depth-of-field are in focus, while other objects are blurred. Multi-focus image fusion is developed to solve this problem. There are various approaches have been performed in the literatures. These approaches can be divided into two types, spatial domain method and multi-scale fusion method. Spatial domain fusion method is performed directly on the source images. In spatial domain techniques, we directly deal with the image pixels. The pixel values are manipulated to achieve desired result. The fusion methods such as Principal Component Analysis (PCA) [1], Bilateral gradient-based methods [2] fall under spatial domain approaches. The disadvantage of spatial domain approaches is that they produce spatial distortion in the fused image. Spatial distortion can be very well handled by multi-scale approaches on image fusion. In multi-scale fusion methods the fusion process is performed on the source images after decomposing them into multiple-scales. The discrete wavelet transform (DWT) [3]-[5], Laplacian pyramid image fusion [6]-[8], Discrete cosine transform with variance calculation (DCT+var) [9], saliency detection based method (SD) [10] are examples of image fusion techniques under transform domain.

In this work, we modify the Laplacian Pyramid (LP) image fusion. Indeed, Laplacian Pyramid use two basic operation: reduce and expand based on low-pass filter. The filter used in LP is Gaussian. As in [11], we substitute Gaussian by alpha stable filter and we use as integration rule in the core of LP fusion a new measure “Neighbor alpha stable distance” based on the distance between each pixel and its neighbor’s pixels.

As we know that Gaussian density distribution is a particular cases of alpha-stable distribution (α =2). Alpha stable distribution best describes noises that are impulsive in nature. Alpha stable distribution has been used to model many phenomena where the Gaussian is not a reasonable choice (when the variance is very large). Noises of such class contain sharp or occasional burst spikes. Impulsive noises, which can be modeled with alpha stable distributions include atmospheric noise in radio links, switching transients and accidental hits in telephone lines [12]. Alpha Stable distribution have also modeled phenomena in economics [13], physics [14], electrical engineering [15], and image processing [16].

This paper is organized as follow: Section 2 describes the Alpha Stable filter that used in the fusion process. In section 3, we provide an explanation about Laplacian Pyramid fusion method. Section 4 presents the proposed method: Laplacian Pyramid fusion method using Alpha Stable filter where the integration rule is Neighbor alpha stable distance. We compare our method to some recent methods. And section 5 gives conclusion of this work.

II. Alpha Stable Filter and Neighbor Distance

The Alpha-stable distribution is widely used in the processing of impulsive or spiky signals. It also has been applied in image processing field. [17] and [13] give the model of the sea clutter in SAR images using alpha

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stable distribution for ship detection while [18] removes speckle noise using alpha stable based bayesian algorithm in the wavelet domain. Furthermore, alpha stable distribution is also used in image segmentation [19] and compressive image fusion [17]. Both [18], [19], and [13] and Wan employ alpha stable in wavelet domain. This section provides a brief of the alpha-stable distribution.

A. Alpha Stable Filter

In this work, we deal with filter generated by alpha stable distribution. The symmetric α-stable (SαS) distribution is best defined by its characteristic function

$$\varphi(\omega) = \exp\left(j\delta\omega - \gamma|\omega|^{\alpha}\right) \tag{1}$$

where

$\alpha =$ the characteristic exponent, $0 < \alpha \le 2$

$\delta =$ the location parameter, $-\infty < \delta < \infty$

$\gamma =$ the dispersion of the distribution.

As we work with image, two dimensional case, alpha-stable filter used is from bivariate stable distribution. Bivariate stable distributions much like the univariate stable distributions are characterized by the stability property and the generalized central limit theorem [20]-[22]. However, It is more difficult to describe. Bivariate stable distribution appropriates for modeling signals and noise.

The characteristic function of bivariate isotropic α-stable has the form

$$\varphi(w_1, w_2) = \exp\left(j(\delta_1 w_1 + \delta_2 w_2) - \gamma|w|^{\alpha}\right) \tag{2}$$

where $w = (w_1, w_2)$ and $|w| = \sqrt{w_1^2 + w_2^2}$.

