The Modified Model of Unknown Correlated Noise Fields under the Polarization Sensitive Array

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***Abstract*—Currently for the polarization vector array of unknown noise covariance matrix still use scalar matrix model, resulting in low precision of source number estimation Estimation and a series of problems. This paper defines the polarization sensitive array under unknown correlated noise field of the noise covariance matrix, it is derived under the assumption that the spatial noise covariance is block diagonal or banded, covariance matrix of the original noise model is modified. Three models are compared based on scalar matrix, polarization sensitive vector array, modified polarization array vector array underlying unknown correlated noise field. The simulation results of MATLAB show that the correct detection probability of the modified model can be significantly improved. Furthermore the analysis of noise suppression is envisaged due to the fact that modified noise covariance matrix has Toeplitz structure.**

***Index Terms*—polarization sensitive array, unknown correlated noise, modified covariance matrix, estimation of source number**

I. Introduction

The source number estimation method based on information theory, such as the AIC criterion, MDL criterion and EDC criterion, is the premise of the white noise model, which is not applicable to the unknown correlated noise model. Solutions for unknown correlated noise background, literature [1]-[4] were used prior knowledge, diagonal loading technique, the multistage Wiener filter, modify function criterion method improved the MDL criterion, literature [5] proposed uses the characteristics of polarization sensitive array to feature processing method for source number estimation, but these are not binding polarization sensitive array of unknown noise covariance matrix correction, still using the literature [6] mentioned in unknown correlated noise model.

Polarization sensitive array signal processing is widely concerned, but its covariance matrix in the unknown correlated noise field still needs to be redefined. The determination of the number of sources in the array is a necessary condition for the application of many signal parameter estimation methods. Scalar matrix of each antenna placed in different ways, polarization sensitive array consists of two mutually perpendicular electric dipole, the part is located in the same spatial location and has the same phase center, to form a plurality of electromagnetic vector sensor, the correlation between the antenna changed. Using the characteristic of polarization sensitive array to reconstruct the covariance matrix of the noise, the correct estimation of the source number can be effectively improved.

This paper is summarized as follows. In the second section, the source number estimation criterion of unknown correlated noise fields is introduced, and the model of polarization sensitive array signal is used in this paper. In the third section, the simulation analysis of the unknown correlated noise scalar matrix, unknown correlated noise field polarization vector matrix and modified unknown correlated noise under polarization vector array of three model of source number estimation probability of correct, and draw the conclusion after correction model to estimate the probability can be significantly improved correctly. The fourth section summarizes the conclusion, and puts forward some ideas for the suppression of the special unknown correlated noise model.

II. Theoretical Derivation

The model of uniform linear array composed of polarization sensitive array is shown in Fig. 1. The space uniform linear array (ULA) is composed of N polarization sensitive array elements, the polarization sensitive array element is composed of two dipoles which are composed of 2 dipoles along the axis direction and the direction of the axis, Among them, is the array element spacing, and is the incident angle.

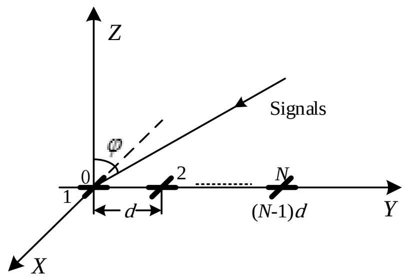

Figure 1. Structure of polarization sensitive array.

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In order to analyze conveniently and without losing generality, the direction of arrival of all the incoming signals is restricted to the YOZ plane. Polarization state constraints in the Poincare sphere η=90° circle track, that all the incoming signal to meet η=90° , one-dimensional polarization sensitive uniform linear array the array steering vector, as in (1).

$$\boldsymbol{a}(\theta,\gamma)=\boldsymbol{a}_S(\theta)\otimes\boldsymbol{a}_p(\theta,\gamma) \quad (1)$$

where

"$\otimes$" denotes the Kronecker product;

$\boldsymbol{a}_p(\theta,\gamma)=$

$[a_{px}(\theta,\gamma)\ \ a_{py}(\theta,\gamma)]^{\mathrm{T}}=[-\cos(\gamma)\ \ \mathrm{j}\cos(\theta)\sin(\gamma)]^{\mathrm{T}}$

denotes the polarization steering vector;

$a_{px}(\theta,\gamma)$ and $a_{py}(\theta,\gamma)$ respectively denote the 2 orthogonal components of the polarization steering vector;

$\boldsymbol{a}_S(\theta)$ denotes spatial vector, as in (2).

$$\boldsymbol{a}_S(\theta)=\left[1\ \ \exp(-\mathrm{j}\frac{2\pi}{c}d\sin\theta)\ \ \cdots\ \ \exp(-\mathrm{j}\frac{2\pi}{c}(N-1)d\sin\theta)\right]^{\mathrm{T}} \quad (2)$$

where

c denotes the speed of electromagnetic wave propagation;

f denotes the frequency of incident signal.

