Delayless Identification in ANC Systems Using Subband Adaptive Techniques

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Abstract—Subband techniques have been developed to use low order subfilters instead of full band higher order filters and consequently reduce the complexity and increase the convergence speed of the adaptive algorithm. In this paper the performance of two delayless subband adaptive algorithms for identification of an unknown system in an active noise control scheme are compared. This is carried out by using a common speech signal as the excitation input for identification of the secondary path model. The performances of the algorithms are measured in terms of the achieved minimum mean square error and misalignment error. The results are also compared to the time domain NLMS algorithm. The compared delayless structures are working in the closed loop form with DFT analysis filterbanks. Adaptation in the auxiliary loop and with help of weight transformation eliminates signal path delay and hence the unknown secondary path can be modelled accurately.

Index Terms—Active Noise Control (ANC), delayless subband adaptive filter, frequency domain adaptive filter

I. INTRODUCTION

Increase in the number of industrial equipment has made acoustic noise a major problem in modern societies. Although these are traditionally handled by using passive techniques like enclosures, barriers and silencers they are large, costly and ineffective at low frequencies. To overcome these shortcomings active noise control has been proved to be a viable technique.

A basic block diagram of the ANC system is shown in Fig. 1. The reference signal \( u(n) \) is fed to the adaptive filter by the reference microphone. Error microphone receives the superposition of primary noise from the noise source through unknown plant and secondary noise through the canceling loudspeaker. The residual noise shown as \( e(n) \) in Fig. 1 is used to update the weights of the adaptive filter [1]. Based on the principle of superposition the primary noise \( d(n) \) is canceled by the secondary noise of equal amplitude but opposite phase [2]. In most of real active noise control systems not only the primary noise has a non-stationary nature but also the surrounding environment is time varying and prone to change [3]-[5]. This necessitates development of an adaptive mechanism to cope with these two types of changes. One of the commonly used adaptive algorithms to update the filter coefficients in ANC systems is the so called Filtered-x Least Mean Square algorithm (Fx-LMS). In case that the incident noise is highly correlated and non-stationary it is shown that filtered-x LMS algorithm performs poorly [6]. On the one hand increase in the length of the adaptive filter results in increase in the complexity of the system, increased level of minimum mean square error, and reduction in convergence speed of the Least Mean Square (LMS) algorithm. On the other hand, FxLMS algorithm has limited capability in tracking non-stationary signals and its stability robustness is subject to changes based on the accuracy of the estimated model of the secondary path [7]-[9]. To be able to address the problems with respect to inaccuracies of the secondary path online identification of secondary path is proposed [10]-[12]. Using robust control approaches is proved to be effective in dealing with uncertainties in the secondary path [13], [14]. To improve the convergence speed of the filtered-x algorithms RLS-based approaches is proposed [15], [16]. Introduction of IIR filters and fast array implementation of the RLS algorithm will reduce computational complexity of the algorithm substantially [17], [18]. Another approach to overcome the above mentioned problems with respect to the properties of the incident noise is to use subband signals. The frequency contents of the subband signals concentrate on the frequency range corresponding to the passband of the analysis filter [1]. Further downsampling of subband...
signals results in reduced spectral dynamic range. At the same time the length of the adaptive filters in subband techniques is shorter allowing larger step sizes to be used and again increase the convergence rate.

Early approaches in subband structures relied on overlapping filterbanks and critical subsampling [19]. However, it resulted in aliasing components in the output. In [20], non-overlapping filterbanks were introduced but it resulted in spectral gaps. Subband structure based on the polyphase decomposition is introduced in [21]. Filterbank structures with critical sampling of subband signals with sparse subfilters have been discussed in [22]. It results in better convergence behavior. In [23], adaptive filtering in subbands has been discussed for computational savings and better convergence rate. Adaptive cross filter between the subbands have been employed for the perfect reconstruction. Adaptive filtering at a lower decimation rate, due to subband processing, reduces the computational complexity. Also, the reduction of spectral dynamic range in each subband leads to faster convergence. However, the main anomaly of such kind of algorithm is the delay introduced in the signal path due to bandpass filters in the subband used to derive bandpass signals. The structure developed in [24], reduces the delay significantly. Here adaptive weights are calculated in subband domain and then collectively transformed into the full band filter coefficients. An additional advantage of this technique is reduction in the aliasing effects. An improvement of this structure by introducing the fractional delays in the polyphase component of the prototype filter is proposed in [25]. This eliminates the need for adaptive cross filters and hence the unknown system is modeled more accurately in a closed loop scheme.

