# Analysis of Blind Source Separation Method for Weak Communication Signals Based on $\ell_{2,1} - RsPCA$ Algorithm

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***Abstract*—With the electromagnetic environment becoming more and more complicated, the signal received by the communication receiver always overlaps much noise and interference. The problem of the weak signal separation has gradually become more and more important. To solve this problem, we propose an algorithm called $\ell_{2,1} - RsPCA$ by combining some research results in the field of communication and mathematics. Through the separation experiment for random number and spread spectrum BPSK signal, we prove that the algorithm of $\ell_{2,1} - RsPCA$ can separate single-channel mixed signal effectively and outperforms some other classical algorithms**

***Index Terms*—RPCA, SCBSS, IALM, signal separation, weak signal, low rank characteristic**

## I. Introduction

In the information age, the wide application of various electronic devices, such as communications and radar, forms a complex electromagnetic environment [1]. It has been an important topic in the field of communication reconnaissance that how to effectively separate the received time-frequency aliasing signals when dealing with passive signals with inaccurate apriori knowledge.

Blind Source Separation (BSS) can obtain the maximum information with the least conditions without the above restrictions. In particular, Single Channel Blind Source Separation (SCBSS) is an extreme case of BSS for separating the multi-channel signals [2]. It's widely used in mechanical fault diagnosis, biomedical signal processing, speech signal separation, wireless communication technology and many other fields [3]. In practical applications, due to the constraints of cost and installation conditions, the number of conventional observation signals is usually far less than the number of source signals. It's difficult to recover the source signal by using the traditional BSS method [4]. However, SCBSS has obvious advantage for it can identify multiple mixed signals in only one channel. And it helps to solve problems like the separation of the aim signal and the Co-Channel Interference in the communication field and the satellite AIS reception [5]. Therefore, we propose $\ell_{2,1} - RsPCA$ to make some contributions to the SCBSS problem.

## II. Communication Signal Separation Theory

### A. Low Rrank Characteristic Theory of Time Series of Communication Signals

Based on the theory of communication, most of the digital modulation signals can be embedded in the subspace $\mathcal{X}_0$ of complex space $L_1$ [6]. It can be discretized and embedded into the subspace $\mathcal{X} \subset \mathcal{C}^N$ and be decomposed into the Karhunen-Loeve expansion as [7]:

$$\boldsymbol{x}(n) = \sum_{k=1}^{K} \alpha_k \boldsymbol{p}_k(n), \quad n = 1, 2, \ldots, N \tag{1}$$

where $\alpha_1, \ldots, \alpha_K$ denotes a set of independent and identically distributed random variables, $K$ denotes the dimension of the linear space $\mathcal{X}$.

To simplify the model, the (1) can be modified to (2). As a result, the support set of $\boldsymbol{p}(n)$ is limited to $[0, T-1]$, where $T$ denotes the symbol period of the modulated signal.

$$\boldsymbol{x}(n) = \sum_{k=1}^{K} \alpha_k \boldsymbol{p}(n - kT), \quad n = 1, 2, \ldots, KT \tag{2}$$

Define reorder mappings as:

$$R : \mathcal{C}^{KT} \to \mathcal{C}^{K \times T} \tag{3}$$

The signal can be reordered as:

$$\boldsymbol{X} = R(\boldsymbol{x}) = \boldsymbol{a} \cdot \boldsymbol{p}^{\mathrm{T}} \tag{4}$$

where $\boldsymbol{a} = [\alpha_1, \ldots, \alpha_K]^{\mathrm{T}}$ and $\boldsymbol{p} = [\boldsymbol{p}(1), \ldots, \boldsymbol{p}(T)]^{\mathrm{T}}$.

Fig. 1 shows the relationship between reordered mappings $R(\bullet)$ and low rank structure $\boldsymbol{X}$. The theoretical rank of the reordered modulation signals is usually small and only related to the specific communication system but not to the size of the observed data. For example, the theoretical rank of a matrix-rized DS-PSK/QAM modulated signal is 1 when the number of

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rows equals the number of signal symbols and the number of columns equals the length of PN sequence [8].