Let $D(D\le N)$ signals impinging on the ULA, and the array receiving parameters, as in (3).

$$\boldsymbol{X}(t)=\boldsymbol{A}(\theta,\gamma)\boldsymbol{S}(t)+\boldsymbol{N}(t) \quad (3)$$

where

$\boldsymbol{X}(t)=[x_1(t)\ \ x_2(t)\ \ \cdots\ \ x_{2N}(t)]^{\mathrm{T}}$ denotes array received data vector that is a 2N*1 dimensional array;

$\boldsymbol{S}(t)=[s_1(t)\ \ s_2(t)\ \ \cdots\ \ s_D(t)]^{\mathrm{T}}$ denotes signal vector that is a D*1 dimensional array;

$\boldsymbol{N}(t)=[n_1(t)\ \ n_2(t)\ \ \cdots\ \ n_{2N}(t)]^{\mathrm{T}}$ denotes noise vector that is a 2N*1 dimensional array;

$\boldsymbol{A}(\theta,\gamma)=[\boldsymbol{a}(\theta_1,\gamma_1)\ \ \boldsymbol{a}(\theta_2,\gamma_2)\ \ \cdots\ \ \boldsymbol{a}(\theta_D,\gamma_D)]$ denotes array oriented matrix that is a 2N*D dimensional array.

When the array signal model is unknown, the covariance matrix of the array receiving data is as in (4).

$$\boldsymbol{R}=\mathrm{E}[\boldsymbol{X}(t)\boldsymbol{X}^{\mathrm{H}}(t)]=\boldsymbol{A}\boldsymbol{R}_S\boldsymbol{A}^{\mathrm{H}}+\boldsymbol{R}_N \quad (4)$$

where

$\boldsymbol{R}_N=E[\boldsymbol{N}(t)\boldsymbol{N}^{\mathrm{H}}(t)]$ denotes noise covariance matrix.

It is the diagonal elements of the diagonal elements when the array noise is white noise, as in (5).

$$\boldsymbol{R}_{N\,w}=\sigma^2\boldsymbol{I} \quad (5)$$

The rank of the signal covariance matrix is determined by the information theory. Assuming that $\boldsymbol{X}(t_1)$, $\boldsymbol{X}(t_2)$, …, $\boldsymbol{X}(t_M)$ is a set of independent and identically distributed observation data. Joint probability density function of observation vectors using covariance matrix, as in (6).

$$f(\boldsymbol{X}(t_1),\boldsymbol{X}(t_2),\ldots,\boldsymbol{X}(t_M)\,|\,\Theta)=\prod_{i=1}^{M}\frac{1}{\pi^{2N}\det(\boldsymbol{R})}\exp[-\boldsymbol{X}^{\mathrm{H}}(t_i)\boldsymbol{R}^{-1}\boldsymbol{X}(t_i)] \quad (6)$$

where

Θ denotes the parameter vector of the model;

M denotes the length of observation data.

A Minimum Description Length (MDL) criterion is used to detect the number of targets, as in (7).

$$\mathrm{MDL}=-\log f(x\,|\,\hat{\Theta})+\frac{1}{2}k\log M \quad (7)$$

where

$\hat{\Theta}$ denotes the maximum likelihood estimation of parameter vector;

k denotes the degree of freedom of parameter vector.

However, it is not equal to the diagonal elements when the array noise is unknown, as in (8).

$$r_n(i,k)=\sigma_n^2\rho^{|i-k|}\exp[\mathrm{j}(i-k)\phi\pi] \quad (8)$$

where

$i,k=1,2,3,\cdots,2N$, and where σ_n^2 denotes power of unknown correlated noise;

$\rho\in[0,1]$ denotes spatial correlation coefficients between adjacent elements;

ϕ denotes any real number.

