DOA Estimation for a Passive Synthetic Array Based on Cross-Correlation Matrix

Wei Jiang
Science and Technology on Electronic Information Control Laboratory, Chengdu, China
Email: 15114086529@163.com

Guodong Qin and Jian Dong
School of Electronic Engineering, Xidian Univ., Xi'an, China
Email: gdqin@mail.xidian.edu.cn, 847274312@qq.com

***Abstract*—Obtaining higher bearing resolution with an aperture limited array has been more and more important in array signal processing. In this paper, a novel MUSIC algorithm based on cross-correlation matrix is proposed to obtain a high angle resolution by using passive synthetic aperture (PSA) array which is formed with a single antenna. To decrease the computational complexity, the noise subspace is calculated by orthogonal projector instead of eigendecomposition of covariance matrix. The proposed algorithm has a better angle resolution with the utility of the PSA technique. Meanwhile, numerical simulations indicate that the DOA estimation performance of the proposed algorithm is superior to the traditional MUSIC in estimation accuracy and computational complexity.**

***Index Terms*—passive synthetic aperture, DOA, virtual array, MUSIC**

I. Introduction

Direction of Arrival (DOA) estimation is a hotspot in array signal processing which can be applied into the field of sonar, radar, and communications [1]-[3]. As is all known in DOA, when employing conventional DOA estimation techniques, angle estimation resolution is proportional to the ratio of aperture length to signal wavelength. However the practical physical size of array is always limited, which makes it difficult to obtain a higher bearing resolution with a traditional algorithm. To obtain a higher resolution, many algorithms are proposed, including higher-order cumulants (HOC) [4], [5], array interpolation method [6] and linear extrapolation method [7], etc.

Application of synthetic aperture technique to passive sensors has received a sustained interest for more than twenty years. This technique exploits the array motion and the temporal coherence of narrow band received signals to build a large synthetic array, resulting in an improvement in beamforming performance compared with that of the short physical array. Recently, many researchers attempt to estimate DOA with the passive synthetic aperture (PSA) array. Autrey [8] proposes an algorithm with passive synthetic aperture array to estimate DOA. The single array element must be moving when it is used to form a PSA array in his algorithm. A Circular Passive Synthetic Array is proposed in [9] with the use of single array element. Meanwhile, the PSA is applied to detect the Indoor GNSS signal in [10]. The Extended towed-array measurements proposed by Yen and Carey [11] is a processing technique in the beam domain which provides coherent processing of subapertures by proper selection of a phase term based on knowledge or the use of a velocity filter concept for the source-receiver relative speed. FFTSA [12] method is same as the method presented by Yen and Carey, but it does not need the velocity information. It performs coherent processing of subaperture signals at successive time intervals in the beam domain via FFT transformations.

In this paper, the PSA technology with single antenna is applied to estimate 2-dimension DOA. The L-shaped virtual array in x-y plane is formed through the movement of the single antenna. And the signal model is established in Section II. A novel algorithm is proposed based on cross-correlation matrix to estimate 2-D angles in Section III. The proposed algorithm calculates the noise subspace by using orthogonal projector instead of eigendecomposition of the covariance matrix in MUSIC [13]. The numerical simulations in Section IV indicate that the DOA estimation performance of the proposed algorithm outperform the traditional MUSIC in estimation accuracy and computational complexity.

II. Geometric Model

In this paper, we suppose K narrow-band far-field signal sources with wavelength λ impinge on the synthetic array which is formed by a single sensor moving at a constant velocity of v, according to the motion trajectory in Fig. 1, where black dots represent to the synthetic virtual array elements. It's supposed that the far-field signal sources are static, and forming the PSA is a short-time process, so that signal angles are nearly invariant. The kth signal is located at position $(\theta_k,\varphi_k)(k=1,2,...,K)$ with $\theta_k(0\le\theta_k\le\pi)$ denoting the signal's azimuth angle, which is measured clockwise to the x-axis, and $\varphi_k(0\le\varphi_k\le\pi/2)$ denoting the elevation

