Mixed Spectra Estimation for Stable P-adic Random Fields

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***Abstract*—Alpha stable processes with p-adic time are considered. These signals whose variance is very large are often encountered in signal processing. P-adic time is chosen because it is a discrete space and infinitely small which is much requested when one discretizes an image with a high resolution. The problem encountered in the concrete case is that the spectral measurement of such a process is not always absolutely continuous with respect to the Lebesgue measure. In general, it is the sum of a continuous part and a discrete part (some jumps). In this work, we provided a solution to this problem by offering a new method to estimate the spectral density of the continuous part. Indeed, we selected two overlapping windows whose widths are well chosen to vanish the bias at jump points. We showed that the proposed estimator was asymptotically unbiased and consistent. The rate of convergence of our estimator has been studied in order to prove the effectiveness of the method used.**

Index Terms—spectral density, p-adic numbers, stable processes, periodogram, spectral windows

I. Introduction

In recent years, there has been a growing interest on p-adic numbers. A certain number of papers shows the interest of p-adic numbers to answer some questions in physics, as in string theory (connected with p-adic quantum field) and in the other natural sciences where there are complicated fractal behaviors and hierarchical structures (turbulence theory, dynamical systems, statistical physics, biology, see [1]-[3]. Particularly 2-adic numbers are useful for computer construction see [4]. Cianciri [5] presented the main ideas to interpret a quantum mechanical state by means of p-adic statistics. He was interested in limits of probabilities when the number of trials approaches infinity. However, these limits are considered with respect to the p-adic metric. Khrennikov [6] found a new asymptotic of the classical Bernoulli probabilities. Khrennikov developed the theory of p-adic probability to describe the statistical information processes [7]. Kamizono [8] defined the symmetric stochastic integrals with respect to p-adic Brownian motion and provided a sufficient condition for its existence. The properties of the trajectories of a p-adic Wiener process were studied using Vladimirov's p-adic differentiation operator, see [9]-[11].

Brillinger [12] and Bachman studied central limit theorem for finite Fourier transforms and for a family of quadratic statistics based on stationary processes $X(t) \quad t \in Q_p$, where Q_p is the field of p-adic numbers. He also studied the spectral representation of theses processes and gave a spectral density estimation by constructing the periodogram similar to the real case. Rachdi and Monsan [13] gave another estimator built from discrete-time observations $X(\tau_k)_{k\in Z}$ where $(\tau_k)_{k\in Z}$ is the sequence of random variables taking their values in Q_p, associated to a Poisson process.

This work is devoted to a class of symmetric α -stable processes. These processes have applications in various areas where the variance increases especially for the modeling of signals and images see [14]-[17]. The estimate of the spectral density of theses processes in univariate case is given by Masry and Cambanis [18] when the time of processes is continuous real, by Sabre [19]-[21] when the time is discrete and by Sabre [22] when the time is p-adic. The estimation where the process is observed with a additive constant error is given in Sabre [23], [24].

In this paper, we consider a harmonizable symmetric stable p-adic random field :

$$X(t_1,t_2)=\int_{Q_p^2} e^{i\left[<t_2\lambda_2>+<t_1\lambda_1>\right]} dM(\lambda_1,\lambda_2),\, t_1,t_2\in Q_p,$$

where Q_p is the field of the p-adic numbers and M is a symmetric α stable random measure with control measure m called spectral measure.

We consider the case where this measure consists of an absolutely continuous part with respect to the Lebesgue measure and a discrete part defined as follows:

$$dm(x_1,x_2)=f(x_1,x_2)\,dH(x_1)\,dH(x_2)+\sum_{i=1}^{q} a_i\delta_{(\beta_{i,1},\beta_{i,2})}$$

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where δ is the Dirac measure, the function f is the density of the continuous part. It is supposed that f is nonnegative uniformly continuous function. The real a_i is the amplitude of the jump at λ_i and is assumed unknown. The jump point λ_i is unknown p-adic number.

The motivation for the choice of such model is illustrated through the concrete example studied in Sabre [25]. This example concerns the study of structural fissure of the agricultural soil. On a homogenous soil, measures of the spatial resistance are taken on several location at a depth of 30cm. The spectral measure is distributed according to a continuous law, except in certain locations where the experimentalist finds small galleries where measurement values of resistance decrease (the presence of jumps).

The aim of this paper is to give an estimator of the spectral density f for every $\lambda \in Q_p$. For that, we start by constructing a periodogram, that we then modify by multiplying it by a normalization constant and putting it on power. We show in proposition 1 that this modified periodogram is an asymptotically unbiased estimator but it is not consistent.

