Optimization of Spectrum Hole Utilization in Rayleigh Faded Cognitive Radio Networks

Mohsin Ali and Haewoon Nam
Department of Electrical Engineering, Hanyang University, Ansan, S. Korea
Email: {mohsin0987, hnam}@hanyang.ac.kr

***Abstract*—This paper investigates the optimization of the probability of spectrum hole utilization (PSHU) given minimum sensing error rate in Rayleigh fading channel. An optimal detection threshold is calculated to minimize the sensing error. Analytically it is very complex to solve optimal threshold, so numerically optimal threshold is calculated which gives the minimum sensing error (P_e). Based on the analytical formulations of sensing error and PSHU, it is shown that there exists an optimal sensing time duration which gives maximum spectrum utilization, in a given mean idle time of PU. The numerical and simulation results show that using the optimal detection threshold to minimize sensing error, PSHU can be maximized on an optimal sensing time.**

***Index Terms*—cognitive radio, spectrum utilization, sensing error, Rayleigh fading**

I. INTRODUCTION

Wireless technologies are growing rapidly and more and more spectrum resources are needed to support numerous emerging wireless services. Within the current spectrum regulatory framework, however, all of the frequency bands are exclusively allocated to specific services and no violation from unlicensed users is allowed. In order to solve the issues of spectrum scarcity and spectrum under-utilization, Cognitive Radio (CR) technology has been proposed. It can improve the spectrum utilization by allowing secondary users to borrow unused radio spectrum from primary licensed users or to share the spectrum with the primary users [1].

One of the most critical components of cognitive radio technology is spectrum sensing. By sensing and adapting to the environment, a cognitive radio is able to fill in spectrum holes and serve its users without causing harmful interference to the licensed user. Probability of detection and probability of false alarm are performance parameters associated with spectrum sensing, where high probability of detection and low probability of false alarm are desired to avoid interference to PUs [2].

Spectrum sensing may face the challenges due to channel fading and shadowing. These challenges can cause spectrum sensing error in the form of probability of false alarm and miss detection. Total sensing error is considered as a combination of both these probabilities. For better utilization of spectrum opportunities total sensing error should be minimum as possible. There are several works done by the researchers which aims to minimize the spectrum error. In [3] authors have investigated the optimality of cooperative spectrum sensing by optimizing the detection performance in the form minimum sensing error. A collaborative spectrum sensing in different fading channels is studied in [4]. Authors suggested that collaboration between secondary users in fading environments significantly improves the sensing performance. A chrip-z transform method is used in [5] for accurate spectrum sensing and also a comparison of system performance through complementary ROC is done for AWGN and Rayleigh channels. Ref. [6] also investigated the spectrum sensing in low SNR channels. Authors focused on derivation of close form expression for miss detection probability in fading channels and to minimize the total error rate by optimizing the detection threshold.

Successful spectrum hole utilization [7] takes place when primary signal’s absence is correctly detected with no sensing error and SU’s transmission is successful without interfering the PU during the given transmission duration of SU. Authors in [8] investigated the spectrum opportunity utilization probability in a dynamic radio environment for different network topologies, however they did not reflect the effect of sensing time duration on utilization of spectrum opportunity. A Trade-off between spectrum sensing and probability of spectrum opportunity utilization for simultaneously increasing sensing and transmission time is discussed in [9]. Authors in [10] discussed the effect of spectrum sensing and transmission duration on spectrum hole utilization. Authors have considered the fixed time frame within which variable sensing and transmission time. Spectrum resource allocation and scheduling problem as a linear 0–1 integer programming optimization problem to maximize the available spectrum resources utilized by the CR based high speed vehicle network is discussed in [11]. An opportunistic feedback based CSS scheme is proposed in [12], to reduce the feedback overhead due to many SUs in the network. Performance of the proposed scheme is analyzed in terms of the channel utilization of an SU, the average feedback load, and the average false alarm probability. A heuristic channel-allocation/power-control algorithm based on constructing a dynamic interference graph is proposed in [13]. Proposed algorithm claimed to maximize the spectrum utilization of a cognitive radio

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network that employs opportunistic spectrum access. Our work differs from the existing work in such a way that existing work supposed error free sensing channel but in our work a Rayleigh fading channel is supposed in sensing. Also this work utilizes optimal detection threshold to minimize the total sensing error and so to maximize the spectrum utilization probability.