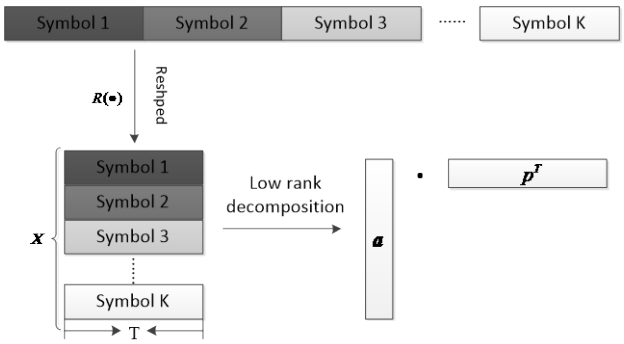



Figure 1. Reordered mappings and low rank structure of signals.

### B. Design of Object Function and Optimization Algorithm Based on 2,1 Norm

A data matrix $\mathbf{X}$ usually contains a lot of structural information and noise. Therefore, it can be decomposed into two forms of matrix summation. One matrix is of low rank structure for it has some structural information inside, which makes some linear correlation between rows or columns high. The other matrix is a sparse structure for it contains noise and the noise is sparse. This problem is considered by Robust principal component analysis ( RPCA) exactly as:

$$\min_{A,E} rank(A) + rank(B) + \lambda \|E\|_0 \quad s.t. X = A + B + E \tag{5}$$

where A denotes a structured matrix and E denotes noise.

In practical communication systems, the rank of matrices can't fully reflect the low rank structure of signals. Because of the influence of noise and wireless channel, the rank of signal matrix will increase or even become full in traditional sense. To solve the problem, we ignore the rank minimization problem to the kernel function minimization problem [9].

On the basis of the theoretical analysis above, we propose an algorithm named $\ell_{2,1}$ -Reshaped Principal Component Analysis ( $\ell_{2,1} - RsPCA$ ) to solve the problem of single-channel weak source separation for most of the low rank characteristics of communication signals.

Design objective function as:

$$\min f(a,b) = \|R_1(a)\|_* + \|R_2(b)\|_* \quad s.t. \|x - a - b\| < \varepsilon \tag{6}$$

where $a$ and $b$ are source signal and $x$ is observed signal.

Then we design the optimization algorithm as (7) based on optimized IALM.

$$L(a,b,y,\lambda) = \|R_1(a)\|_* + \lambda \|R_2(b)\|_* + \langle y, x - a - b \rangle + \frac{\mu}{2} \|x - a - b\|_{2,1} \tag{7}$$

where $\lambda$ denotes dual vector, $\mu$ and $y$ denote two different parameters.

Update variables alternately by iterative vector a and vector b in turn and contrast two contiguous outputs of a variable as an iteration stop condition. Stop iteration until establishing (8). The algorithm flow process is shown in the following table, where $D_\lambda(\bullet)$ denotes a soft threshold decision for singular values of a matrix, which contains parameter $\lambda$.

$$\frac{\|(a_{k+1} - a_k)\|_2^2 + \|(b_{k+1} - b_k)\|_2^2}{\|a_k\|_2^2 + \|b_k\|_2^2} \le \tau \tag{8}$$

TABLE I. DESCRIPTION FOR $\ell_{2,1} - RsPCA$

| |
|---|
| Initialization: $x \in R^T$<br>$y_0 = \mathrm{sgn}(x)$ , $a_0 = b_0 = x/2$ , $k = 0$ , $\lambda > 0$ , $\mu_0 > 0$ , $\tau > 0$ |
| Iterate until establishing (2-8). |
| Solve: $(a_{k+1}, b_{k+1}) = \arg\min_{a,b} L(a,b,y_k,\lambda)$<br>$a_{k+1} = R_1^{-1}(D_{(2\mu_k g)^{-1}}(R_1(x - b_k + (2\mu_k g_k)^{-1} y_k)))$<br>$b_{k+1} = R_2^{-1}(D_{\lambda(2\mu_k')^{-1}}(R_1(x - a_{k+1} + (2\mu_k g_k')^{-1} y_k)))$ |
| Update: $y_k$ , $\mu_k$<br>$y_{k+1} = y_k + \mu_k(x - a_{k+1} - b_{k+1})$ , $\mu_{k+1} = \rho\mu_k$ , $k = k+1$ |
| Output: $(a_{k+1}, b_{k+1})$ |