In order to have an asymptotically consistent estimator, we propose a new method of smoothing the modified periodogram by a double window. The originality of this work concerns the choice of these windows so that they overlap in the intervals containing the jump points. The widths of these intervals are chosen such that when n tends to infinity the interval converges to a single point which is a jump point.

The paper is organized as follows: Section 2 introduces p-adic processes. Section 3 gives the periodogram constructed from the observations of the process and the modified periodogram, which is shown to be asymptotically unbiased but not consistent (Proposition 1). Section 4 concerns the smoothing of the modified periodogram by a double window and shows that it is asymptotically unbiased and consistent (Proposition 2 and Theorem 2). The rate of convergence of the estimators where studied. Section 5 contains the concluding remarks, the potential applications and the open research problems.

II. Preliminaries

In this section, we define the field Q_p of p-adic numbers and give some properties. Let p be a prime number. Define the following norm: for $a,b \neq 0 \in Z$, $\left|a/b\right|_p = p^{n-m}$ where m is the highest power of p dividing a and n is the highest power of p dividing b. The norm of zero is vanishing. Define Q_p as the completion of Q in the metric defined by the norm $|.|_p$. The addition, product, quotient operations are carried over from Q. It follows that the p-adic norm, $|.|_p$, has the following characteristic properties: $|x|_p = 0$ is equivalent to $x=0$, $|xy|_p = |x|_p |y|_p$ and $|x+y|_p \leq max(|x|_p, |y|_p)$. Note that $|x|_p$ can take only the countably many values p^m, $m \in Z$. An important result given in Ostrowski's theorem namely the Euclidean and the p-adic norms are the only possible non-trivial norms on the field of rational numbers Q Valdimirov [26]). All $x \neq 0 \in Q_p$ can be represented in a unique form (Hansel representation) $x = \sum_{i \geq m} x_i p^i$, with $x_i \in \{0,1,...,p-1\}$ and $m \in Z$. If $x_m \neq 0$ then the norm of this p-adic number x is defined to be $|x|_p = p^{-m}$ and the fractional part of the p-adic number x, denoted $<x>$, is defined by: $<x> = \sum_{i<0} x_i p^i$ if $m<0$ and $<x> = 0$ if $m \geq 0$. Note that $<x> \in (0,1)$ and $<x> \leq p|x|_p$. The ring of p-adic integers, Z'_p is given by the elements of Q_p^2 satisfying $|(x_1,x_2)|_p \leq 1$ where $|(x_1,x_2)|_p = Max\left(|x_1|_p, |x_2|_p\right)$.

The ball with center (x_0, y_0) and radius p^n is defined by: $U_n(x_0,y_0) = \left\{(x,y) \in Q_p^2 / |(x-x_0, y-y_0)|_p \leq p^n\right\}$. In particular when $(x_0,y_0) = (0,0)$ we denote $U_n(0,0) = U_n$ and $U_0 = \left\{(x,y) \in Q_p^2 / |(x,y)|_p \leq 1\right\}$.

Q_p is a complete separable metric space, the stochastic process $X(t_1,t_2;;w)$ for $t_1,t_2 \in Q_p$ and $w \in \Omega$, (Ω, A, P) a probability space, is well defined as a map from $Q_p^2 \times \Omega$ to R. $(Q_p,+)$ is an abelian locally compact group; from Haar's theorem there exists a positive measure H on Q_p, uniquely determined except for a constant. It has the properties:

$$dH(t+a) = dH(t) \text{ and } dH(at) = |a|_p \, dH(t)$$

Define a positive measure H' on Q_p^2 by

$dH'(t_1,t_2) = dH(t_1) dH(t_2)$. The measure will be normalized by $H'(Z'_p) = 1$, see (Hewitt and al [10]), Vladimirov [27]). Let (t_1,t_2) be in Z'_p, then $t_1 = a_0 + a_1 p + a_2 p^2 + ...$ and $t_2 = b_0 + b_1 p + b_2 p^2 + ...$ writing $h(t_1,t_2) = g(a_0,a_1,a_2...;b_0,b_1,b_2...)$ and taking $(T_0^1, T_1^1, T_2^1...)$ and to be two sequences of i.i.d. random variables on the sample space $\{0,1,2,...\}$ with equal probability. Then, it has $\int_{Z'_p} h(t_1,t_2) dH'(t_1,t_2) = g(T_0^1, T_1^1, ...; T_0^2, T_1^2, ...)$ and

$$\int_{Q_p^2} h(t_1,t_2) dH'(t_1,t_2) = \lim_{n\to\infty} p^{2n} \int_{Z'_p} h(p^{-n}s_1, p^{-n}s_2) dH'(s_1,s_2)$$