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angle, which is measured clockwise to the *z*-axis. The unit length of coordinate axis is half-wavelength $\lambda/2 = c/2f$, then the time required for single antenna moving along the unit length of coordinate axis is denoted by $T_c = c/2fv$. Considering that space between synthetic array elements may be longer than half-wavelength, which leads to angle estimation ambiguity, the passive synthetic array is designed as Minimum Redundancy Array (MRA). The position vectors of synthetic array elements on the *x*-axis and *z*-axis are defined as *x* and *z*, respectively.

$$\begin{aligned} x &= [x_0, \cdots, x_m, \cdots, x_{P-1}] = [0,1,2,6,10,13]\lambda/2 \\ z &= [z_0, \cdots, z_m, \cdots, z_{P-1}] = [0,1,2,6,10,13]\lambda/2 \end{aligned} \tag{1}$$

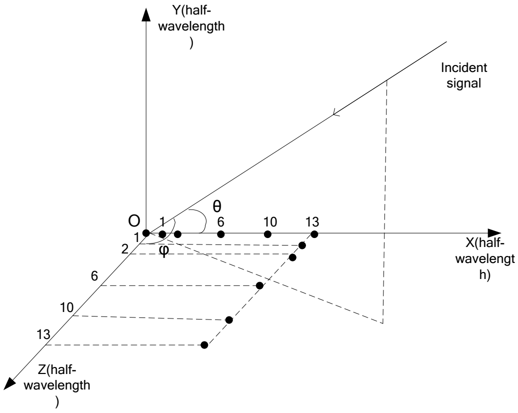

Figure 1. Three-dimensional DOA estimation PSA of a moving single sensor model.

III. ESTIMATION OF 2-D DOA

We divide the process of receiving signals into *M* segments, and there are *M*+1 synthetic virtual array elements in total. The beginning time of the *m*th $(m = 1, \cdots, M)$ segment is defined as T_m. $T_{m+1} - T_m$ stands for the interval between *m*th and $(m+1)th$ segment, and τ which is generally defined as the pulse width for pulse signals denotes the sampling time in *m*th segment. There are *P* (*M*=2*P*-2) synthetic array elements on the *x*-axis and *z*-axis respectively, as seen in Fig. 1. The signals of the moving single sensor on the *x*-axis and *z*-axis receiving from *K* signal sources at $t + T_m (t \in [0, \tau], m = 1, \cdots, M)$ are defined as $x_X(t+T_m)$ $(m = 1, ..., P-1)$, $x_Z(t+T_m)$ $(m = P, ..., \mathrm{M})$ respectively.

$$x_X(t+T_m) = \sum_{k=1}^{K}\left[A_k \exp\left(j(2\pi f_k(t+T_m) + 2\pi f_k \boldsymbol{x}_m \cos\theta_k / c + \phi_k + \varphi_k(t))\right)\right] + \varepsilon_X(t+T_m) \tag{2}$$

$$x_Z(t+T_m) = \sum_{k=1}^{K}\left[A_k \exp\left(j(2\pi f_k(t+T_m) + 2\pi f_k z_m \cos\varphi_k / c + \phi_k + \varphi_k(t))\right)\right] + \varepsilon_Z(t+T_m) \tag{3}$$

where A_k, f_k, ϕ_k and $\varphi_k(t)$ are amplitude, carrying frequency, initial phase and modulation phase of *k*th signal, respectively. $\varepsilon_X(t+T_m)$ and $\varepsilon_Z(t+T_m)$ are Gaussian white noise vectors with zeros mean and covariance σ^2. Rewriting (2) and (3) in the vector $x_X(t), x_Z(t)$ $(t \in [0, \tau])$, that is

$$\begin{aligned} x_X(t) &= \left[x_X(t+T_1), \cdots, x_X(t+T_{P-1})\right]^T \\ &= \boldsymbol{A}_X(\theta)\boldsymbol{s}(t) + [\varepsilon_X(t+T_1), \cdots, \varepsilon_X(t+T_{P-1})]^T \end{aligned} \tag{4}$$

$$\begin{aligned} x_Z(t) &= \left[x_Z(t+T_P), \cdots, x_Z(t+T_M)\right]^T \\ &= \boldsymbol{A}_Z(\varphi)\boldsymbol{s}(t) + [\varepsilon_Z(t+T_P), \cdots, \varepsilon_Z(t+T_M)]^T \end{aligned} \tag{5}$$