## III. RANDOM NUMBER SEPARATION EXPERIMENT

To verify the feasibility of the algorithm above and test the influence of the value of the rank to the result, we have carried out a random number separation experiment. Assume the length of the signal is $T$ , $T = 10000$ symbols. Two reordering mappings is defined as $R_1 : C^{10000} \to C^{50\times200}$ and $R_2 : C^{10000} \to C^{200\times50}$ . Source signal $a_0$ and $b_0$ with the rank of r satisfy:

$$x = \sigma a_0 + b_0 (\sigma > 0) \tag{9}$$

where $\sigma$ is a constant to simulate the energy differences between potential components.

To evaluate the performance of the algorithm, we use the value of $SIR$ (Signal to Interference Ratio) between the estimated signal $\tilde{a}_1$ and the source signal $a_1$ as the performance index [10].

$$\mathrm{SIR} = 10\log\left[\frac{a_1^2(k)}{[a_1(k) - \tilde{a}_1(k)]^2}\right] \tag{10}$$

Fig. 2 shows the influence of the size of rank to the algorithm. It can be seen that under the condition of the same $\sigma$ , with the increase of rank of the random data matrix, the value of $SIR$ decreases and the separation performance become worse. In addition, under the same value of rank, the larger the value of $\sigma$ is, the better the separation performance is. It is easier to separate the data when the value of $\sigma$ is larger because the greater the weight of $a_0$ is, the more the energy of $a_0$ in mixed signals is. Although the value of $SIR$ decreases gradually with the increase of rank, the separation precision can

still keep high. Even if it's under the condition of a lower rank, the energy difference between the two potential components has no obvious effect on the separation effect of the algorithm.

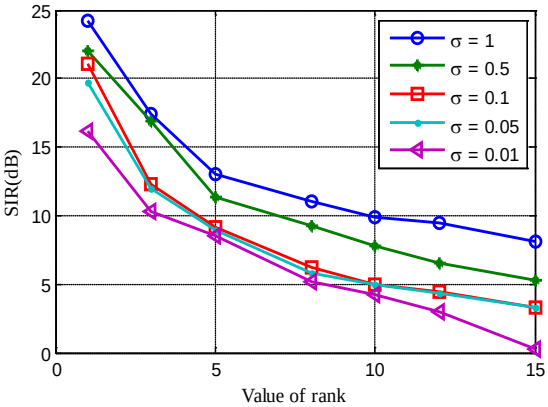
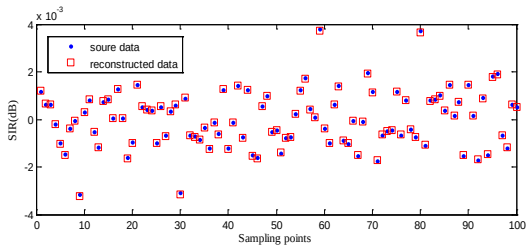
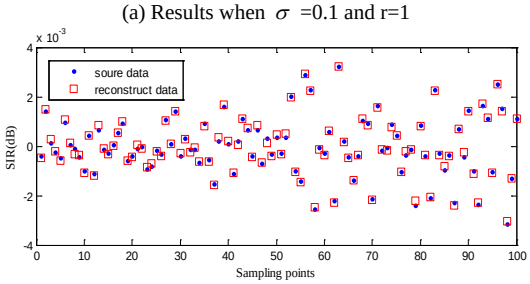


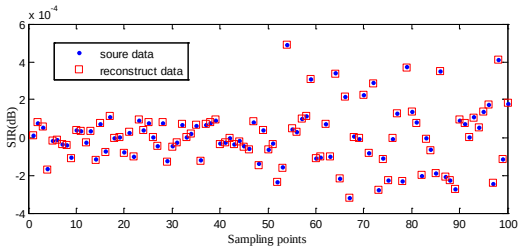

Figure 2. Relationship among value of rank, $\sigma$ and $SIR$ .