III. PERIODOGRAM AND SPECTRAL DENSITY ESTIMATION

Consider a process $X = \{X(t_1,t_2): t_1,t_2 \in Q_p\}$ where Q_p is the field of p-adic numbers having the following integral representation: $\forall t_1,t_2 \in Q_p$

$$X(t_1,t_2) = \int_{Q_p^2} e^{i[<t_1\lambda_1>+<t_2\lambda_2>]} dM(\lambda_1,\lambda_2) \quad (1)$$

where M is a symmetric α stable $S\alpha S$ random measure with independent and isotropic increments. There exists a control measure m that is defined by: $m(A) = [M(A), M(A)]_\alpha^{1/\alpha}$.

Assume that the measure m is the sum of an absolutely continuous measure with respect to Haar measure and a discrete measure:

$$dm = f(x_1,x_2)dH(x_1)dH(x_2) + \sum_{i=1}^{q} a_i \delta_{(\beta_{i,1},\beta_{i,2})} \quad (2)$$

As in the classical works of esimations of the spectral density we construct a periodogram defined on the ball : $U_n = \{(x_1,x_2) \in Q_p^2 ; |(x_1,x_2)|_p \le p^n\}$ as the time of observation of the process. The periodogram :

$$d_n(\lambda_1,\lambda_2) = A_n Re \int_{U_n} e^{-i[<t_1\lambda_1>+<t_2\lambda_2>]} p^{2-n} \times$$

$$h(t_1 p^n, t_2 p^n) X(t_1,t_2) dH'(t_1,t_2) \quad (3)$$

For all $\lambda_1,\lambda_2 \in Q_p$, where h satisfied

$$B_\alpha = \int_{Q_p^2} |L(\lambda_1,\lambda_2)|^\alpha dH(\lambda_1)dH(\lambda_2) < +\infty$$

where the function L is defined as follows:

$L(\lambda_1,\lambda_2) = 0$ if $(\lambda_1,\lambda_2) \notin Z'_p$ and if $(\lambda_1,\lambda_2) \notin Z'_p$

$$L(\lambda_1,\lambda_2) = \int_{Z'_p} h(t_1,t_2) e^{-i[<t_1\lambda_1>+<t_2\lambda_2>]} dH'(t_1,t_2)$$

Assume that $L(\lambda_1,\lambda_2) \le C$ for all $(\lambda_1,\lambda_2) \in Z'_p$.

$$L_n(\lambda_1,\lambda_2) = \left(\frac{p^{2n}}{B_\alpha}\right)^{\frac{1}{\alpha}} L(p^{-n}\lambda_1, p^{-n}\lambda_2)$$
$$= A_n L(p^{-n}\lambda_1, p^{-n}\lambda_2)$$

Therefore, from (1) we obtain

$$\int_{Q_p^2} |L_n(\lambda_1,\lambda_2)|^\alpha dH(\lambda_1)dH(\lambda_2) =$$

$$\frac{p^{2n}}{B_\alpha} p^{-2n} \int_{Q_p^2} |L(v_1,v_2)|^\alpha dH(v_1)dH(v_2)$$

From the definition of the norm $|.|_p$, we have

$$\int_{Q_p^2} |L_n(\lambda_1,\lambda_2)|^\alpha dH(\lambda_1)dH(\lambda_2) = 1.$$

Denote $A = \{(\lambda_1,\lambda_2) \in Q_p^2 : (\lambda_1,\lambda_2) \in [b_{1,i},c_{1,i}] \times [b_{2,i},c_{2,i}]\}$ where the bloc $[b_{1,i},c_{1,i}] \times [b_{2,i},c_{2,i}]$ contains the jump point $(\beta_{1,i},\beta_{2,i})$.

Following the study realized by Sabre [22] we can show easily that for $\lambda \in Q_p$ the characteristic function of $d_n(\lambda_1,\lambda_2)$, $E\exp\{ird_n(\lambda_1,\lambda_2)\}$ converges to $\exp\{-C_\alpha |r|^\alpha f(\lambda_1,\lambda_2)\}$.