In this paper the problem of identification of secondary path in ANC systems using delayless subband adaptive filtering is investigated. To avoid injection of auxiliary noise in ANC applications it is highly desirable to identify the secondary path using the existing music or speech signals. Due to the non-stationary nature of these signal adaptation of filter weights is challenging task. Here the performances of two adaptive subband filters for such applications in terms of speed of convergence, achievable minimum mean square error, and computational complexity are compared. The remainder of the paper is organized as follows. In Section II, the structure of the delayless frequency domain adaptive system identification is introduced. Different weight transformation schemes and computational complexity of different algorithms are compared in Section III. Simulation results are presented in Section IV and finally concluding remarks are given in Section V.

II. DELAYLESS FREQUENCY DOMAIN ADAPTIVE FILTERING

The block diagram of the delayless frequency domain adaptive sound control system is shown in the Fig.2. The adaptive filter \(W(z)\) is implemented directly in the time domain to avoid delay caused by collecting \(N\) samples. The convolution is performed by multiplication in the frequency domain. It is to be noted that although full band filter weights is in the time domain all the filter updating is performed in the frequency domain. In Fig. 1 \(P(z)\) in the block diagram represents the transfer function from the noise source to the error source. Convolution of the reference signal \(x(n)\) with the primary path impulse response, gives the desired signal \(d(n)\). The length of the primary adaptive filter is \(L\). The reference signal in the secondary path is decomposed into subband signals by using polyphase FFT. A DFT filter bank, is constructed from the \(K\) length prototype filter by modulation. The analysis filters of an \(M\)-channel DFT filter bank are obtained as:

\[
H_i(z) = H(ze^{-j2\pi i/M}), i = 0, 1, \ldots M - 1
\]

where, \(H(z)\) is the real valued prototype low pass filter with a cutoff frequency of \(
\pi/M\). Shifting of low pass filters to the right by the multiples of \(2\pi/M\) gives the complex modulated bandpass filters. The impulse response coefficients of \(H_i(z)\) and \(H_{M-1}(z)\) are complex conjugates of each other. Therefore, for real valued signals only the first \(M/2+1\) subbands need to be processed. The pseudo error signal is also decomposed into number of subbands using the same DFT filter bank as above. Here, \(e(n)\) is the residual noise from the error sensor. The weight adaptation is applied on the subfilters using the subband signals \(u_i,D(k)\) and \(e_i,D(k)\). A subband regressor \(u\) for the subfilters \(w_i(k)\) of length \(M_s\) is defined as follows:

\[
u_i(k) \equiv [u_i,D(k), u_i,D(k - 1), \ldots u_i,D(k - M_s + 1)]^T
\]

for \(i=0,1,\ldots M-1\), where \(D=M/2\) is the decimation factor. Each column of \(u\) holds a subband regression vector. The use of FFT to decompose the signals into subbands leads to significant amount of computational savings; however, it introduces circular convolution and circular correlation. This can be further nullified by overlapping of input samples.

![Figure 2. Delayless active sound control using subband.](image)

Delayless subband system eliminates the signal path delay caused by the analysis and synthesis filter banks. The fullband filtered reference signal and the pseudo error signal is decomposed into number of subbands...
using analysis filters in the DFT filterbanks. All the subband signals are downsamples by the decimation factor \( D \). Subband weight adaptation is done by closed loop feedback mechanism. The fullband error signals are fed to subband adaptive filters which finally converges to optimal Weiner solution. The filter weights in each subband are adjusted using complex normalized LMS algorithm defined as:

\[
\hat{w}_i(k + 1) = \hat{w}_i(k) + \frac{\mu u_i^*(k)}{|\hat{u}_i(k)|^2} e_i(k)
\]  

(3)

here, \( \mu \) is the step size for the adaptation algorithm. Its value affects the convergence speed, steady state error and stability of the adaptive filter. The subbands adaptive weight vector of \( mth \) subband are defined as:

\[
w_m(n) = [w_{m_0}(n) \ w_{m_1}(n) \ldots w_{m_{M-1}}(n)]^T
\]  

(4)

These weights are then transformed from subband to fullband by weight transformation scheme. The filter weights are transformed into frequency domain by \( M \) point FFT; this results in

\[
W(k) = \text{FFT}[W(k)] = [W_0(k) \ W_1(k) \ldots W_{M-1}(k)]^T
\]  

(5)

where \( W_i(k) \) is the subband adaptive weights. These weights are properly stacked and then inverse transformed every \( N \) samples to get the wide band filter coefficients [1].

\[
w(k) = \text{IFFT}[W(k)] = [w(kN - 1)w(kN - 2) \ldots w(kN - N)]^T
\]  

(6)

### III. DIFFERENT WEIGHT TRANSFORMATION SCHEMES

The weight transformation is greatly dependent on the characteristics of the analysis filter bank used for the subband decomposition. Two different weight transformation schemes are explained below.