The parameters δ_1, δ_2 are the location parameters. The distribution is isotropic with respect to the point (δ_1, δ_2). Note that the two marginal distributions of the isotropic stable distribution are SαS with parameters $(\delta_1, \gamma, \alpha)$ and $(\delta_2, \gamma, \alpha)$. The bivariate isotropic Cauchy and Gaussian distributions are special cases for $\alpha = 1$ and $\alpha = 2$, respectively.

In this paper we consider that the isotropic stable distribution is centered at origin, $(\delta_1, \delta_2) = (0,0)$. As in the case of the univariate SαS density function, when $\alpha \ne 1$ or $\alpha \ne 2$, no closed form expressions exist for the density function of the bivariate stable random variable. By using polar coordinate $r = |x| = \sqrt{x_1^2 + x_2^2}$, the density function can be written as $f_{\alpha,\gamma}(x_1, x_2) = f_{\alpha,\gamma}(r)$, and can be expressed in a power series expansion form:

$$f_{\alpha,\gamma}(r) = \begin{cases} \dfrac{1}{\pi^2\gamma^{2/\alpha}} \displaystyle\sum_{k=1}^{\infty} \dfrac{2^{\alpha k}(-1)^{k-1}}{k!} \left(\Gamma(\alpha k/2+1)\right)^2 \sin\left(\dfrac{k\alpha\pi}{2}\right)\left(\dfrac{r}{\gamma^{1/\alpha}}\right)^{-\alpha k-2} & \text{for } 0 < \alpha < 1 \\ \dfrac{\gamma}{2\pi\left(r^2+\gamma^2\right)^{3/2}} & \text{for } \alpha = 1 \\ \dfrac{1}{\pi\alpha\gamma^{2/\alpha}} \displaystyle\sum_{k=0}^{\infty} \dfrac{(-1)^k}{2^{2k+1}(k!)^2} \Gamma\left(\dfrac{2k+2}{\alpha}\right) s\left(\dfrac{r}{\gamma^{1/\alpha}}\right)^{2k} & \text{for } 1 < \alpha < 2 \\ \dfrac{1}{4\pi\gamma}\exp\left[-\dfrac{r^2}{4\gamma}\right] & \text{for } \alpha = 2. \end{cases} \tag{3}$$

B. Neighbor Alpha Stable Distance

In this work, we develop a novel fusion method that we use as selection rule in Laplacian pyramid method. This consists of weighting each pixel of each image by exponential of Neighbor Alpha Stable Distance (NASD). This neighbor alpha stable distance generalized, when $\alpha = 2$, the quadratic difference between the value of the pixel (x, y) and the all pixel values of its neighbors.

We use in this work the neighbor, with the size "a", of a pixel (x, y) defined as follows: $(x+i, y+j)$ where $i = -a, -a+1, \ldots, a-1, a$ and $j = -a, -a+1, \ldots, a-1, a$.

For example the neighbor with the small size ("a" = 1) contains: $(x-1, y-1)$, $(x-1, y)$, $(x-1, y+1)$, $(x, y-1)$, $(x, y+1)$, $(x+1, y-1)$, $(x+1, y)$, $(x+1, y+1)$.

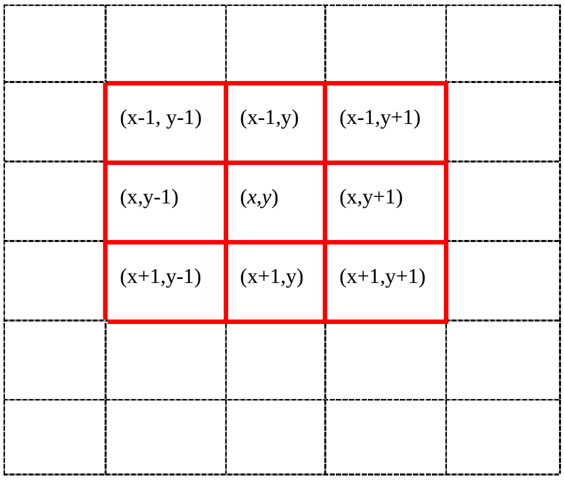

Figure 1. Pixel at *(x,y)* within its neighborhood, a = 1.