We modify this periodogram as follows:

$$I_n(\lambda_1,\lambda_2) = C_{q,\alpha} |d_n(\lambda_1,\lambda_2)|^q , \quad (4)$$

where $0 < q < \frac{\alpha}{2}$ and the normalization constant is $C_{q,\alpha} = \frac{D_q}{F_{q,\alpha} C_\alpha^{\frac{q}{\alpha}}}$, where $D_q = \int \frac{1-cos(u)}{|u|^{1+q}} du$; $F_{q,\alpha} = \int \frac{1-e^{-|u|^\alpha}}{|u|^{1+q}} du$ and $C_\alpha = \frac{1}{2\pi}\int_0^\pi |cos(u)|^\alpha du$.

The following theorem shows that the modified periodogram is an estimator asymptotically unbiased of the spectral density but not consistent.

Proposition 1 *Let* $\lambda \in Q_p$, then $E(I_n(\lambda_1,\lambda_2)) = (\psi_n(\lambda_1,\lambda_2))^{q/\alpha}$ where $\Psi_n(\lambda_1,\lambda_2) \stackrel{\Delta}{=} \int_{Q_p^2} |L_n(\lambda_1-u_1,\lambda_2-u_2)|^\alpha f(u_1,u_2)dH'(u_1,u_2)$ and $I_n(\lambda_1,\lambda_2)$ is an asymptotically unbiased estimator of the spectral density but not consistent : $E(I_n(\lambda_1,\lambda_2)) - (f(\lambda_1,\lambda_2))^{q/\alpha} = o(1)$ *and* $Var(I_n(\lambda_1,\lambda_2)) = V_{\alpha,q} f^2(\lambda_1,\lambda_2) + o(1)$, *with* $V_{\alpha,q} = \frac{C_{q,\alpha}^2}{C_{2q,\alpha}} - 1.$

The *proof* of this result is similar to that given in Masry and Cambanis [17] and Sabre[19]-[21].

IV. THE SMOOTHING ESTIMATE

In order to obtain an asymptotically unbiased estimator consisting of the point where there is a jump, we smooth the modified periodogram by two spectral windows. These spectral windows are judiciously chosen. In fact, they intersect at intervals containing each the jump points and depending on the width of the windows. The characteristic of these intervals is that they are asymptotically reduced to a single point (jump point). The estimator is defined according to the value of order to have an asymptotically and consistent estimate, we smooth the modified periodogram using spectral windows according to the value of (λ_1, λ_2) ($(\lambda_1,\lambda_2)\in A$ or not).

- If $f\ \ (\lambda_1,\lambda_2)\notin A$

$$f_n(\lambda_1,\lambda_2)=\int_{Q_p^2}W_n^{(1)}(\lambda_1-u_1,\lambda_2-u_2)I_n(u_1,u_2)\,d\mathcal{H}(u_1,u_2)$$

- If $(\lambda_1,\lambda_2)\in A,$

$$f_n(\lambda_1,\lambda_2)=$$

$$\int_{Q_p^2}W_{1n}^{(1)}(\lambda_1-u_1,\lambda_2-u_2)W_n^{(2)}(\lambda_1-u_1,\lambda_2-u_2)I_n(u_1,u_2)\,d\mathcal{H}$$

where $W_n^{(1)}(x_1,x_2)=\left|M_n^{(1)}\right|_p^2 W\left(x_1M_n^{(1)},x_2M_n^{(1)}\right)$ and $W_n^{(2)}(x_1,x_2)=\left|M_n^{(2)}\right|_p^2 W\left(x_1M_n^{(2)},x_2M_n^{(2)}\right)$ such that

$$M_n^{(i)}\to\infty\,;\ \frac{M_n^{(i)}}{n}\to 0\,;|\,M_n^{(i)}\,|_p\to 0,$$

$$\frac{|\,M_n^{(i)}\,|_p}{p^n}\to 0\ \ and\ \ \frac{M_n^{(2)}}{M_n^{(1)}}\to 0.$$

$W^{(i)}$ are an even nonnegative function vanishing outside Z and $\int_{Q_p}W^{(i)}(v)\,d\mathcal{H}(v)=1$

Moreover $W^{(i)}$ satisfying the following equality:

$$W^{(2)}(x_1M_n^{(2)},x_2M_n^{(2)})-W^{(1)}(x_1M_n^{(1)},x_2M_n^{(1)})=0$$

$$\forall(x_1,x_2)\in\left]-\frac{1}{M_n^{(1)}},\frac{1}{M_n^{(1)}}\right[\times\left]-\frac{1}{M_n^{(1)}},\frac{1}{M_n^{(1)}}\right[$$

The following result shows that the smoothed peridogram is an asymptotically unbiased estimator of the spectral density.