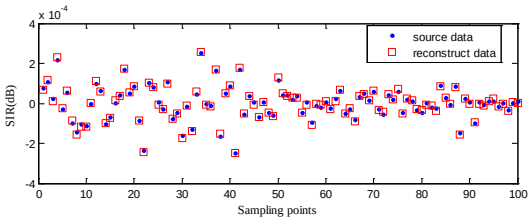

(a) Results when $\sigma$ =0.1 and r=1




(b) Results when $\sigma$ =0.1 and r=3




(c) Results when $\sigma$ =0.01 and r=1




(d) Results when $\sigma$ =0.01 and r=3

Figure 3. Results of data separation under different values of $\sigma$ and rank.

Fig. 3 shows the comparison of random number source data and reconstructed data when $\sigma = 0.1$ and $\sigma = 0.01$ when the rank is respectively 1 and 3. Although the reconstructed data have some bias compared with the source data, it is still within the acceptable range. It proves that the algorithm of $\ell_{2,1} - RsPCA$ can separate signals with high accuracy under low rank condition.

## IV. Experiment on the Extraction of Spread Spectrum BPSK Signal

The source signal is a mixed of baseband direct spread BPSK signals of two ways as:

$$\boldsymbol{x} = \sigma \boldsymbol{a} + \boldsymbol{b} (\sigma \le 1) \tag{11}$$

As the length of symbols increases, the low rank property of the matrix is more obvious and the separation performance of the algorithm will be improved in a certain. In this simulation experiment, we pick the symbol length of 100. Set $\sigma = 0.1 \sim 1$ and signal-to-noise ratio 20 $dB$ . The results are shown in Fig. 4.

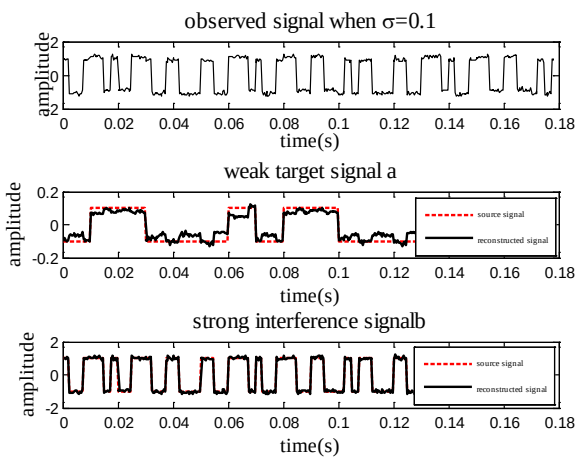



Figure 4. Time domain waveform contrast diagram.

Among them, the dashed line represents the source signal and the solid line represents the reconstructed signal. From the comparison of the time domain waveforms between the source signal and the reconstructed signal, we can see this separation method based on $\ell_{2,1} - RsPCA$ can effectively separate the mixed DSSS-BPSK signals even if the intensity of the interference signal is greater than the target signal.

In the analysis of digital modulation signal, the "amplitude-phase" relation of a digital modulation signal is often illustrated by means of two-dimensional plane signal constellation diagram, so that the "minimum signal distance" related to anti-jamming ability can be qualitatively indicated. BER is the ratio of the number of error bits in the digital signal received by the receiver to the total number of bits received by the receiver in the same period of time during signal transmission. In order to measure the performance of this signal separation method, we refer to the constellation and the bit error rate of the separated signals.

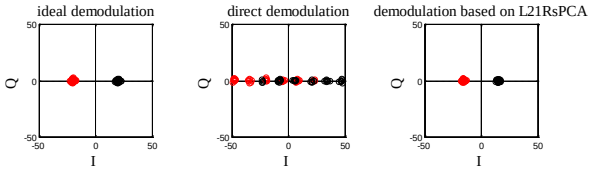



Figure 5. Planisphere.