Then, the steps of image fusion with size "a" are as follows:

Suppose there are M original source images, I1, ..., IM, with different focus to be fused. The images here have the same size $(N_1 \mathrm{x} N_2)$. The general principle of making fusion rules are:

Step 1: For each pixel of each image, calculate the neighbor alpha stable distance (NASD) of every source image, da,k(x,y):

$$d_{a,k}(x, y) = \left(\frac{1}{R}\sum_{m=-a}^{a}\sum_{n=-a}^{a}\left|I_k(x, y) - I_k^{'}(x+m, y+n)\right|^{\alpha}\right)^{\frac{1}{\alpha}} \tag{4}$$

where

$$I_k^{'}(x+m, y+n) = \begin{cases} I_k(x+m, y+n), \ if \ 1 \le x+m \le N_1 \ and \ 1 \le y+n \le N_2 \\ I_k(x, y), \ otherwise \end{cases},$$

$$A = (2a+1)^2 - card(S),$$

$$S = \left\{(m, n) \in \left([-a, a]^2 - \{(0,0)\}\right) \middle| I_k^{'}(x+m, y+n) = I_k(x, y)\right\}.$$

$k = 1, \ldots, M$.

Step 2: The fused image image proposed, F, is calculated in the following model:

$$F(x,y)=\frac{\sum_{i=1}^{M}\exp(d_{a,k}(x,y))I_i(x,y)}{\sum_{i=1}^{M}\exp(d_{a,k}(x,y))} \quad (5)$$

Obviously, this method depends on the size "a". We have remarked for almost image that a=4 gives the better result. In the following we take a=4.

III. Laplacian Pyramid Fusion Method

The Laplacian pyramid was first introduced by [6] and [13]. The basic idea of Laplacian Pyramid fusion method is to perform a pyramid decomposition on each source image, then integrate all these decompositions to form a composite representation, and finally reconstruct the fused image by performing an inverse pyramid transform [23]-[25].

Alpha stable Laplacian pyramid decomposition is done by taking the difference of levels in the Alpha stable pyramid. This process involves two main operations: reduce and expand. Schematic diagram of the Laplacian Pyramid fusion method is shown in Fig. 2.

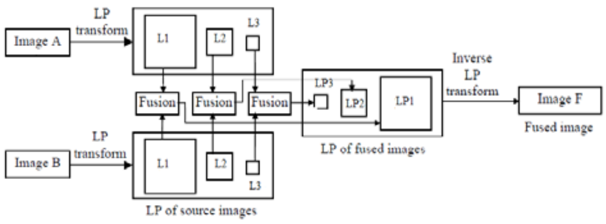

Figure 2. Scheme of LP fusion method.

Selection fusion rule used in this Laplacian Pyramid is Neighbor Alpha Stable Distance.

A. Alpha Stable Pyramid Decomposition

Suppose g_0 is the original image with size RxC. This image becomes the bottom or zero level of pyramid. Pyramid level 1 contains image g_1, which is reduce and low-pass filtered version of g_0. Pyramid level 2, g_2 is obtained by applying reduce and low-pass filtered version of g_1. The level-to-level process is as follow

$$g_l = reduce(g_{l-1})$$

which means, for level $0<l<N$ and nodes $i,j, 0<i<C_l, 0<j<R_l$

$$g_l(i,j)=\sum_{m=-2}^{2}\sum_{n=-2}^{2}w(m,n)g_{l-1}(2i+m,2j+n) \quad (6)$$

N refers to the number of level in the pyramid and C_l and R_l are the dimension of the lth level. $w(m,n)$ is generating kernel generated from alpha stable distribution.

Iterative pyramid generation is equivalent to convolving the image g_0 with a set of equivalent functions h_l:

$$g_l = h_l \otimes g_0$$

or

$$g_l=\sum_{m=-M_l}^{M_l}\sum_{n=-M_l}^{M_l}h_l(m,n)g_0(i2^l+m, j2^l+n) \quad (7)$$

The sequence image $g_0, g_1, g_2, \ldots, g_N$ is called alpha stable pyramid.

