An Improved Algorithm of Modulation Index Estimation for FM/PM Signal

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***Abstract*—FM/PM signal is widely used in communications, so the detection of FM/PM signal is very important. Modulation index estimation is of great significance for identification and demodulation of FM/PM signal. Based on the analysis of existing algorithms, this paper proposes an improved algorithm by introducing wavelet transform into FM/PM modulation index estimation. The existing modulation index estimation algorithms can be easily affected by noise and the estimation accuracy is too poor to meet the actual demand. The improved algorithm using wavelet transform to reconstruct signal can effectively suppress noise and improve the accuracy of modulation index estimation. Simulation shows that using the proposed algorithm, the estimation accuracy for modulation index is greatly improved with RMSE (the root-mean-square error) of the modulation index estimation being less than 0.05 when SNR≥15dB (the signal-to-noise ratio).**

***Index Terms*—information processing technology, modulation index estimation, wavelet transform, FM/PM signal**

I. Introduction

FM (Frequency Modulation) and PM (Phase Modulation) are often grouped together under the name of angle modulation, which is a common used nonlinear analog modulation scheme in which the baseband modulated signal amplitude changes the frequency or phase of the carrier to achieve the purpose of information transmission [1]. FM and PM signals are widely used in communication, so the detection of FM and PM signals is of great significance in communication reconnaissance.

Modulation index is an important parameter of FM and PM signals. Modulation index estimation is of great significance to FM/PM signal modulation recognition and demodulation. In 1999, Jeremy et al put forward an identification method for FM and PM signals [2]. In the same year, Shintaro et al proposed an automatic classification method of analogue modulation signals (including FM and PM signals) by using statistical parameters [3]. In 2003 and 2005, Zhong Xingwang and He Jia introduced an automatic classification method of FM and PM signals for TT&C (tele-control and tele-measure communication) signals of satellite, respectively [4]-[5]. In 2007, EI-Mahdy at al put forward an automatic classification of composite FM/PM speech signals in sensor arrays over flat fading channel [6]. In 2009, support vector machine was employed for FM/PM signal recognition by Engin at al [7]. However, for all the methods mentioned above, the modulation index was utilized to realize the recognition of FM and PM signals. If the modulation index can be accurately estimated, it can be used to identify the FM and PM modulation. Also, FM and PM signals demodulation can be possible.

For the modulation index estimation, there exist relatively few researches currently. The latest research was blind estimation of modulation index for angle modulation signals put forward by Peng Geng in 2010 [8]. This method calculates the modulation index according to the time frequency structure of the angle signals. Therefore, this method is easy to be disturbed by noise and has low recognition accuracy. To overcome the drawbacks, an improved algorithm for modulation index estimation utilizing wavelet transform is proposed in this paper. This paper compares these two methods via simulation and finds that the improved algorithm can effectively reduce noise interference and improve the accuracy of modulation index estimation. The improved algorithm is more suitable for practical engineering.

II. Signal Model

Angle modulation signal [9] is described as:

$$s(t) = A\cos\left[w_c t + \varphi(t)\right] \tag{1}$$

where A is the signal amplitude, w_c is the signal carrier frequency, and $\varphi(t)$ determines the modulation type of the signal.

The frequency of FM signal varies with the baseband signal amplitude. It can be expressed as follows:

$$\varphi_{FM}(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau \tag{2}$$

$$s_{FM}(t) = A\cos\left[w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau\right] \tag{3}$$

where k_f is the frequency sensitivity, and $m(t)$ is the baseband information signal.

The phase of PM signal varies with baseband signal amplitude. It can be written as:

$$\varphi_{PM}(t) = k_p m(t) \tag{4}$$

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$$s_{PM}(t) = A\cos\left[w_c t + k_p m(t)\right] \tag{5}$$

where k_p is the phase sensitivity.