where the array manifolds on the *x*-axis and *z*-axis are respectively

$$A_X = \begin{bmatrix} e^{j2\pi f_1 x_1 \cos\theta_1/c} & \cdots & e^{j2\pi f_K x_1 \cos\theta_K/c} \\ \vdots & \ddots & \vdots \\ e^{j2\pi f_1 x_{P-1} \cos\theta_1/c} & \cdots & e^{j2\pi f_K x_{P-1} \cos\theta_K/c} \end{bmatrix} \tag{6}$$

$$A_Z = \begin{bmatrix} e^{j2\pi f_1 z_P \cos\varphi_1/c} & \cdots & e^{j2\pi f_K z_P \cos\varphi_K/c} \\ \vdots & \ddots & \vdots \\ e^{j2\pi f_1 z_M \cos\varphi_1/c} & \cdots & e^{j2\pi f_K z_M \cos\varphi_K/c} \end{bmatrix} \tag{7}$$

and,

$$\boldsymbol{s}(t) = [s_1(t), \cdots, s_K(t)] \tag{8}$$

where $s_k(t) = A_k \exp(j(2\pi f_k t + \varphi_k(t) + \phi_k))$.

Then we define a new $2P \times 1$ combined vector as

$$Y = \begin{bmatrix} x_X(\mathrm{t}) \\ x_Z(\mathrm{t}) \end{bmatrix} = A(\theta, \varphi)\mathrm{s}(t) + \varepsilon(t) \tag{9}$$

According to *Y*, we can deduce

$$R_{YY} = \begin{bmatrix} R_{XX} & R_{XZ} \\ R_{ZX} & R_{ZZ} \end{bmatrix} = \frac{1}{L}\sum_{t=1}^{L} YY^H = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \begin{matrix} \} K \\ \} 2P-K \end{matrix} \tag{10}$$

Then, we can get

$$\prod = Q(\mathrm{I}_{2P-K} - \mathrm{PP}^H(\mathrm{PP}\cdot\mathrm{PP}^H + \mathrm{I}_K)^{-1}PP)Q^H \tag{11}$$

where $PP = (G_1 G_1^H)^{-1} G_1 G_2^H$, $Q = \left[PP^T, -I_{2P-K}\right]^T$. Based on the $\prod$, we can get 2-dimensional MUSIC spectrum as following,

$$\mathrm{P}_{2\mathrm{DMUSIC}}(\theta, \varphi) = 1/\left[\boldsymbol{a}(\theta, \varphi)^H \prod\prod^H \boldsymbol{a}(\theta, \varphi)\right] \tag{12}$$

IV. SIMULATIONS AND ANALYSIS

A. Estimating Azimuth and Elevation of Target

For the first simulation, we suppose that there are *K*=4 uncorrelated targets with frequency $f_0 = 1GHz$, and the number of snapshots is 991 for the synthetic L-array formed by a moving single sensor at a constant velocity of v=300m/s. In the simulation of MUSIC algorithm, Beamspace MUSIC (BS-MUSIC) algorithm [14] and CCM-MUSIC algorithm, we define that the signals are located at $(\theta_1, \phi_1) = (4^\circ, 20^\circ)$, $(\theta_2, \phi_2) = (10^\circ, 26^\circ)$, $(\theta_3, \phi_3) = (16^\circ, 32^\circ)$ and $(\theta_4, \phi_4) = (22^\circ, 38^\circ)$, and the single sensor moves as seen in Fig. 1. Fig. 2 and Fig. 3 are DOA estimation results of CCM-MUSIC algorithm, where the angles of targets are distinguished precisely and correctly estimated.