A function expand is the reverse of function reduce. Its effect is to expand an (M+1)-by-(N+1) array into a (2M+1)-by-(2N+1) array by interpolating new node values between the given values. Thus, expand applied to array g_l of the alpha stable pyramid would yield an array $g_{l,1}$ which is the same size as gl. Let $g_{l,n}$ be the result of expanding g_l n times. Then $g_{l,0}=g_0$ and $g_{l,n}$ =expand (g_l ,n - 1) by expand it means, for level $0<l\le N$ and $0\le n$ and nodes $i,j, 0<i<C_{l-n}, 0<j<R_{l-n}$,

$$g_{l,n}(i,j)=4\sum_{m=-2}^{2}\sum_{n=-2}^{2}w(m,n)g_{l,n-1}\left(\frac{i-m}{2},\frac{j-n}{2}\right) \quad (8)$$

where

$$g_{l,n-1}\left(\frac{i-m}{2},\frac{j-n}{2}\right)=\begin{cases} g_{l,n-1}\left(\frac{i-m}{2},\frac{j-n}{2}\right), & for\ \frac{i-m}{2},\frac{j-n}{2}\ \text{integer} \\ 0, otherwise \end{cases}$$

B. Alpha Stable Laplacian Pyramid Fusion

We define the alpha stable laplacian pyramid as a sequence of error images $L_0, L_1, L_2, \ldots, L_N$. Each is the difference between two levels of the alpha stable pyramid

$$L_l = g_l - expand(g_{l+1,}1)\ \text{for}\ l = N-1, N-1, \ldots 0 \quad (9)$$

The equation (9) can be written as follow

$$L_l = g_l - g_{l+1,1} \quad (10)$$

and for L_N, $L_N = g_N$.

The original image, g_0, can be obtained by expanding then summing all the levels of alpha stable laplacian pyramid:

$$g_l=\begin{cases} L_l + expand(g_{l+1,}1) & \text{for}\quad l=N-1, N-2, \ldots, 0 \\ L_N & \text{for}\quad l=N \end{cases} \quad (11)$$

Alpha stable Laplacian pyramid can be used for multi-focus image fusion. It started with two or more images focused on different distances and fuse them in a way that retains the sharp regions of each. Let L_A and L_B be Laplacian pyramids for the two original images. Thus, in focus image components can be selected pixel-by-pixel in the pyramid. A pyramid L_C is constructed for the composite image using the Neighbor Alpha Stable Distance as selection rule :

$$L_C(x,y) = \frac{\exp(d_{4,A}(x,y))L_A(x,y) + \exp(d_{4,B}(x,y))L_B(x,y)}{\exp(d_{4,A}(x,y)) + \exp(d_{4,B}(x,y))}$$

and again the fusion image is obtained by expanding and summing L_C's.

IV. Experimental Results

First, to generate a blurred image g we use, as proved in [26]-[27], the convolution of Gaussian filter applied on the reference image g_f:

$$g_0(i,j) = \begin{cases} \sum_{n=-2}^{2}\sum_{m=-2}^{2} h(m',n')g_f(i-m',j-n'), (i,j) \in blurred\ area \\ g_f(i,j), (i,j) \in object\ focus\ area \end{cases} \quad (12)$$

where $h(m',n')$ is Gaussian filter.

We applied the method on 150 sets of multi focus images on a datasets of images [28]. In this paper, as the number of pages is limited, we present only one set of multi focus images. Fig. 3 shows the multi focus images and Fig. 4 shows the fused image by proposed method and the reference image.

Blurred image1 Blurred image2

Figure 3. Multi focus images.

Fusion, proposed method Reference image

Figure 4. Fused image by proposed method and reference image.

We also compare the proposed method to other methods like: Principal Component Analysis (PCA) [1], Decomposition Wavelet Tansform (DWT) [29]-[31], Bilateral Gradient-based (BG) [2], Laplacian Pyramid with Average as selection rule (LP_AV), Laplacian Pyramid with PCA as selection rule (LP_PCA)

We analyze the performance of the results using quantitative analysis root mean square error (RMSE) which gives the information how the pixel values of fused image deviate from the reference image. Let $F(i,j)$ be the gray level intensity of pixel (i,j) of the fused image and $R(i,j)$ be the gray level intensity of pixel (i,j) of the reference image. RMSE between the reference image and fused image is computed as:

$$RMSE = \sqrt{\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}\left[R(i,j) - F(i,j)\right]^2} \quad (13)$$

where m x n is the size of the input image and i, j represents to the pixel locations. A smaller value of RMSE shows good fusion result. If the value of RMSE is 0 then it means the fused image is exactly the same as reference image.

