Uncertainty Quantification of Underwater Sound Propagation Loss Integrated with Kriging Surrogate Model

Xianpeng Guo, Dezhi Wang, Lilun Zhang, Yongxian Wang, Wenbin Xiao, and Xinghua Cheng
National University of Defense Technology, Changsha, China
Email: guo_xianpeng, wang_dezhi} @hotmail.com

***Abstract*—In this study, an efficient Monte Carlo like method integrated with a Kriging surrogate model is proposed to estimate the uncertainties of underwater sound propagation loss with respect to multiple uncertain environmental parameters. The Kriging model is trained to simulate and replace the classical Kraken underwater acoustic propagation model in order to significantly reduce the computational cost but ensuring the accuracy. Monte Carlo process is performed by means of continuously sampling from the trained Kriging surrogate model, by which we finally achieve a quantification of the sound propagation loss. The Kriging models are respectively employed within different segments of a full calculation range in order to overcome the ill-conditioning problem of Kriging algorithm and increase the spatial resolution. 90% confidence bands are also calculated to illustrate the spatial distribution of uncertainties of the sound propagation loss.**

***Index Terms*—Kriging surrogate model, kraken model, underwater sound propagation loss, uncertainty quantification**

I. Introduction

The practical underwater environment is complex and full of uncertainties. For example, the water temperature, salinity, and other environmental parameters are time-varying and cannot be accurately predicted, which significantly affect the sound velocity profile and the sound propagation process. The underwater sound propagation loss is a significant factor for applications like sonar performance prediction and normally shows a strong uncertainty. It is very important to estimate existing uncertainties of the propagation loss for better sonar applications.

Normally uncertainties of the underwater sound propagation loss are estimated based on the Monte Carlo method with repeatedly calling the acoustic propagation model to generate samples. Since the calculation of acoustic propagation is highly computation intensive, the quantification of propagation uncertainties generally results in a significant time cost. In view of this point, the Kriging surrogate model is introduced into the sound-field calculation to replace these traditional acoustic calculation models (e.g. Kraken model) to balance the accuracy and efficiency for an efficient computation in practical applications. As a Best, Linear and Unbiased Prediction (BLUP) method, Kriging is a statistical model that contains autocorrelation (i.e. statistical relationships between measurement points). The model is originally used in geoscience [1]. Cheng, etc. [2] introduced the Kriging method into the field of underwater acoustics and preliminarily validate the practicability of the Kriging surrogate model for acoustic propagation model. Robinson, etc. [3] from Harvard University coupled the acoustic model with the Harvard Oceanographic System to achieve dynamic prediction of the underwater acoustic environment. Zhang, etc. [4] from the Ocean University of China, used the reverberation correlation to invert the submarine parameters, and analyzed the influence of the uncertainty of parameters on the acoustic propagation loss.

In this paper, the Kriging model is employed to replace the Kraken model to greatly reduce the calculation cost of sound propagation. On this basis, the Kriging model is further integrated into the Monte Carlo process to quantify the uncertainties of sound propagation loss in terms of obtaining the confidence intervals and probability density distribution. The related results show the effectiveness and computational efficiency of our proposed method.

II. Methods

A. Kriging Method

The Kriging model is a statistical model that contains correlation (i.e. statistical relationships between measurement points) [5]. The whole model runs through a two-step process. Firstly, we create a mutation function and a covariance function to estimate the statistical dependency (called spatial correlation) values that depend on the correlation model (fitting model). Secondly, unknown values are predicted (predictions). The fitting model is a polynomial fit while the covariance model is based on semi-variogram and covariance function. The semi-variogram is a function of geostatistical analysis [6], which is:

$$\gamma(x, h) = \frac{1}{2} Var[Z(x) - Z(x + h)] \tag{1}$$

Manuscript received July 1, 2017; revised November 24, 2017.

doi: 10.18178/ijsps.5.4.141-145

Z(x) is the measured value at position x and Z(x+h) is the measured value at position x+h . At the same time, satisfies the stationary hypothesis, so:

$$(x, h) = \frac{1}{2} E[Z(x) - Z(x + h)]^2 \quad (2)$$

The covariance function is defined as:

$$Cov(X, Y) = E\{[X - E(Y)][Y - E(Y)]\} \quad (3)$$

In the kriging method, a number of functions for fitting the semi-variogram model are provided, such as exponential function (EXP), linear function (LIN), Gaussian function (GAUSS), cubic spline, etc. for different applications.