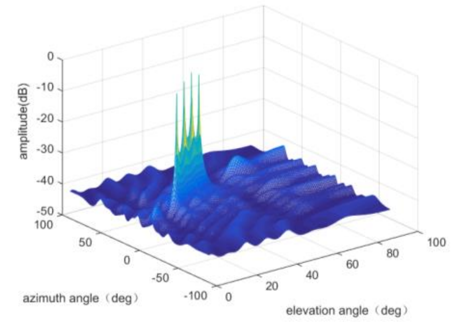

Figure 2. Three-dimensional graph of DOA estimation

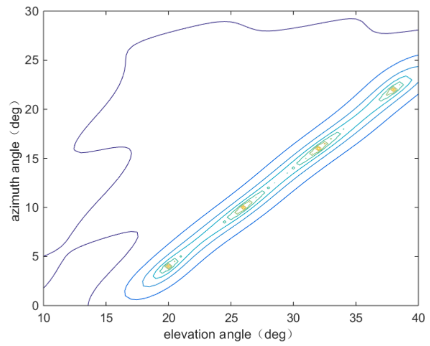

Figure 3. Contour plot of DOA estimation

B. *Root Mean Square Error (RMSE) of Angle Estimation with SNR and Snapshot*

Fig. 4 and Fig. 5 shows the relationship between RMSE of angle estimation and SNR (from 0dB to 20dB), while Fig. 7 and Fig. 8 shows the relationship between RMSE of angle estimation and snapshot (from 100 to 1000), in the MUSIC, BS-MUSIC and CCM-MUSIC algorithm, respectively. The other simulation conditions are the same as A. As seen in Fig. 4 and Fig. 5, when SNR is higher than 10dB, the RMSE of angle estimation of proposed algorithm is similar as MUSIC and BS-MUSIC algorithm, while the performance of MUSIC and BS-MUSIC is superior to the proposed algorithm in low SNR. As seen in Fig. 6 and Fig. 7, when the snapshot is lower than 900 the RMSE of angle estimation of CCM-MUSIC algorithm is better than that of MUSIC and BS-MUSIC algorithm.

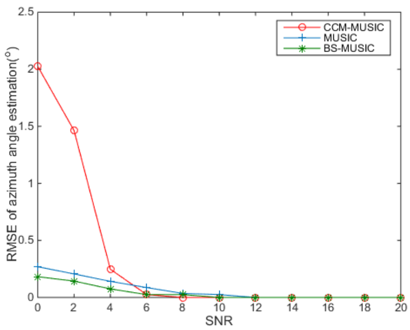

Figure 4. RMSE of azimuth angle estimation with SNR

C. *RMSE of Angle Estimation with Angle-Interval*

The relationship between RMSE of angle estimation of the fast algorithm and angle-interval (from 4^{o} to 12^{o}) is shown in Fig. 8 and Fig. 9. The other simulation condition is same as A. The estimation results indicate that the estimation accuracy becomes higher and higher with the angle-interval expanding, and the RMSE of angle estimation tends to zero when the interval is higher than 6^{o}. When the targets are too close below 6^{o}, MUSIC has a better performance than CCM-MUSIC, while BS-MUSIC performs worst.