The following table1 gives the RMSE of the studded methods. The results show that our method has a smaller RMSE almost in all cases. In exceptional cases where the DWT method gives a smaller RMSE than our method the difference between RMSE is very small.

TABLE I. RMSE of Studded Fusion Methods on 150 Images on a Datasets of Images [22].

N° image	LP_AV	PCA	BG	LP_PCA	DWT	proposed method
1	6,92	6,92	8,84	6,90	3,57	**1,51**
2	10,48	10,47	14,70	10,42	6,78	**2,56**
3	7,46	7,44	10,30	7,35	3,67	**1,93**
4	5,14	5,07	5,21	4,64	2,96	**0,61**
5	8,54	8,43	8,45	7,78	4,12	**1,73**
6	6,02	6,00	6,85	5,89	3,28	**1,35**
7	6,33	6,33	7,69	6,30	3,28	**1,16**
8	9,44	9,30	13,41	8,64	3,77	**2,21**
9	8,57	8,43	12,37	7,79	3,38	**1,40**
10	6,47	6,11	10,56	5,31	2,65	**1,19**
11	6,29	6,28	9,00	6,14	3,13	**2,37**
12	7,87	7,87	10,25	7,85	4,21	**3,93**
13	1,40	1,39	1,49	1,39	0,72	1,06
14	6,90	6,89	8,25	6,85	3,40	**2,22**
15	5,90	5,89	6,72	5,81	4,46	**1,43**
16	11,99	11,99	15,35	11,95	5,52	**2,90**
17	10,21	10,21	12,38	10,18	3,38	**1,88**
18	9,10	9,10	11,69	9,05	4,56	**1,53**
19	1,58	1,58	1,91	1,57	0,92	**0,52**
20	4,92	4,92	6,07	4,90	2,49	**1,37**
21	8,42	8,41	11,41	8,31	5,93	**2,35**
22	9,03	9,01	10,91	8,86	5,06	**1,20**
23	6,96	6,95	8,01	6,87	3,77	**0,92**
24	2,93	2,91	3,12	2,84	2,08	**0,70**
25	10,01	9,98	10,85	9,59	6,06	**1,67**
26	9,54	9,54	12,76	9,52	4,71	**2,13**
27	13,99	13,94	18,20	13,76	5,00	**3,19**
28	8,54	8,52	10,26	8,47	4,27	**3,36**
29	7,05	7,05	8,63	7,04	3,54	4,39