That is

$$\boldsymbol{R}_{N\,uc}=\sigma_n^2\begin{bmatrix}1 & \rho\,\mathrm{e}^{\mathrm{j}\phi\pi} & \cdots & \rho^{2N-1}\mathrm{e}^{\mathrm{j}(2N-1)\phi\pi}\\ \rho\,\mathrm{e}^{-\mathrm{j}\phi\pi} & 1 & \cdots & \rho^{2N-2}\mathrm{e}^{\mathrm{j}(2N-2)\phi\pi}\\ \vdots & \vdots & & \vdots\\ \rho^{2N-1}\mathrm{e}^{-\mathrm{j}(2N-1)\phi\pi} & \rho^{2N-2}\mathrm{e}^{-\mathrm{j}(2N-2)\phi\pi} & \cdots & 1\end{bmatrix} \quad (9)$$

The unknown correlated noise field is modified in the polarization sensitive array, as in (10).

$$\boldsymbol{R}_{N\,new}=\boldsymbol{I}_2\otimes\boldsymbol{R}_{N\,uc} \quad (10)$$

where $\boldsymbol{I}_2$ denotes unit array of 2 rows and 2 columns.

To expand the matrix, as in (11)

$$\boldsymbol{R}_{N\,new}=\sigma_n^2\begin{bmatrix}1 & 1 & \rho\,\mathrm{e}^{\mathrm{j}\phi\pi} & \rho\,\mathrm{e}^{\mathrm{j}\phi\pi} & \cdots & \rho^{2N-1}\mathrm{e}^{\mathrm{j}(2N-1)\phi\pi} & \rho^{2N-1}\mathrm{e}^{\mathrm{j}(2N-1)\phi\pi}\\ 1 & 1 & \rho\,\mathrm{e}^{\mathrm{j}\phi\pi} & \rho\,\mathrm{e}^{\mathrm{j}\phi\pi} & & \rho^{2N-1}\mathrm{e}^{\mathrm{j}(2N-1)\phi\pi} & \rho^{2N-1}\mathrm{e}^{\mathrm{j}(2N-1)\phi\pi}\\ \rho\,\mathrm{e}^{-\mathrm{j}\phi\pi} & \rho\,\mathrm{e}^{-\mathrm{j}\phi\pi} & 1 & 1 & \cdots & \rho^{2N-2}\mathrm{e}^{\mathrm{j}(2N-2)\phi\pi} & \rho^{2N-2}\mathrm{e}^{\mathrm{j}(2N-2)\phi\pi}\\ \rho\,\mathrm{e}^{-\mathrm{j}\phi\pi} & \rho\,\mathrm{e}^{-\mathrm{j}\phi\pi} & 1 & 1 & & \rho^{2N-2}\mathrm{e}^{\mathrm{j}(2N-2)\phi\pi} & \rho^{2N-2}\mathrm{e}^{\mathrm{j}(2N-2)\phi\pi}\\ \vdots & & \vdots & & & \vdots & \\ \rho^{2N-1}\mathrm{e}^{-\mathrm{j}(2N-1)\phi\pi} & \rho^{2N-1}\mathrm{e}^{-\mathrm{j}(2N-1)\phi\pi} & \rho^{2N-2}\mathrm{e}^{-\mathrm{j}(2N-2)\phi\pi} & \rho^{2N-2}\mathrm{e}^{-\mathrm{j}(2N-2)\phi\pi} & \cdots & 1 & 1\\ \rho^{2N-1}\mathrm{e}^{-\mathrm{j}(2N-1)\phi\pi} & \rho^{2N-1}\mathrm{e}^{-\mathrm{j}(2N-1)\phi\pi} & \rho^{2N-2}\mathrm{e}^{-\mathrm{j}(2N-2)\phi\pi} & \rho^{2N-2}\mathrm{e}^{-\mathrm{j}(2N-2)\phi\pi} & & 1 & 1\end{bmatrix} \quad (11)$$

After the new noise model is redefined, it is more accurate to describe the antenna correlation in the polarization sensitive array, so that it can improve the accuracy of the following problems.

III. Step Summary

Taking the source number estimation as an example, this paper shows that the improvement of the accuracy of the detection probability can be significantly improved after the noise model is modified in the unknown correlated noise field. According to the previous theoretical analysis, the implementation steps of this algorithm are as follows:

- Established the polarization sensitive array signal model, as in (3).
- Estimated covariance matrix as in (4), in which the noise covariance matrix as selection of different models.
- Assume that the scalar array is unknown and correlateded with the noise covariance matrix as in (9). In such a case, it is based on the information theory, the model with the minimum description length is selected as the estimation result by using the MMDL criterion.
- In the second case, the scalar array is replaced by the polarization sensitive array, however, the noise covariance matrix is still as in (9).
- In the third case, Assume that the polarization sensitive array is unknown and correlateded in which the noise covariance matrix as in (10).
- Comparison of three different algorithms to obtain the accurate detection probability.