Spectrum utilization can be maximized when SU senses the spectrum correctly and sensing error is as low as possible. In this paper we optimize the probability of spectrum utilization in the Rayleigh fading channel. First we optimize the detection threshold under low SNR to minimize the total sensing error in Rayleigh fading channel. By using this optimal threshold probability of false alarm is calculated, which then used with total sensing error to optimize the probability of spectrum hole utilization. A fixed time frame with variable sensing and transmission time in a given idle time of primary user, is considered in the whole process.

The remainder of this paper is comprised as: In section 2, a system model containing energy based local spectrum sensing and Rayleigh fading is explained. Optimization of sensing error rate for optimal detection threshold to optimize the PSHU, is discussed in section 3. Simulation and numerical results are explained in section 4 and in section 5, conclusion of the work is shown.

II. SYSTEM MODEL

A. *Energy Detection Based Local Spectrum Sensing*

Consider a cognitive radio network consisting of secondary users (SUs) and primary users (PUs). Energy detector is used to measure the energy of the signal in the frequency band of interest. Each SU collects N samples and measures the average energy of those samples, so the test statistics of energy detector can be represented as

$$Y = \frac{1}{N}\sum_{n=1}^{N} |y(n)^2|. \tag{1}$$

where $n = 1, 2, \ldots N$. Spectrum sensing in cognitive radio networks follows a binary hypothesis testing problem: hypothesis H_0 (primary signal absent) and hypothesis H_1 (primary signal present). Received signal for the binary hypothesis can be given as

$$y(n) = \begin{cases} w(n) & H_0 \\ hs(n) + w(n) & H_1 \end{cases} \tag{2}$$

where $s(n)$ is the transmitted signal, h is the wireless channel gain, and $w(n)$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2.

Output (Y) of energy detector may be shown to have the central and non central chi-square distributions under the hypothesis H_0 and H_1, respectively. Assuming large N, according to central limit theorem PDF of Y can be approximated by Gaussian distribution given as

$$f(y) = \begin{cases} \mathcal{N}\left(\sigma_n^2, \frac{\sigma_n^4}{N}\right) & H_0 \\ \mathcal{N}\left((\gamma+1)\sigma_n^2, \frac{(2\gamma+1)\sigma_n^2}{N}\right) & H_1 \end{cases} \tag{3}$$

where γ is signal to noise ratio (SNR).

In a non-fading environment where h is deterministic, probabilities of detection and false alarm are given by the following formulas,

$$P_f(\lambda) = \Pr(Y > \lambda | H_0) = Q\left(\left(\frac{\lambda}{\sigma_n^2} - 1\right)\sqrt{N}\right). \tag{4}$$

$$P_d(\lambda) = \Pr(Y > \lambda | H_1) = Q\left(\left(\frac{\lambda}{\sigma_n^2} - \gamma - 1\right)\sqrt{\frac{N}{2\gamma+1}}\right). \tag{5}$$

where λ is predefined detection threshold and $Q(.)$ is known as Q-function, defined as

$$Q(z) = \frac{1}{\sqrt{2\pi}}\int_z^\infty exp\left(-\frac{v^2}{2}\right) dv. \tag{6}$$

B. *Rayleigh Fading*

From (4) it is noticeable that P_f is independent of γ, since under H_0 there is no primary signal present, so P_f will not be affected due to fading. On the other hand, when h is varying due to fading, probability of detection is conditioned on the γ. In this case, average probability of detection may be derived by averaging (5) over fading statistics as follow,

$$\overline{P}_d(\lambda) = \int_x P_d(\lambda) f_\gamma(x)\, dx. \tag{7}$$

where $f_\gamma(x)$ is the probability density function (PDF) of γ under fading.