30	4,90	4,89	5,17	4,89	3,56	**2,57**
31	7,90	7,90	9,80	7,88	3,63	3,99
32	10,20	10,20	13,51	10,14	4,58	**4,90**
33	8,90	8,86	9,67	8,77	4,83	**4,45**
34	7,85	7,85	10,25	7,84	5,00	**2,63**
35	8,61	8,61	10,96	8,59	4,07	**1,65**
36	10,87	10,87	12,67	10,89	3,35	4,03
37	9,62	9,59	12,81	9,54	4,19	**3,92**
38	8,68	8,68	10,86	8,60	4,72	**2,54**
39	10,66	10,66	13,64	10,64	6,15	**3,13**
40	9,35	9,34	11,26	9,33	5,11	**3,03**
41	7,24	7,24	9,96	7,22	4,94	**2,43**
42	10,28	10,27	12,72	10,24	5,46	**3,78**
43	5,37	5,31	9,10	4,81	3,05	**2,86**
44	8,01	8,00	10,43	7,98	4,13	**2,98**
45	19,57	19,56	22,44	19,54	4,45	**3,65**
46	4,05	4,05	5,47	4,03	2,38	2,70
47	13,21	13,21	17,21	13,16	7,66	**2,70**
48	12,03	12,03	14,83	12,05	5,31	**2,55**
49	20,78	20,78	24,31	20,78	5,77	**3,90**
50	19,27	19,26	24,50	19,20	5,98	**2,71**
51	16,53	16,53	19,65	16,51	7,13	**3,80**
52	14,06	13,71	19,56	13,06	4,04	**3,31**
53	24,32	24,24	26,48	24,18	6,07	**4,42**
54	14,47	14,46	17,74	14,29	6,30	**3,52**
55	12,59	12,34	18,25	12,14	6,66	**3,55**
56	18,10	18,09	22,67	18,05	5,59	**3,11**
57	4,53	4,51	4,45	4,45	4,02	**1,76**
58	16,21	15,82	16,44	14,91	5,19	**3,62**
59	5,49	5,49	6,54	5,44	2,91	**2,73**
60	7,91	7,87	8,15	7,55	4,40	**2,70**
61	5,02	5,03	6,52	5,00	3,04	**1,86**
62	5,80	5,77	5,99	5,59	3,23	3,97
63	8,17	8,16	11,87	8,25	4,07	**3,13**
64	4,94	4,92	7,01	4,82	3,01	**1,59**
65	13,03	13,03	16,62	13,00	5,96	**2,47**
66	4,20	4,20	5,41	4,19	2,71	**1,37**
67	13,89	13,89	17,31	13,86	6,38	**3,57**
68	2,86	2,86	3,98	2,84	1,64	**0,82**
69	9,31	9,31	11,61	9,30	4,43	**2,45**
70	12,57	12,57	16,23	12,54	5,26	**3,14**
71	8,77	8,77	10,58	8,74	3,76	**3,11**
72	11,41	11,40	14,14	11,37	5,04	**2,29**
73	7,31	7,31	8,52	7,26	3,86	**2,23**
74	5,80	5,80	6,37	5,78	2,19	**2,14**
75	9,87	9,84	11,59	9,80	4,20	**2,15**

76	7,66	7,65	9,75	7,57	5,97	**0,83**
77	4,68	4,63	4,37	4,61	3,66	**0,91**
78	12,30	12,20	19,16	11,83	5,73	**2,68**
79	8,03	8,03	10,10	8,00	2,85	**1,90**
80	6,86	6,86	8,61	6,84	3,05	**2,01**
81	4,02	4,01	5,82	4,00	1,95	2,34
82	8,23	8,22	10,51	8,19	3,52	**1,99**
83	8,40	8,40	11,23	8,39	4,76	**2,49**
84	10,88	10,86	12,80	10,78	4,63	**2,85**
85	15,08	15,01	20,38	14,55	5,38	**2,79**
86	6,99	6,99	8,99	6,97	2,94	**1,68**
87	6,27	6,27	8,24	6,25	4,07	**1,79**
88	5,73	5,72	6,53	5,70	2,78	**2,77**
89	5,35	5,35	6,80	5,32	3,99	**1,62**
90	8,17	8,13	12,72	7,98	5,56	**3,02**
91	4,16	4,15	4,24	4,08	2,05	**1,38**
92	7,28	7,27	9,51	7,21	3,07	**2,84**
93	8,83	8,83	11,20	8,80	6,22	**1,58**
94	10,03	10,00	12,01	9,84	5,24	**1,77**
95	7,09	7,07	8,00	6,97	3,46	**1,74**
96	11,73	11,71	15,89	11,96	6,18	**4,32**
97	6,81	6,81	8,67	6,78	2,73	**1,02**
98	6,20	6,20	8,15	6,18	3,68	4,00
99	3,99	3,99	5,69	3,97	3,15	**1,06**
100	8,17	8,17	10,66	8,12	4,35	**2,32**
101	6,69	6,69	8,37	6,65	3,69	**1,86**
102	8,32	8,32	10,87	8,28	3,81	**2,67**
103	4,27	4,26	5,99	4,21	1,60	**1,54**
104	8,04	7,98	8,68	8,04	2,92	**0,76**
105	9,78	9,78	12,42	9,74	4,64	**2,54**
106	5,95	5,94	7,93	5,93	2,61	**1,35**
107	5,85	5,73	9,31	4,31	2,94	**1,35**
108	9,06	9,05	12,32	8,95	4,49	**2,15**
109	7,18	7,18	8,92	7,16	5,82	**0,79**
110	17,18	17,17	21,84	17,13	5,91	**2,95**
112	8,52	8,49	9,04	8,40	5,17	**2,04**
113	5,17	5,17	7,13	5,16	3,33	**1,91**
114	13,20	13,15	17,92	13,03	6,97	**2,55**
115	6,23	6,23	7,86	6,18	2,78	3,73
116	6,23	6,23	7,86	6,18	2,78	3,73
117	9,54	9,53	11,75	9,52	4,23	**2,98**
118	3,62	3,61	4,04	3,56	2,04	**0,55**
119	2,12	2,12	2,75	2,12	0,77	1,45
120	13,89	13,85	15,64	13,74	5,20	**3,41**
121	10,06	10,03	11,50	9,95	5,29	**3,02**
122	13,81	13,80	16,37	13,77	4,42	4,42