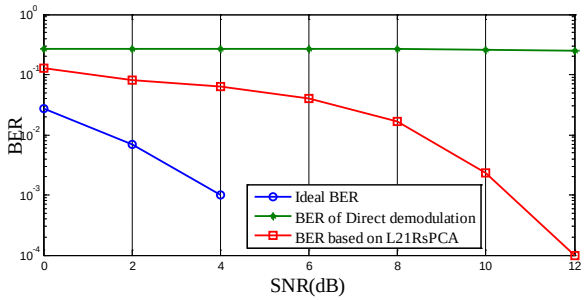



Figure 6. Contrast diagram.

From the above diagrams, we can see that a large number of erroneous information will be generated after demodulation without the algorithm. However, although the BER of the signal separated by the algorithm is still far from the ideal result, the demodulation effect has been significantly improved compared with that without the algorithm. Moreover, the BER of the signals separated by the algorithm is in a reasonable range.

To observe the effect of different values of $\sigma$ on the separation effect of the algorithm, we still use the SIR mentioned above as a judgment index.

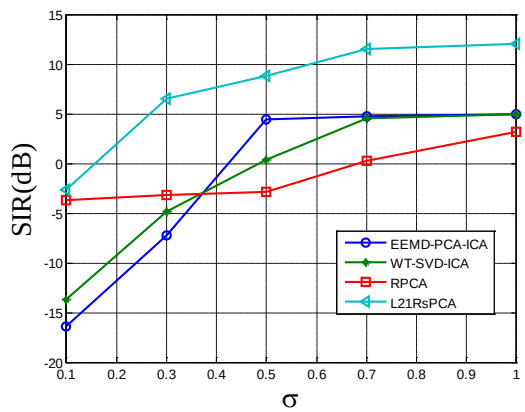



Figure 7. Signal separation results under different values of $\sigma$ and performance comparison of different algorithms.

Obviously, with the increase of the value of $\sigma$ , the separation effect of the signal is getting better and better because the proportion of the target signal in the mixed signal increases. When $\sigma$ = 0.1, the energy of the interfering signal is 100 times the energy of the target signal. Nevertheless, the SIR of the separated signal is still as high as 10 dB.It proves that the algorithm has good performance in weak signal separation.

For better comparison, we select WT-SVD-ICA, EEMD-PCA-ICA and RPCA, which are three of classical SCBSS algorithms, to conduct signal separation experiments under the conditions above. Results are showed in Fig. 7. It can be seen that the value of SIR of $\ell_{2,1} - RsPCA$ is obviously higher than other three methods, which proves the superiority of the algorithm in separating weak and blind communication signals.

## V. Conclusion

To solve the problem of weak signal decomposition from strong interference, we propose the algorithm of $\ell_{2,1} - RsPCA$ applied to single-channel blind communication signal separation under the RPCA model.

Through the random number separation experiment, we prove that the separation effect is gradually increased by not only the effect of the reduction of the rank of the random number matrix but also the effect of the increase of the energy of the target signal. It shows a good separation result even if it's under the condition of low rank and weak source.

Through the spread spectrum BPSK signal separation experiment, we prove that as the number of signal symbols increases, the performance of the algorithm is gradually improved. In addition, we prove the algorithm of $\ell_{2,1} - RsPCA$ can separate single-channel mixed signal effectively and outperforms algorithms like WT-SVD-ICA, EEMD-PCA-ICA and RPCA, which are three of classical signal separation methods in the professional field. Under the condition that the interference intensity is obviously higher than the target signal, the conventional method are not able to demodulate the target signal. However, the target signal can still be effectively demodulated based on $\ell_{2,1} - RsPCA$.

All of these prove that the algorithm is an excellent solution to the isolation of weak communication signals. Furthermore, because of the superiority of single channel blind source separation, this algorithm has good application prospects.

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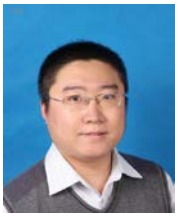

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