Under Rayleigh fading, γ would have an exponential distribution $f_\gamma(x) = \frac{1}{\gamma}\exp(-\frac{x}{\gamma})$. In this case, a closed-form formula for $\overline{P}_d(\lambda)$ after some manipulations, can be obtained by substituting $f_\gamma(x)$ in (7) as follow [14],

$$\overline{P}_d^{Ray}(\lambda) = Q\left((\lambda-1)\sqrt{N}\right) + exp\left(\frac{1}{2\lambda^2\gamma^2 N} - \frac{\lambda-1}{\lambda\gamma}\right) \times Q\left(\frac{1}{\lambda\gamma\sqrt{N}} - (\lambda-1)\sqrt{N}\right). \tag{8}$$

III. SENSING ERROR AND PROBABILITY OF SPECTRUM UTILIZATION

Spectrum utilization can be affected if the SU's sensing results are not reliable regarding primary signal's presence or absence. Sensing error causes the wastage or miss use of spectrum opportunity.In practical systems sensing error may occur due to probability of false alarm (P_f) and miss detection (P_{md}) in the local sensing phase. Probability of miss detection in Rayleigh fading can be expressed as,

$$P_{md}^{Ray}(\lambda) = 1 - \overline{P}_d^{Ray}(\lambda). \tag{9}$$

So the total sensing error rate (P_e) can be expressed as follow [6],

$$P_e(\lambda) = P_f(\lambda) + P_{md}^{Ray}(\lambda). \tag{10}$$

In order to achieve maximum probability of spectrum hole utilization (PSHU), should be as small as possible. So there must be an optimal detection threshold in the faded environment over which P_e can be minimized i.e.

$$\lambda^* = arg\min_{\lambda} P_e(\lambda) \tag{11}$$

Optimal threshold (λ^*) can be achieved by taking partial derivative of P_e with respect to λ and equating it to zero.

$$\frac{\partial P_e(\lambda)}{\partial \lambda} = 0. \tag{12}$$

$$\frac{\partial P_e(\lambda)}{\partial \lambda} = Q\left(\frac{1}{\lambda\gamma\sqrt{N}} - (\lambda - 1)\sqrt{N}\right) exp\left(\frac{1}{2\lambda^2\gamma^2 N} - \frac{\lambda-1}{\lambda\gamma}\right)\times \left(\frac{1}{\lambda^3\gamma^2 N} + \frac{1}{\gamma\lambda^2}\right) - \frac{1}{\sqrt{\pi}} exp\left(\frac{1}{2\lambda^2\gamma^2 N} - \frac{\lambda-1}{\lambda\gamma}\right)\times exp\left(-\frac{1}{\lambda\gamma\sqrt{N}} - (\lambda - 1)\sqrt{N}\right)\left(\frac{1}{\gamma\lambda^2\sqrt{N}} + \sqrt{N}\right). \tag{13}$$

It is very complex to solve (13) for the exact solution λ^*, so optimal threshold which gives minimum (P_e) can be obtained numerically. Optimal detection threshold (λ^*) and minimum sensing error rate $P_e(\lambda^*)$ calculated numerically are then use to maximize the probability of spectrum hole utilization.

Fig. 1 shows the frame structure of SU. It is supposed that time frame (T) of SU lies in the given idle time of PU. The transition of idle and busy time durations of PU denoted by T_I and T_B can be considered as poison process and modeled as exponentially distributed with mean α_I and α_B and with probability density functions (PDFs) as $f_{T_I}(t) = \frac{1}{\alpha_I} e^{-\frac{t}{\alpha_I}}$ and $f_{T_B}(t) = \frac{1}{\alpha_B} e^{-\frac{t}{\alpha_B}}$, respectively [15]. From the means of idle and busy states of PU, the probabilities of being idle and busy can be calculated as

$$\theta_I = \frac{\alpha_I}{\alpha_I + \alpha_B}, \quad \theta_B = \frac{\alpha_B}{\alpha_I + \alpha_B} \tag{14}$$

Successful or acceptable transmission of SU is only possible when PU remains idle or inactive during the SU transmission. In other words, PU idle time should be longer than SU transmission time. SU transmission can be marked as successful if there is no reappearance of PU in the spectrum during the SU transmission duration (τ_{tr}). This can be defined as probability of no interference (P_{NI}) and is expressed as follows:

$$P_{NI}(\tau_s) = Pr\big(u > (T - \tau_s)\big). \tag{15}$$

where u is the duration of idle time of PU, which is assumed to be started just after the SU sensing time (τ_s) until PU reappears in the spectrum. Since u can be modeled as exponentially distributed with mean value $\alpha_0 = \alpha_I - \tau_s$ and its PDF is $f_u(t) = \frac{1}{\alpha_0} e^{-\frac{t}{\alpha_0}}$, as shown in the Fig. 1, so (15) can be re-written as

$$P_{NI}(\tau_s) = \int_{T-\tau_s}^{\infty} \frac{1}{\alpha_0} exp\left(-\frac{t}{\alpha_0}\right) dt = exp\left(-\frac{T-\tau_s}{\alpha_0}\right). \tag{16}$$

Since we consider the fixed length of SU time frame (T) and dynamic sensing time (τ_s), which is liable to change in transmission time as, $\tau_{tr} = T - \tau_s$. Variation in sensing time has a large impact on probability of no interference by SU to PU. Higher probability of no interference is one of the factors to maximize the PSHU.