In the process of replacing the underwater acoustic model, the input of the Kriging model are the environmental parameters, the meshing and the results of the sound field calculation [2]. The output P is composed of the polynomial F and the random distribution Z which are developed by the marine environmental parameters [6]

$$P(\varepsilon) = F(\beta, \varepsilon) + z(\varepsilon) = f^T(\varepsilon)\beta + Z(\varepsilon) \quad (4)$$

where F(β,ε) is a regression equation, which is used to describe the global approximation of the output i.e. the sound field. β is the regression coefficient, f(ε)is a polynomial containing ε, which can be 0-order, first-order, second-order polynomial. Z(ε) is the approximation error, which reflects the randomness of the sound field. The related physical quantity has the following statistical characteristics:

$$E(Z(\varepsilon)) = 0 \quad (5)$$

$$Var(Z(\varepsilon)) = \delta_z^2 \quad (6)$$

$$Cov[Z(\varepsilon_i), Z(\varepsilon_j)] = \delta_z^2[R_{ij}(\theta, \varepsilon_i, \varepsilon_j)] \quad (7)$$

$R_{ij}(\theta, \varepsilon_i, \varepsilon_j)$ is a correlation function, that is, to interpolate from the random deviations of the polynomial fit i.e. After comparison, the GAUSS correlation function and the second order polynomial was used to establish the semi-variogram model in this paper.

B. Kriging Integrated Monte Carlo Method

As an important approach in the field of computational science, Monte Carlo method can be used to solve the actual problem from a statistical perspective using a large number of statistical tests [7]. In the Kraken calculation process, once any environment variable changes, the Kraken model needs to be called to do a new calculation. In order to do the Monte Carlo simulations, Kraken calculations have to be carried out for millions of times especially for simulating the changes of multi parameters, which requires a very high computational capacity, On the other hand, Kriging surrogate model has been proved to be an effective replacement of the traditional Kraken model, which can very efficiently compute the sound propagation results. Thus, it is significant integrate the Kriging surrogate model into the framework of Monte Carlo simulation to estimate the posterior probability distribution of target variable for uncertainty analysis and other further applications. The efficiency will be greatly improved for practical usage.

III. Experiments

A. Kraken Model for Sound Propagation Calculation

The Kraken program is an acoustic propagation software based on a normal-model theory [8]. The normal-model theory is used to solve the depth - dependent wave equation. The specific implementation of Kraken calculation can be found from a sets of modeling tools called Acoustic Toolbox. [8] The acoustic environmental model and related parameters applied in this study are demonstrated as follows:

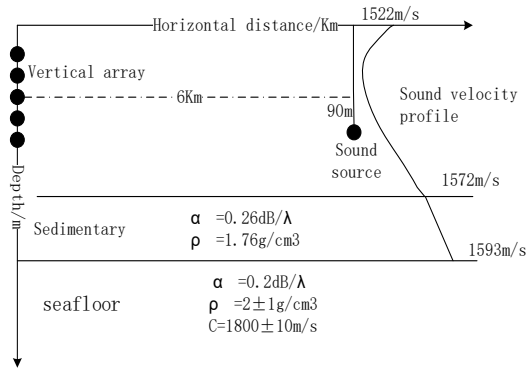

Figure 1. Environmental model and parameters.

The experiment uses a two-layer submarine model for the sedimentary layer and the seabed. Sound source position, sound velocity profile and other environmental parameters are shown in the Fig. 1. The depth of sound source is 90 meters and the horizontal distance between the vertical array and the sound source is 6 km. The sound velocity profile is shown is the figure. The range of submarine sound velocity is defined as 1790m/s to 1810m/s. Besides, the range of submarine density is defined as 1g/cm^3 to 3 g/cm^3.The α is the acoustic attenuation coefficient. In this paper, the influence of environmental parameters on acoustic propagation loss is analyzed by Kriging integrated Monte Carlo simulation.