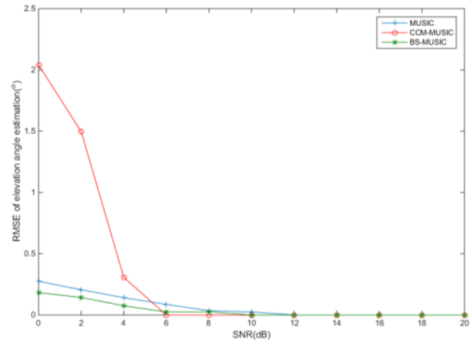

Figure 5. RMSE of elevation angle estimation with SNR

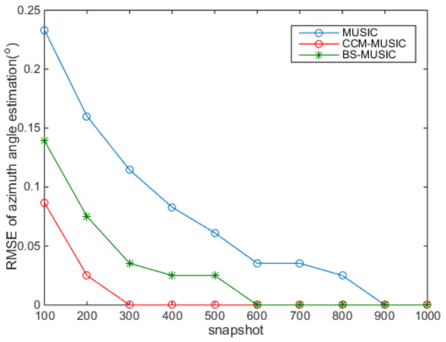

Figure 6. RMSE of azimuth angle estimation with snapshot

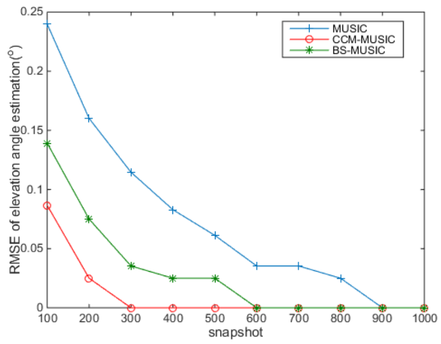

Figure 7. RMSE of elevation angle estimation with snapshot

D. *Runtime Comparing with MUSIC Algorithm*

Fig. 10 shows the relationship between runtime and snapshot(from 400 to 2000) with 2 signals, located at $(\theta_1,\phi_1)=(4^{\circ},20^{\circ})$, $(\theta_2,\phi_2)=(10^{\circ},26^{\circ})$, in BS-MUSIC algorithm, MUSIC algorithm and CCM-MUSIC algorithm. The other simulation conditions are the same as simulation A. From Fig. 10, we can see that the runtime becomes higher as snapshot increases, and the runtime in CCM-MUSIC algorithm is obviously lower than that in MUSIC algorithm and BS-MUSIC algorithm. Because beam number is given as 10, which is close to the antenna number, and the computational complexity of transforming matrix in BS-MUSIC contributes to the runtime in BS-MUSIC algorithm. As a result, the computational complexity of BS-MUSIC is higher than MUSIC.

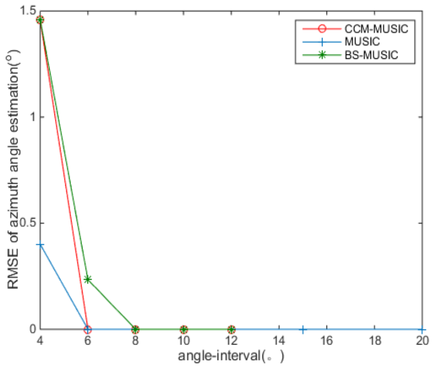

Figure 8. RMSE of azimuth angle estimation with angle-interval

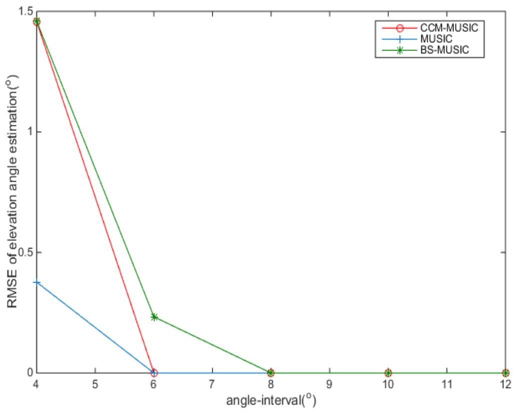

Figure 9. RMSE of elevation angle estimation with angle-interval

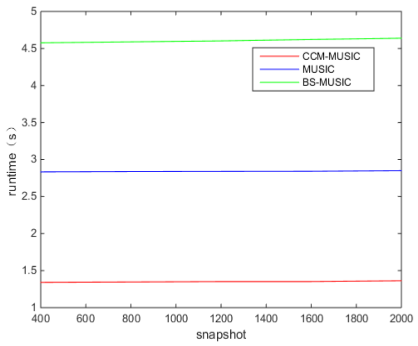

Figure 10. Runtime with snapshot

V. CONCLUSIONS

In this paper, a novel MUSIC algorithm based on cross-correlation matrix is proposed to obtain a high angle resolution by using passive synthetic aperture (PSA) array which is formed with a single antenna. The proposed algorithm has a better angle resolution with the utility of the PSA technique. Numerical simulations show that the proposed algorithm has a better angle resolution and a low computational complexity.

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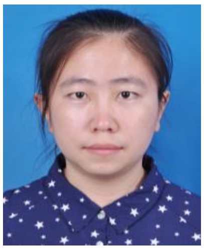
Wei Jiang received the Ph.D. degrees in communication and information engineering from Harbin Institute of Technology, Harbin, China, in 2011. He is working in Science and Technology on Electronic Information Control Laboratory. His research interests include array signal processing, passive direction-finding and location.

Guodong Qin received the M.S. and Ph.D. degrees in electrical engineering from Xidian University, Xi'an, China, in 2006 and 2009, respectively. He is currently a Lecturer with Xidian University. His research interests include DOA estimation, passive location and implementation of algorithms for array signal processing.



Jian Dong received the B.S. degrees in information engineering from Xi'an Jiaotong University, Xi'an, China, in 2015. She is currently a graduate student with Xidian University. Her research interests focus on DOA estimation.