Spectrum hole utilization takes place when primary signal's absence is correctly detected with no sensing error and SU's transmission is successful without interfering the PU during the given transmission duration probability $(\frac{T-\tau_s}{T})$ of SU. So, probability of spectrum hole utilization (PSHU) can be defined as follow,

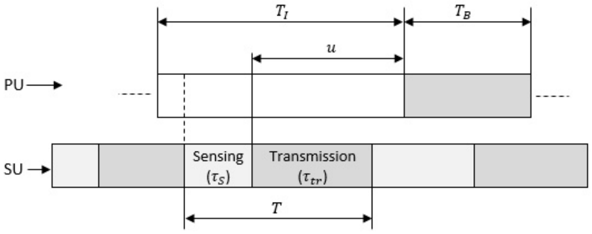

Figure 1. PU idle (T_I) and busy (T_B) time representation and SU time frame (T) structure.

$$P_{SHU}(\tau_s, \lambda^*) = \theta_I\big(1 - P_e(\lambda^*)\big)P_{NI}(\tau_s)\left(\frac{T-\tau_s}{T}\right). = \theta_I\big(1 - P_e(\lambda^*)\big)exp\left(-\frac{T-\tau_s}{\alpha_0}\right)\left(\frac{T-\tau_s}{T}\right). \tag{17}$$

IV. SIMULATION AND NUMERICAL RESULTS

In this section we provide simulation results of proposed problem. Rayleigh fading channel is considered with other system parameters as, T=10ms, f_s =6MHz, θ_I=0.9 and α_0=650ms. Results are also taken for γ = -10dB and -12dB.

In Fig. 2 optimal sensing error is shown over a range of detection threshold. Figures shows that there is an optimal value of threshold in the range of detection threshold, which gives the minimum sensing error in the Rayleigh faded environment. Figure also shows that sensing error can be further minimized if the received SNR by SU is improved.

Detection threshold has a major part in sensing process as primary signal absence or presence is decided by comparing sensed energies with threshold. In different channel conditions different values of threshold are required. In Fig. 3 optimal detection threshold is shown against different signal to noise ratio. Figure shows that higher optimal threshold can be achieved at higher SNR values. This optimal threshold is useful to get maximum spectrum utilization in different channel conditions.

An optimal sensing time over which maximum probability of spectrum hole utilization by using the optimal detection threshold is shown in Fig. 4. By using the analytical results of (17), figure shows that by using optimal detection threshold correct sensing of primary signal's absence can be improved. Similarly optimal threshold is helpful in minimizing the sensing error in fading channel. So overall an optimal sensing time is achieved to maximize the PSHU. Simulated results are also plotted, which show a fair similarities with analytical or theoretical results. Graphs are plotted for received SNR of -10dB and -12dB, which show that higher PSHU can be achieved at higher SNR.

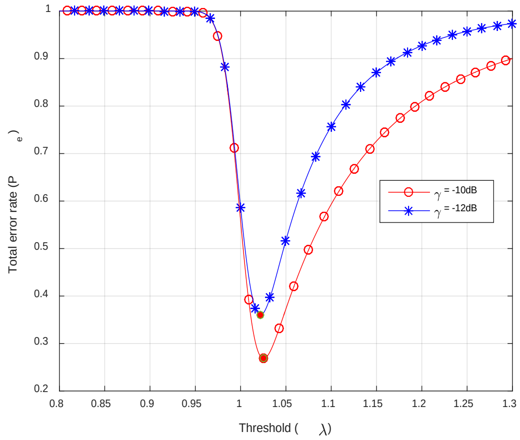

Figure 2. Total sensing error (P_e) vs detection threshold.