#### A. Frequency Sampling Method [25]

In this method DFT filterbank consists of complex modulated bandpass filters. As the subband signals are complex valued, subband adaptive weights are also complex valued. Weight transformation maps the complex subband tap weights into an equivalent set of real valued full band tap weights. The weight transformation consists of the following steps:

1. For the first \( M/2+1 \) subbands the weight vectors are transformed by the FFT to obtain \( M \) real DFT coefficients for each subband.

2. The DFT coefficients obtained above are stacked to form the first \( L/2 \) points of an \( L \) element vector from index 0 to \( L/2-1 \). It is completed by setting \( L/2 \)th point to zero and then using the complex conjugate values from index 1 to \( L/2-1 \) in reversed order. The inverse FFT of the \( L \) element vector gives the fullband tap weights.

The frequency stacking rules are listed as follows:

1. For \( l \in \{0; L/2-1\}, W(l) = W_p(q) \), where \( W(l) \) and \( W_p(q) \) denote the FFT coefficients of the fullband filter and the \( pth \) subband filter, respectively; \( p = \lfloor ML/L \rfloor \) where \( \lfloor . \rfloor \) denotes the rounding towards the nearest integer; and \( q = l_{2l/M} \), where \( a_b \) denotes a modulus \( b \).

2. For \( l/L/2, W(L/2) = 0 \)

3. For \( l \in \{L/2 + 1; L - 1\}, W(l) = W(L - 1)^* \).

#### B. DFT Filter Bank with Fractional Delays [26]

Weight transformation for critically decimated subband adaptive filtering can be done by using DFT filterbank by using lowpass prototype filter as \( M \)th band filter. Analysis DFT filter bank with fractional delays can be obtained by using last polyphase component as 

\[
E_{N-1}(z) = z^{-\Delta_{\text{int}}} \]

where \( \Delta_{\text{int}} \) denotes the integer part of the delay. Also, the length of the adaptive subfilters needs to be increased by one sample for accurate modelling of the unknown system. The subband tap weights to full band weight transformation can be done through the following steps:

1. Compute an \( N \) point IFFT on each of \( M \) columns of the matrix formed by impulse response of the adaptive subfilters \( W_i(z) \). This result gives the impulse response of the fractionally delayed polyphase component \( G_i'(z) = G_i(z)z^{\Delta_{\text{int}}} \), where \( G_0(z), \ldots, G_{N-1}(z) \) are the polyphase component of the fullband filter \( W(z) \).

2. Take \( G_0(z) = G_0'(z) \), for the first polyphase component. For the consequent components the impulse response of \( G_i'(z) \) is convolved with the fractional delay \( E_{i-1}(z) \) as:

\[
G_i(z)z^{-\Delta_{\text{int}}+1} = G_i'(z)E_{i-1}(z)
\]  

(7)

3. Discard the first polyphase component, discard the last sample and for the subsequent samples discard the first \( \Delta_{\text{int}} + 1 \) samples and retain the next \( M - 1 \) samples. The fullband filter can be constructed from these polyphase components as:

\[
W(z) = \sum_{i=0}^{N-1} G_i(z^{N})z^{-1}
\]  

(8)

The computational complexity of the delayless subband structure can be divided into these parts:

- Filter bank operations
- Subband weight adaptation
- Fullband filtering
- Weight transformation