III. The Existing Algorithm

Take the FM modulation signal shown in (3) as an example. First, the signal's amplitude is normalized. Then, differentiate the signal over time which can be represented as:

$$\begin{aligned}\frac{ds_{FM}(t)}{dt} &= \frac{dA}{dt}\cos\left[w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right] \\ &= -A\left[w_c + 2\pi k_f m(t)\right]\sin\left[w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right]\end{aligned} \tag{6}$$

Because the signal is normalized firstly, the value of $\boldsymbol{A}$ is 1. Equation (6) can be expressed as:

$$\begin{aligned}\frac{ds_{FM}(t)}{dt} &= -\left[w_c + 2\pi k_f m(t)\right] \\ &\bullet\sin\left[w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right]\end{aligned} \tag{7}$$

From (7), it can be seen that the FM signal is essentially an amplitude & frequency modulated signal. Both the amplitude and frequency of FM signal carry information. The Hilbert transform of the signal is described as:

$$\begin{aligned}H\left[\frac{ds_{FM}(t)}{dt}\right] &= \left[w_c + 2\pi k_f m(t)\right] \\ &\bullet\exp\left\{j\left[w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau + \frac{\pi}{2}\right]\right\}\end{aligned} \tag{8}$$

Due to the fact that w_c is not always greater than $2\pi k_f m(t)$, equation (8) can be described as:

$$\begin{aligned}H\left[\frac{ds_{FM}(t)}{dt}\right] &= \left[w_c + 2\pi k_f m(t)\right] \\ &\bullet\exp\left\{j\left[w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau + \frac{\pi}{2} + \varphi(t)\right]\right\}\end{aligned} \tag{9}$$

$$\varphi(t) = \begin{cases} 0, w_c + 2\pi k_f m(t) \geq 0 \\ 1, w_c + 2\pi k_f m(t) < 0 \end{cases} \tag{10}$$

Obviously, $\left|w_c + 2\pi k_f m(t)\right|$ can be obtained by using (8); We can get $\phi(t)$ by taking the angle form (9) and unwrapping the phase [10]-[11].

Hence, $\theta(t)$ can be written as:

$$\begin{aligned}\theta(t) &= \left|w_c + 2\pi k_f m(t)\right|\exp\left\{\phi(t)\right\} \\ &= w_c + 2\pi k_f m(t)\end{aligned} \tag{11}$$

Substitute the baseband modulated signal $m(t)$ in (11). Subtract its average score and integrate it:

$$\begin{aligned}&\int_0^t 2\pi k_f \left\{m(\tau) - mean\left[m(\tau)\right]\right\}d\tau \\ &= 2\pi k_f \frac{V}{2\pi f_m}\sin(2\pi f_m \tau)\Big|_0^t \\ &= m_f \sin(2\pi f_m t)\end{aligned} \tag{12}$$

where $m_f = k_f \bullet V / f_m$ is the modulation index of FM signal [12].

The FM signal modulation index can be evaluated by the amplitude difference of the signal $m_f \sin(2\pi f_m t)$.

For PM signal, carry out the same process. It can be obtained:

$$\begin{aligned}&\int_0^t 2\pi k_p \left\{m'(\tau) - mean\left[m'(\tau)\right]\right\}d\tau \\ &= k_p V \bullet \cos(2\pi f_m \tau)\Big|_0^t \\ &= m_p \bullet \left[\cos(2\pi f_m t) - 1\right]\end{aligned} \tag{13}$$

where $m'(t)$ is the derivative of $m(t)$; $m_p = k_p \bullet V$ is the modulation index of PM signal.

Similarly, the PM signal modulation index can be evaluated by the amplitude difference of the signal $m_p\left[\cos(2\pi f_m t) - 1\right]$.

IV. The Proposed Algorithm

For FM signal, its amplitude is normalized firstly, i.e. A becomes 1. Then, the FM signal is described as:

$$s_{FM}(t) = \cos\left[w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right] \tag{14}$$

Hilbert transformation is performed on the above FM signal. It has the form:

$$H\left[s_{FM}(t)\right] = \exp\left\{j\left[w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right]\right\} \tag{15}$$

The instantaneous phase can be obtained from (15). Unwrapping the phase, we have:

$$\varphi(t) = w_c t + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau \tag{16}$$

Then, differentiating (16) yields:

$$\theta(t) = w_c + 2\pi k_f m(t) \tag{17}$$

$m(t) = V\cos(2\pi f_m t)$ is the baseband signal of the FM signal. Substract the integral of the mean of (17) from (16). We have:

$$\begin{aligned}&\varphi(t) - \int_0^t mean\left[\theta(\tau)\right]d\tau \\ &= 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau \\ &= 2\pi k_f \frac{V}{2\pi f_m}\sin(2\pi f_m \tau)\Big|_0^t = m_f \sin(2\pi f_m t)\end{aligned} \tag{18}$$

Wavelet transform is employed to reconstruct the signal $m_f \sin(2\pi f_m t)$ [13]-[16].