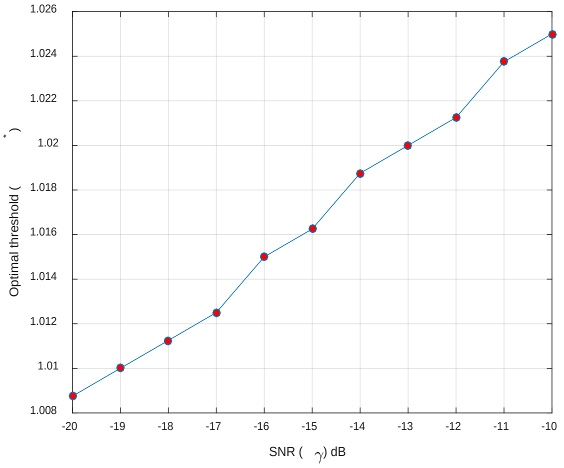

Figure 3. Optimal detection threshold (λ^*) vs SNR (γ).

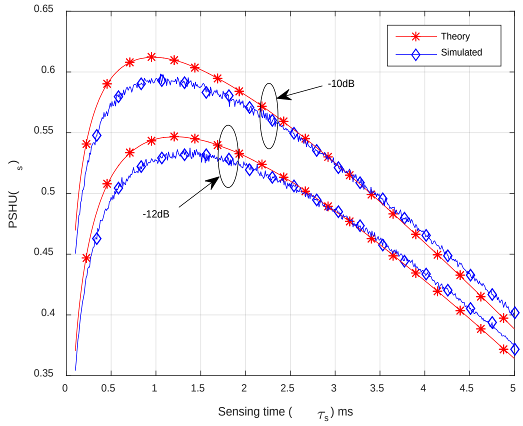

Figure 4. Probability of spectrum hole utilization (PSHU) vs sensing time.

V. CONCLUSIONS

Spectrum utilization can be maximized when SU senses the spectrum correctly and sensing error is as low as possible. In this paper we optimize the probability of spectrum utilization in the Rayleigh fading channel. First we optimize the detection threshold under low SNR to minimize the the total sensing error in Rayleigh fading channel. By using this optimal threshold probability of false alarm is calculated, which then used with total sensing error to optimize the the probability of spectrum hole utilization. A fixed time frame with variable sensing and transmission time in a given idle time of primary user, is considered in the whole process. The proposed work can be extended further in future for cooperative spectrum sensing for multiple SUs by considering different network topolgies and channels between PUs, SUs, and FC.

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Mohsin Ali received the B.S and M. S degree in computer engineering and electrical engineering from COMSATS Institute of Information Technology, Abbottabad, Pakistan, in 2007 and 2012 respectively. He is currently working towards the Ph.D degree in electronics and communication engineering with Hanyang University, Seoul, Korea. He has achieved scholarship for higher studies from Higher Education Commission of Pakistan.

His current area of research includes cognitive radio networks.

Haewoon Nam received the B.S degree from Hanyang University, Seoul, Korea, in 1997; the M.S. degree from Seoul National University, Seoul, in 1999; and the Ph.D degree in electrical and computer engineering from the University of Texas at Austin, Austin, TX, USA, in 2006.

From 1999 to 2002, he was with Samsung Electronics, Suwon, Korea, where he was engaged in the design and development of code-division multiple access and GSM/GPRS baseband modem processors. In the summer of 2003, he was with the IBM T. J. Watson Research Center, Yorktown Heights, NY, USA, where he performed extensive radio channel measurements and analysis at 60 GHz. In the fall of 2005, he was with the wireless mobile system group, freescale semiconductor, Austin, where he was engaged in the design and test of the WiMAX MAC layer. His industry experience also includes work at Samsung Advanced Institute of Technology, Kiheung, Korea, where he participated in the simulation of multiple-input–multiple-output systems for the 3GPP LTE standard. In October 2006, he joined the Mobile Devices Technology Office, Motorola Inc., Austin, where he was involved in algorithm design and development for 3GPP LTE mobile systems, including modeling of the 3GPP LTE modem processor. Later in 2010, he was with Apple Inc., Cupertino, CA, USA, where he worked on the research and development of next-generation smart mobile systems. Since March 2011, he has been with the Department of Electrical Engineering, College of Engineering and Science, Hanyang University, where he is an associate professor.

Nam received the Korean government overseas scholarship for his doctoral studies in the field of electrical engineering.