### TABLE I. COMPUTATIONAL COMPLEXITIES OF CLOSED LOOP DELAYLESS SUBBAND ADAPTIVE FILTER STRUCTURE

<table>
<thead>
<tr>
<th>Sections</th>
<th>Morgan and Thi</th>
<th>Merched et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter bank operations</td>
<td>( 4L^2/M + \log_2(M) )</td>
<td>( 2L^2/M + \log_2(M) )</td>
</tr>
<tr>
<td>Subband weight adaptation</td>
<td>( L/M )</td>
<td>( L/M )</td>
</tr>
<tr>
<td>Fullband filtering</td>
<td>( L )</td>
<td>( L )</td>
</tr>
<tr>
<td>Weight transformation</td>
<td>( 2\log_2(2L/M) + \log_2(L) )</td>
<td>( \left(1 + \frac{M}{L} \right)\log_2(1 + \frac{K(M - 1)}{M^2}) )</td>
</tr>
</tbody>
</table>
The detailed breakdown of computational requirements of two weighting transformation schemes is listed in Table I. In both algorithms by increase in the number of subbands delayless structure impose less computational burden. Besides, computational complexity for Merched algorithm [25] is less than Morgan algorithm [26]. In Table I, $K$ is length of analysis filterbanks, $M$ is number of subbands, $L$ is length of fullband filter, and $J$ is the value ranges from 1 to 8.

IV. SIMULATION RESULTS

Simulations were done in MATLAB to verify the performance of the proposed delayless subband identification algorithm. Two different types of signals were used as the excitation signal to identify the secondary path. Speech signal with sampling frequency of 16KHz was considered for this purpose. The reason for selection of this signal type is that, unlike white noise, speech signal can be sent through the secondary loudspeakers in an integrated audio and ANC system without disturbing the listeners. This aids to avoid injection of auxiliary annoying noise during the operation of system. Further to that it is worthy to note that this signal has a non-stationary nature that makes adaptive identification of secondary path a challenging task. The plot of spectrum of speech signal in Fig. 3 shows that the energy of the signal is mostly concentrated around lower frequencies.

![Single-sided Magnitude spectrum (Hertz)](image)

Figure 3. Single sided amplitude spectrum of speech.

The primary and secondary path transfer functions $P(z)$ and $S(z)$ are chosen as the state space models identified using the techniques described in [27]. These models are then transferred into respective transfer functions. The reference signal $x(n)$ in Fig. 2 is subjected to polyphase FFT in a block of 1024 samples in each iteration. The prototype low pass filter used to achieve DFT filterbank has the order of 255. It has passband edge frequency at 6 KHz and cutoff frequency of $\pi/8$. The band pass filters in the subbands are the frequency shifted version of the prototype low pass filter. The length of the full band adaptive weight vector is 1024. $M=32$ subbands filter bank is used with each subband having 8M weight vectors. The value of the stepsize $\mu$ has been set differently for different structures. For Merched et al., it has been set to 0.3, however, the stepsize for Morgan structure is set to be 0.2 for the speech signal. Step sizes are chosen such that fastest possible response is achieved before the adaptive algorithm goes unstable.

The minimum mean square error of the identification error after convergence of the adaptive algorithms for the speech signal compared to the time-domain NLMS algorithm are plotted in Fig. 4 and Fig. 5. The advantage of using the subband structure is that one achieves a faster convergence rate compared to full band NLMS scheme. In the open loop structure MSE is high as the fullband error signal is not minimized. Therefore, it is significantly reduced by closed loop subband structure. To compare the algorithms in terms of the speed of convergence the normalized misalignment calculated as the norm of the adaptive weight vectors below:

$$Normalized\ misalignment = 20\log_{10}\frac{\|w(k) - b\|}{\|b\|} \quad (9)$$

where $b$ is the optimum weights. The learning curves for both algorithms by plotting the normalized misalignment
in front of speech signal are shown in Fig. 6. The results are compared with respect to the time-domain NLMS algorithm. As can be seen, Merched algorithm performs much better than the two others in terms of achievable minimum mean square error and speed of convergence.

![Normalized misalignment learning curves for the speech signal.](image)

**Figure 6.** Normalized misalignment learning curves for the speech signal.

**V. CONCLUSION**

Analysis and synthesis filter banks are essential parts of the subband adaptive filtering. However, they tend to increase the overall signal path. In this paper, the performances of two frequency domain delayless adaptive filters for identification of the secondary path in an ANC system are compared. Two common types of audio signals, i.e. music and speech signals are used for this purpose. Simulation results show that due to the wideband and non-stationary nature of such signals commonly used time-domain NLMS algorithm will fail to give an accurate result. Nevertheless, among subband algorithms Merched is performing significantly better than the others both in terms of convergence speed and achievable minimum mean square error. This is due to the fact that the use of fractional delay in the weight transformation scheme will cancel aliasing effects in subbands. It is also proved that this algorithm is less computational complex than the others.

**REFERENCES**


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