Proposition 2 Let $\lambda_1,\lambda_2\in Q_p$ *then* $Bias\left(f_n(\lambda_1,\lambda_2)\right)=o(1)$. *Moreover if f verify* $\left|f(x_1,x_2)-f(y_1,y_2)\right|\le cste\left|(x_1-y_1,x_2-y_2)\right|_p^{-k}$, with $0<k<1$, *then*

$$\left|Bias\left(f_n(\lambda)\right)\right|=O(\max\left(\frac{1}{\left|p^n\right|_p^{-kq/\alpha}},\frac{1}{\left|M_n^{(1)}\right|_p^{-kq/\alpha}}+\frac{1}{\left|M_n^{(2)}\right|_p^{-kq/\alpha}}\right)$$

Proof:

From the proposition 1, we have

$$E\left(f_n(\lambda_1,\lambda_2)\right)=\int_{Q_p^2}W(v_1,v_2)\times$$

$$\left\{\Psi_n\left(\lambda_1-\frac{v_1}{M_n},\lambda_2-\frac{v_2}{M_n}\right)\right\}^{q/\alpha}dH'(v_1,v_2).$$

As $q<\dfrac{\alpha}{2}$, then we obtain

$$\left|Bias\left(f_n(\lambda_1,\lambda_2)\right)\right|\le\int_{Q_p^2}W(v_1,v_2)\times$$

$$\left|\Psi_n\left(\lambda_1-\frac{v_1}{M_n},\lambda_2-\frac{v_2}{M_n}\right)-\left(f(\lambda_1,\lambda_2)\right)\right|^{q/\alpha}dH'(v_1,v_2)$$

On the other hand,

$$\left|\Psi_n\left(\lambda_1-\frac{v_1}{M_n^{(1)}},\lambda_2-\frac{v_2}{M_n^{(2)}}\right)-\left(f(\lambda_1,\lambda_2)\right)\right|\le$$

$$\left|U_n\left(\lambda_1-\frac{v_1}{M_n^{(1)}},\lambda_2-\frac{v_2}{M_n^{(2)}}\right)-f(\lambda_1,\lambda_2)\right|+$$

$$\left|J_n\left(\lambda_1-\frac{v_1}{M_n^{(1)}},\lambda_2-\frac{v_2}{M_n^{(2)}}\right)\right|$$

where

$$\left|U_n\left(\lambda_1-\frac{v_1}{M_n^{(1)}},\lambda_2-\frac{v_2}{M_n^{(2)}}\right)-f(\lambda_1,\lambda_2)\right|=$$

$$\frac{1}{B_\alpha}\int_{Q_p^2}\left|H_n(u_1,u_2)\right|^\alpha\left[f\left(\lambda_1-\frac{v_1}{M_n^{(1)}}-\frac{u_1}{p^n},\lambda_2-\frac{v_2}{M_n^{(2)}}-\frac{u2}{p^n}\right)-f(\lambda_1,\lambda_2)\right]d\mathcal{H}$$

$$J_n\left(\lambda_1-\frac{v_1}{M_n^{(1)}},\lambda_2-\frac{v_2}{M_n^{(2)}}\right)=$$

$$\sum_{i=1}^{q}a_i\left|H_n\left(\lambda_1-\frac{v_1}{M_n^{(1)}}-\beta_{1,i},\lambda_2-\frac{v_2}{M_n^{(2)}}-\beta_{2,i}\right)\right|^\alpha \tag{5}$$

Using the fact that f is uniformly continuous, and the following equality proved in Vladimirov [23] page 25

$$\int_{|x|_p\le 1}|\,x\,|^{r-1}\,dx=\frac{1-p^{-1}}{1-p^{-r}},\qquad for\ all\ positif\ real\ r$$

we get

$$\left|U_n\left(\lambda_1-\frac{v_1}{M_n^{(1)}},\lambda_2-\frac{v_2}{M_n^{(2)}}\right)-\left(\Phi(\lambda_1,\lambda_2)\right)\right|\le$$

$$Max\left(\frac{1}{B_\alpha}\frac{|v|_p^{-k}}{|M_n|_p^{-k}},\frac{1}{B_\alpha\left|p^n\right|_p^{-k}}\frac{1-p^{-1}}{1-p^{\alpha\beta+\beta-1}}\right)$$

Thus we obtain

$$\left|Bias\left(f_n(\lambda_1,\lambda_2)\right)\right|=O(\max\left(\frac{1}{\left|p^n\right|_p^{-kq/\alpha}},\frac{1}{\left|M_n^{(1)}\right|_p^{-kq/\alpha}}+\frac{1}{\left|M_n^{(2)}\right|_p^{-kq/\alpha}}\right)$$

Now we will show that f_n is an asymptotically consistent estimate.