In the paper, wavelet transform reconstruction is utilized. Wavelet denoising is capable of removing noise from a noise contaminated signal and improves the signal's SNR in order to analyze the signal more accurately and effectively.

The procedures of wavelet transform denoising are shown below:

Firstly, perform the wavelet decomposition. Secondly, select threshold of the high frequency coefficients and quantize. For each layer of the high frequency coefficients, the threshold is selected for quantization. Finally, according to the low and high frequency coefficient after quantitation, wavelet reconstruction of the signal is performed.

The modulation index of the FM signal can be calculated according to the amplitude difference of the signal $m_f \sin(2\pi f_m t)$ after reconstruction of wavelet transform.

For PM signal, perform the same process as in (18). We have:

$$\begin{aligned} &\varphi(t) - \int_0^t mean\left[\theta(\tau)\right]d\tau \\ &= w_c t + k_p m(t) - \int_0^t mean\left[w_c + k_p m'(\tau)\right]d\tau \\ &= k_p m(t) = k_p V \bullet \cos(2\pi f_m t) \\ &= m_p \bullet \cos(2\pi f_m t) \end{aligned} \tag{19}$$

The modulation index of the PM signal can be calculated according to the amplitude difference of the signal $m_p \cos(2\pi f_m t)$ after reconstruction of the wavelet transform.

V. Simulation and Analysis

In our simulations, FM and PM signals are firstly generated. The sampling frequency is 1200 kHz. The carrier frequency is set at 300 kHz. V is set to be 1V.

Two simulation scenarios are considered: (1) FM: k_f is 1200 Hz/V, f_m is 1000 Hz, and m_f is 1.2. (2) PM: k_f is 1.2 rad/V, f_m is 1000 Hz, and m_p is 1.2. The data length is 600000.

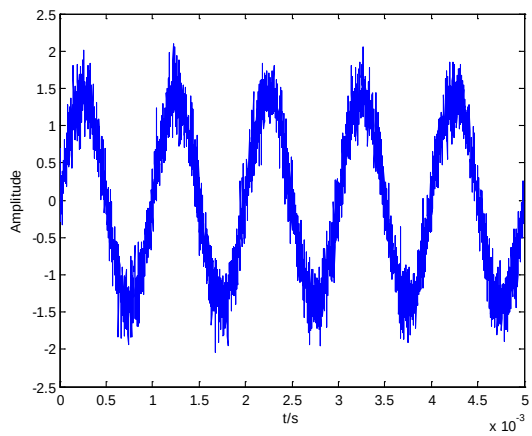

Figure 1. The original of $m_f \sin(2\pi f_m t)$ (SNR=10dB).

As shown in Fig. 1 and Fig. 2, Fig. 1 is the original signal $m_f \sin(2\pi f_m t)$ and Fig. 2 is the corresponding reconstructed signal after wavelet transform. Here we use Haar wavelets and soft threshold function. As PM signal and FM signal are similar after the wavelet transform reconstruction, the analysis for PM signal is omitted here for simplicity.

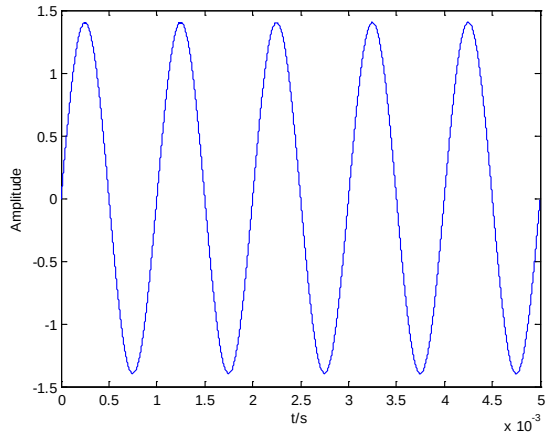

Figure 2. The reconstruction signal $m_f \sin(2\pi f_m t)$ after wavelet transform (SNR=10dB).