B. Kriging Surrogate Model for Sound Propagation Calculation

The Kriging surrogate model is trained based on the input environmental parameters and output sound pressure data of Kraken calculation model. We run the Kraken model at first to get the acoustic pressure data at the fixed points under certain environmental parameters. The acoustic propagation loss TL can be calculated by using Equation (8).

$$TL = -10 * \log |P| \quad (8)$$

P is the sound pressure calculated by Kraken. After obtaining the acoustic propagation loss, we take the

environmental parameters of the boundary points and the corresponding acoustic propagation loss as input of the Kriging model. The Kriging model produces the corresponding estimation error in the calculation process. We smoothen the error by adding the point where the estimated error is the greatest. The surrogate model is constructed by increasing the input sample points, and the estimation error is finally converged to a smaller value and the relative error of the result is acceptable.

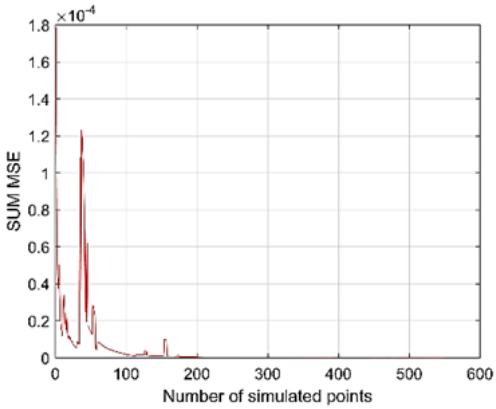

Figure 2. Evolution of MSE summation.

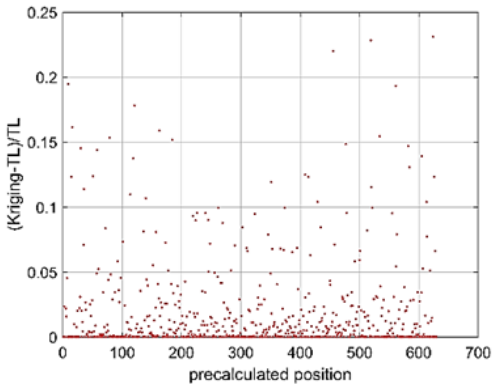

Figure 3. Relative error values for the segmented Kriging surrogate model.

In order to overcome the ill-conditioning problem of Kriging algorithm and increase the spatial resolution, the calculation range of sound propagation are divided into a number of segments. Within each segment, a Kriging surrogate model is trained and used to generate the required samples in Monte Carlo simulation. Fig. 2 shows the change in the MSE of the alternative model for calculating the total computational area [9] with the increasing number of calculations. It can be seen that MSE eventually converge to a minimum, indicating that the model is fully optimized. The environmental parameters of the Kriging models in range for segments are shown in Table I.

TABLE I. ENVIRONMENTAL PARAMETERS

horizontal distance /Km	Depth /m	acoustic velocity m/s	Density g/cm^3	Calculati-on times
1-5（0.5 Km interval）	85/90/95	1790-1810	1 -3	200times for each one

The full horizontal distance from 1 km to 5 km is divided into a series of segments with an interval of 0.5 km. The depth for each segment is also fixed as discrete values as shown in the Table II. As the sampling points are relatively dense, the variation of the propagation loss can be fully simulated. 200 training points based on the Kraken model are used to train the Kriging surrogate model in each segment. Fig. 3 shows the relative error values for the segmented replacement model at a horizontal distance of 4.5Km-5Km. As a result, the error is concentrated within the range of 0-0.1, which indicates the good accuracy of Kriging surrogate model.

C. Uncertainty Quantification of Sound Propagation Loss

The probability density distribution of single-point acoustic propagation loss is firstly studied. The related parameter settings and the total calculation time including both the training of Kriging model and the uncertainty analysis are shown in Table II.