Theorem 1 Let λ_1,λ_2 *be in* Q_p *and* $M_n^1=p^{v_1n}$ and $M_n^2=p^{v_2n}$ *where* $0<v_1,v_2<1$. *Assume that* $f\in L^1_{Q_p}$. *Then*

$$Var\left(f_n(\lambda_1,\lambda_2)\right)=O\left(p^{-n(3v_1+1)}+p^{-n(3v_2+1)}\right)$$

Proof:

From the definition of spectral window, we have

$$Var\left(f_n(\lambda_1,\lambda_2)\right)=\int_{Q_p^2}\int_{Q_p^2}W(x_1,x_2)W(y_1,y_2)\times$$

$$C\left(\lambda_1-\frac{x_1}{M_n^1},\lambda_2-\frac{x_2}{M_n^2},\lambda_1-\frac{y_1}{M_n^1},\lambda_2-\frac{y_2}{M_n^2}\right)\times$$

$$dH'(x_1,x_2)\,dH'(y_1,y_2)$$

We split the integrals as follows:

$$Var\left(f_n\left(\lambda_1,\lambda_2\right)\right)=\iint_{\left\{\left|(x_1-y_1,x_2-y_2)\right|_p<\varepsilon_n\right\}}$$

$$+\iint_{\left\{\left|(x_1-y_1,x_2-y_2)\right|_p>\varepsilon_n\right\}}$$

$$\overset{\Delta}{=}J_1+J_2$$

where ε_n is a positive real converging to zero as n tends to infinity.

$$\left|J_2\right|\le\iint_{\left\{\left|(x_1-y_1,x_2-y_2)\right|_p>\varepsilon_n\right\}}W\left(x_1,x_2\right)W\left(y_1,y_2\right)$$

$$Cov\left(\lambda_1-\frac{x_1}{M_n^1},\lambda_2-\frac{x_2}{M_n^2},\lambda_1-\frac{y_1}{M_n^1},\lambda_2-\frac{y_2}{M_n^2}\right)\times$$

$$dH'\left(x_1,x_2\right)dH'\left(y_1,y_2\right)$$

We show that

$$|J_2|=O\left(\frac{|M_n^{(1)}|_p^3}{p^n}+\frac{|M_n^{(2)}|_p^3}{p^n}\right).$$

and

$$\left|J_1\right|\le const\left(\Phi\left(\lambda_1,\lambda_2\right)\right)^{2p/\alpha}\varepsilon_n.$$

Then

$$Var\left(f_n\left(\lambda_1,\lambda_2\right)\right)=O\left(\frac{|M_n^{(1)}|_p^3}{p^n}+\frac{|M_n^{(2)}|_p^3}{p^n}+\varepsilon_n\right)$$

choosing $\varepsilon_n=\frac{|M_n^{(1)}|_p^3}{p^n}+\frac{|M_n^{(2)}|_p^3}{p^n}$, we obtain

$$Var\left(f_n\left(\lambda_1,\lambda_2\right)\right)=O\left(\frac{|M_n^{(1)}|_p^3}{p^n}+\frac{|M_n^{(2)}|_p^3}{p^n}\right).$$

V. Conclusion

In this paper, we proposed in this paper some results about the estimation of the spectral density for symmetric stable p-adic processes. The approach was based on the technique used by Masry and Cambanis [18] for stable processes combining estimates of p-adic spectrum introduced by Brillinger [12]. This work could be applied to several cases when processes have an infinite variance and have a discrete time, as for example :

• The segmentation of a sequence of images of a dynamic scene, detecting weeds in a farm field.

• The detection of possible structural changes in the dynamics of an economic structural phenomenon .

• The study of the rate of occurrence of notes in melodic music to simulate the sensation of hearing from afar. The prospects for this work are as follows:

- The optimal choice of spectral window widths because the optimal convergence rate depends on it.
- The study of the Central Normality limits for these etimators and the creation of a test to detect the jump points
- The improvement of the convergence rate according to the behavior of the spectral density at the origin (example of long memory process)
- The estimation of the spectral density when the process is observed with random errors (deconvolution problem).
- The extension of this estimation method (double window) to the evolutionary spectral density for non-stationary processes.

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