123	5,10	4,94	8,16	4,78	2,60	**1,52**
124	9,03	9,01	10,91	8,86	5,06	**1,20**
125	8,63	8,62	9,84	8,56	5,72	**2,33**
126	8,11	8,09	10,00	7,95	4,17	**1,57**
127	7,79	7,79	10,89	7,79	5,98	**2,56**
128	8,79	8,78	11,42	8,68	4,62	**1,59**
129	17,13	17,10	22,32	17,08	6,12	**3,41**
130	8,36	8,36	11,42	8,30	4,39	**3,51**
131	7,30	7,30	9,89	7,28	5,49	**2,93**
132	6,46	6,45	8,60	6,40	2,88	**1,69**
133	3,26	3,25	4,89	3,23	2,44	**1,31**
134	5,17	5,17	6,95	5,17	2,92	**1,88**
135	7,60	7,53	8,14	7,27	4,59	**1,72**
136	12,16	11,92	17,95	11,37	5,07	**2,58**
137	4,18	4,18	5,10	4,17	2,93	**0,91**
138	6,09	6,06	6,25	6,00	2,81	**2,79**
139	8,08	8,08	10,13	8,06	5,88	**1,75**
140	8,06	8,06	10,52	8,04	3,33	**2,92**
141	9,90	9,83	14,42	9,60	5,19	**1,94**
142	7,96	7,96	10,06	7,94	3,25	3,56
143	8,95	8,94	11,85	8,91	3,77	**2,74**
144	10,22	10,22	13,27	10,19	5,40	**1,61**
145	13,94	13,92	17,79	13,83	4,78	**2,92**
146	6,96	6,96	8,58	6,93	3,63	**1,50**
147	9,67	9,66	11,86	9,64	4,06	**2,75**
148	15,13	15,11	18,12	14,94	6,18	**3,15**
149	3,62	3,62	4,00	3,60	2,29	**1,52**
150	13,20	13,15	17,92	13,03	6,97	**2,55**
151	10,77	10,24	16,05	9,13	4,13	**2,31**

V. Conclusion

Mutifocus image fusion using Laplacian Pyramid fusion method based on Alpha Stable filter with neighbor alpha stable distance as selection rule gives better result than other studded methods. This method can be used in many applications, such as :

1) Drone is a new technology in digital imaging, it has opened up unlimited possibilities for enhancing photography. Drone can capture images on the same scene that zooms in on different objects, and at various altitudes. It will produces several images on the same scene but with different objects in-focus.

2) In medical imaging, the DST_LV can be used to detect an abnormal object or cell using local variability where the behavior of each pixel with its neighborhood is given.

3) For quality control in of food industry, cameras are used that take pictures. each camera targets one of several objects to detect an anomaly. The objects are on a treadmill. To have a photo containing all the objects in clear we can use Our proposed methods of fusion which gives more details.

The perspectives of this work:

- As many work on image fusion have implemented on grayscale images. In this thesis, all proposed methods are performed on the grayscale image. However, these proposed methods can be extended to color images as color conveys significant information.
- We are also encouraged to fuse more than two images by taking into account the local variability in each image (intra variability) and variability between image (inter variability). This inter variability can detect the ’abnormal pixels’ among the images.

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