TABLE II. PARAMETERS SETTING AND CALCULATION TIME

horizontal distance /Km	depth /m	acoustic velocity m/s	Density g/cm^3	computing time
2-10	200	1800±10	2±1	282.62s

Two environmental parameters i.e. the seabed sound velocity and seabed density are set as the uncertain parameters with an uniform distribution, and the horizontal distance is respectively fixed at 2.5Km, 3.5Km and 4.5Kmcorresponding to the depth of 85m, 90m and 95m. 2000 samples are simulated by the proposed Kriging integrated Monte Carlo method for each set of parameter settings. The simulated frequency distribution is shown in Fig. 4. It can be seen from the results of the calculation that when the calculated depth is 90 meters, the overall propagation loss is more concentrated within a smaller range. Especially when the horizontal distance is 3.5Km, the calculation result is the most concentrated, which indicates a relatively high accuracy or confidence. On the contrary, at a depth of 85 meters, the results are more dispersed. As the distance increases, the data appears two concentrated values and presents a divergent trend. At a depth of 95 m, the prediction results are closer to both values. When the horizontal distance is 4.5Km, the proportion of the two extremes is basically the same. In general, at fixed depth, the uncertainty of propagation loss increases with increasing propagation distance. At the same propagation distance, the uncertainty does not show a clear relationship with depth.

Using the segmented Kriging surrogate models, the distribution of overall propagation loss is predicted. The parameter setting and the total calculation time are shown in Table III.

Through the depth is fixed, we take as many sampling points as possible to simulate the acoustic propagation loss. For the 0.5Km level segment interval, we take 100 points to train the Kriging surrogate model and take 2000 Monte Carlo simulations for each estimation point. The calculation of uncertainty analysis is linear and the

total computational time is within 1 s. Fig. 5- Fig. 7 show the distribution of the sound propagation loss with confidence intervals, where the red line is 90% confidence interval upper limit, blue line is 90% confidence interval lower limit. We determine the confidence interval by sorting the estimates from small to large, taking the 5% and 95% confidence curves [10].

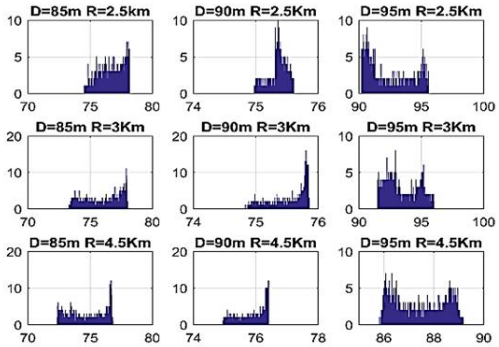

Figure 4. Frequency distribution.

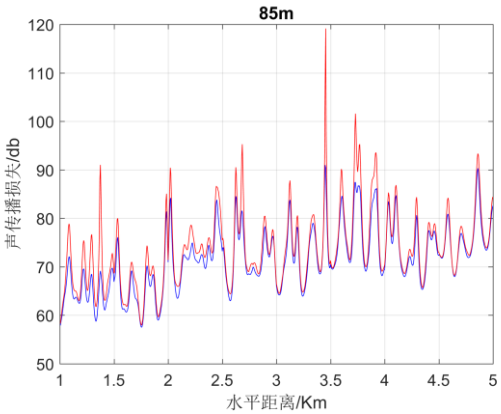

Figure 5. Overall confidence interval distribution at the depth of 85m.

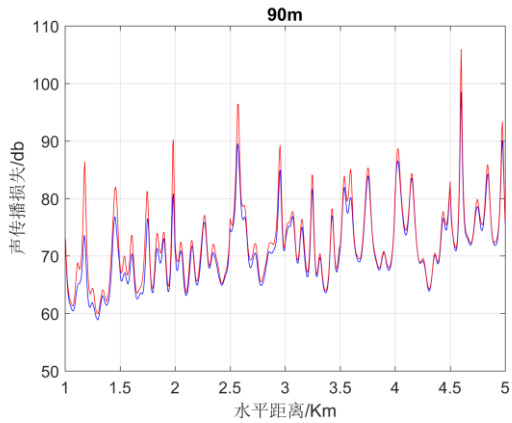

Figure 6. Overall confidence interval distribution at the depth of 90m.