In this paper, we assume that the array receiving noise is unknown, and the noise covariance matrix of the polarization sensitive array is used to improve the accuracy of the model.

IV. Numerical Examples and Concluding Remarks

The section presents some examples that illustrate the performance and verifies some theoretical claims in the previous section.Simulation conditions are compared with the source number estimation of three kinds of noise models. First is unknown correlateded noise [6] of 8 elements uniform linear array used MMDL criterion for source number estimation (MMDL); the second is unknown correlated noise on the polarization sensitive array used MMDL source number estimation (PMDL); the third for the correction of unknown correlated noise model polarization sensitive array used MMDL source number estimation (MPMDL).

The polarization sensitive array is uniform linear array with 8 array element, polarization sensitive array element for orthogonal dipole, array element spacing is half of the wavelength of the incident signal, three narrow-band far-field incident signal to the antenna array, signal incident azimuth angle 10 °, 30 ° and 40 °, number of snapshots set 128, signal to noise ratio from 10 dB in steps of 0.5 dB change to 10dB, the simulation experiment of 100 times Monte Carlo experiment, the changing rule of the comparison of the three kinds of situations of correct detection probability with the signal-to-noise ratio, as shown in Fig. 2.

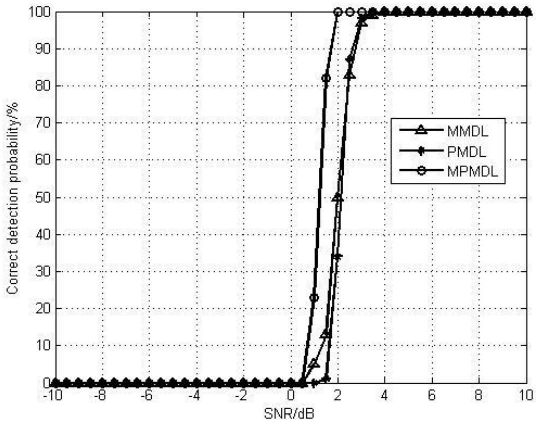

Figure 2. Performance of the MMDL, PMDL and MPMDL models.

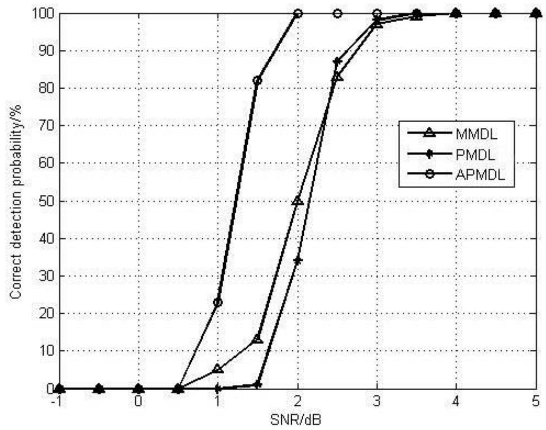

Figure 3. Performance of the MMDL, PMDL and MPMDL models.

The result shown in Fig. 2 is obtained for the cases with three sources and with the three models of unknown correlated noise fields when 128 snapshots of statistically independent data are used to estimate the number of impinging signals. Fig. 3 enlarged the curve of Fig. 2 in order to show the results more clearly.

The follwing comments on the results displayed in Figs. 2-3 are in order:

- A correct detection probability are with the signal-to-noise ratio increased increased, however, the polarization sensitive array to estimate the effect better.
- After correcting the noise model, polarization sensitive array source number estimation detection probability is better than the other two contrast model.
- When the number of snapshots for 128, for correction of unknown correlated noise model using MMDL criterion when the SNR is greater than 2dB correct detection probability can reach 100%. The detection probability of the three models is 100% corresponding to the SNR threshold of 4dB, 3.5dB, 2dB, respectively

V. SUMMARY

Under unknown correlated noise background and for three model proposed in this paper, the probability of correct detection are with the signal-to-noise ratio increased increased, after correcting the noise model, polarization sensitive array source number estimation detection probability is obviously superior to the other two contrasting models. After correcting the correlation of each antenna, the signal to noise ratio is improved obviously.

In the modified before and after the noise model and the noise covariance matrix has Toeplitz structure, in polarization domain of zero weighted smoothing, like literature [7], can achieve noise suppression effect.

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