As can be seen from the above figures, the noise is effectively suppressed. Therefore, the wavelet transform can effectively suppress the noise. Because we need a high accuracy but computational requirements are not high during signal processing, this proposed method is very suitable.

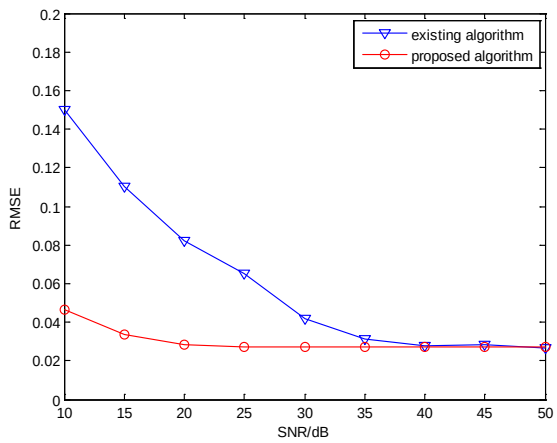

Figure 3. The root-mean-square error of the FM signal modulation index.

1000 Monte Carlo experiments are conducted under different signal-to-noise ratio to evaluate the performance of the proposed algorithm. The change of RMSE of the modulation index is shown in Fig. 3 and Fig. 4.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(K_i - \hat{K}\right)^2} \tag{20}$$

where K_i and $\hat{K}$ are estimated modulation index and the actual value, respectively.

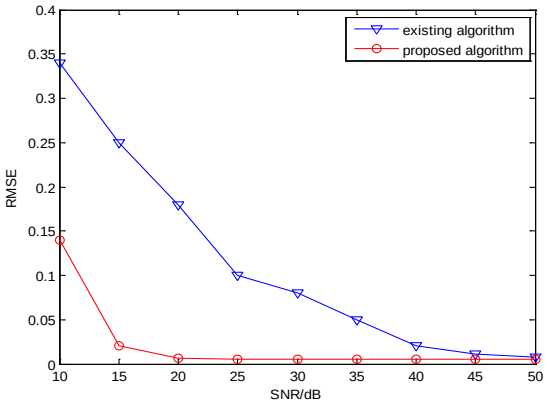

Figure 4. The root-mean-square error of the PM signal modulation index.

Based on the above simulation results, the following conclusions can be made: Firstly, compared with the existing algorithm, the modulation index estimation error of the proposed algorithm is significantly reduced in the case of low SNRs while the modulation index estimation error of the two algorithms is very close at high SNRs. The proposed algorithm can effectively suppress the influence of noise during the modulation index estimation. Secondly, when SNR ≥ 15dB, the root-mean-square error of the modulation index estimation is less than 0.05, which can be applied to practical engineering.

It is worth mentioning that the proposed algorithm outperforms the existing counterpart at the cost of higher complexity. This is due to the fact that wavelet transform is needed for signal reconstruction in the proposed algorithm. For example, the CPU time of the existing algorithm is 0.442s and the CPU time of the proposed algorithm is 0.769s.

VI. Conclusion

This paper mainly proposes an improved algorithm of modulation index estimation for FM/PM signal. The existing algorithms calculate the modulation index directly according to the time and frequency structure, so it can be easily affected by noise and its accuracy is poor. Based on the original algorithm, the improved algorithm uses wavelet transform to reconstruct the signal and effectively reduces the influence of noise. Computer simulation shows that compared with the existing algorithm, the modulation index estimation error of the proposed algorithm is significantly reduced in the case of low SNRs while the modulation index estimation error of the two algorithms is very close at high SNRs. Therefore, the improved algorithm can effectively reduce the noise impact during modulation index estimation, which has certain military and economic value.

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