TABLE III. PARAMETERS SETTING AND CALCULATION TIME

horizontal distance /Km	Depth /m	acoustic velocity m/s	Density g/cm^3	computing time
1-5	85/90/95	1800±10	2±1	149.24s

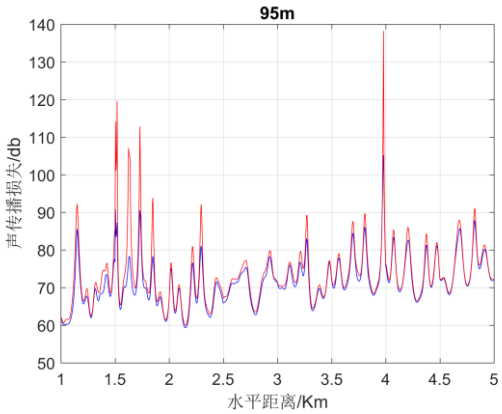

Figure 7. Overall confidence interval distribution at the depth of 95m.

IV. CONCLUSION

In this paper, the Kriging surrogate model is employed to replace the classical Kraken acoustic propagation model to significantly reduce the computational cost but ensuring the accuracy. Based on the Kriging surrogate model, an efficient Kriging integrated Monte Carlo simulation method is developed to quantify the uncertainties of the underwater sound propagation loss in terms of probability distribution and confidence intervals. . Thanks to the proposed method, the total calculation time is reduced by about four orders of magnitude, and the calculation accuracy is still kept. The experiment results also validate the effectiveness of the developed method, which is also of great practical significance.

ACKNOWLEDGMENT

This study was funded by the National Key R & D Program of China (No.2016YFC1401800), the National Natural Science Foundation of China (No.61379056), the Scientific Research Project of NUDT (No.ZK16-03-31, No.ZK16-03-46) and the Aerospace Support Technology Fund of China (2014).

REFERENCES

[1] S. N. Lophaven, H. B. Nielsen, J. Søndergaard, "DACE-A Matlab Kriging toolbox," version 2.0, 2002.

[2] G. L. Cheng and M. M. Zhang, "A fast and universal algorithm for an uncertain acoustic filed in shallow-water," *Acta Physical Sinica*, vol. 63, no. 8, pp. 196-203, 2014.

[3] A. R. Robinson, P. Abbot, P. F. J. Lermusiaux, *et al.*, "Transfer of uncertainties through physical-acoustical-sonar end-to-end systems: A conceptual basis," in *Impact of Littoral Environmental Variability of Acoustic Predictions and Sonar Performance*, Springer Netherlands, 2002, pp. 603-610.

[4] H. Q. Zhang, W. Gao, and H. Z. Wang, "Uncertainty of inverted geoacoustics parameters to sound transmission loss estimation," *Journal of System Simulation*, vol. 21, no. 15, pp. 4907-4911, 2009.

[5] J. P. C. Kleijnen, *Design and Analysis of Simulation Experiments*, 2nd Ed. Springer, 2015.

[6] T. Lu and M. Lin, "Model comparison for water interpolation based on Kriging method," *Science Technology and Engineering*, vol. 15, no. 4, pp. 135-139, 2015.

[7] J. Martin and T. Simpson, "A Monte Carlo simulation of the Kriging model," in *Proc. Aiaa/issmo Multidisciplinary Analysis and Optimization Conference*, New York, 2013.

[8] M. B. Porter, "The kraken normal mode program," Naval Research Lab Washington DC, 1992.
[9] S. Jeong, M. Murayama, and K. Yamamoto, "Efficient optimization design method using Kriging model," *Journal of Aircraft*, vol. 42, no. 2, pp. 413-420, 2005.
[10] A. M. Simundic, "Confidence interval," *Biochemia Medica*, vol. 18, no. 2, pp. 154-161, 2008.

Xianpeng Guo is the author of this article enrolled in the Ocean University of China, the study is marine science, and successfully obtained a Bachelor of Science degree Graduates studying at the National University of Defense Technology, specializing in computer science and technology. The main research direction is underwater acoustic engineering.

Dezhi Wang is a research assistant of National University of Defense Technology, Doctoral Degree, Ph.D. in engineering, University of Liverpool, UK, The main research areas are Uncertainty analysis, underwater acoustic signal processing, deep learning.

Lilun Zhang is a research fellow of National University of Defense Technology, Doctoral Degree, Ph.D. in engineering, National University of Defense Technology, Research Supervisor. The main research areas are large-scale numerical parallel algorithms, load balancing algorithms and high performance computing applications.