# An Efficient Turbo Equalization for Faster than Nyquist Signal

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***Abstract*—In this paper, we analyzed efficient decoding scheme with faster than Nyquist (FTN) signaling that is transmission method faster than Nyquist theory and increase the throughput. We proposed BCJR equalization model to minimize inter symbol interference when faster than Nyquist signal is transmitted. The presented model utilized interference as branch information and iteratively exchange probabilistic information between BCJR and LDPC decoder. In BCJR equalizer, the performance depends on Euclidean distance of branch metrics between possible transitions at each node, in order to maximize Euclidean distances, we proposed FTN re-mapper by reordering the branch matrices on trellis diagram. We confirmed that performance was improved compared to conventional methods as increasing throughput of fast than Nyquist signal.**

***Index Terms*—faster than Nyquist, BCJR equalizer, LDPC codes**

## I. Introduction

Faster-Than-Nyquist (FTN) signalling is a technique of transmitting information at a rate higher than the allowed Nyquist limit [1]-[4]. Systems employing this technique have shown to achieve higher information rates at the cost of increased processing in the transmitter and the receiver. There have been some efforts to apply FTN theory to commercial applications, e.g. DVB-S2 for the satellite broadcasting system. It evaluated the performance of DVB-S2 building block based on FTN transmission, and this result gives an opportunity for the practical implementation. There is increasingly growing demand to send high data rates over satellite channels. We here discuss utilizing time packing or FTN signalling to satisfy this demand in combination with using tight frequency spacing is $(1+\alpha)\cdot R_s$, where $R_s$ is the symbol rate and $\alpha$ is roll-off factor. In FTN signalling, information symbols are transmitted at a rate higher than that suggested by the Nyquist criterion, i.e., $1/\tau > 2W$ and, therefore, ISI is unavoidable. Signaling above the Nyquist rate comes at the expense of higher receiver complexity and higher transmitted power, since more information symbols are sent per second. Since it was first studied, the usefulness of FTN has not yet been determined. Several receivers have been suggested, some implying the feasibility of FTN and others not considering it worthwhile. The presented model utilized interference as branch information and iteratively exchange probabilistic information between BCJR equalizer and LDPC decoder [5]-[8]. In BCJR equalizer, the performance depends on Euclidean distance of branch metrics between possible transitions at each node, in order to maximize Euclidean distances, we proposed FTN re-mapper by reordering the branch matrices on trellis diagram. We confirmed that performance was improved compared to conventional methods as increasing throughput of FTN signal.

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## II. FTN Signal Modeling

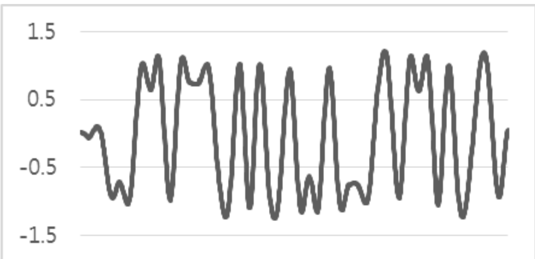
With this background, we turn to FTN signalling. The key aspect of the FTN method is that $h(t)$ is no longer orthogonal with respect to the symbol time. The same $h$ is employed but the symbol time is $\tau T$, $\tau < 1$. The signal becomes:

$$s(t) = \sqrt{E_s}\sum_n \alpha_n h(t - n\tau T) \qquad (1)$$

here $\alpha_n$ is a sequence of independent M-ary data symbols, each with energy $E_s$, and a new unit-energy pulse are $h(t)$ appears each T seconds, the symbol time.

The factor $\tau$ can be thought of as a time acceleration factor since now the pulses come too fast by a factor $1/\tau$. If a filter matched to $h(t)$ is used in the detection, its samples are no longer the M-ary values $a_n$ plus noise, but contain inter symbol interference (ISI) as well.

Above is ordinary orthogonal linear modulation with $h(t) = \left(\sqrt{T}/\pi t\right)\sin\left(\pi t/T\right)$, with the lighter sinc pulses representing symbols -1, +1, -1, +1, +1… that add up to the heavier curve $s(t)$.




(a) Nyquist rate data ($T$=1 and τ=1)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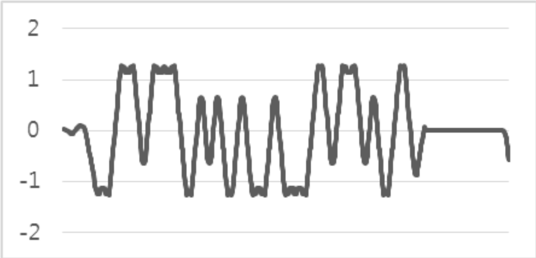



(b) Faster than Nyquist rate data ($T$=1 and τ=0.8)

Figure 1. Illustration of FTN signaling with unit-T sinc pulses and τ=1 and 0.8

Fig. 1 shows an example of FTN signal with orthogonal symbol time $\tau = 1$ and $\tau = 0.8$. Fig. 1(a) is ordinary signal with Nyquist rate, and Fig. 1(b) shows FTN signal with $\tau = 0.8$. It can be seen than five sinc pulses are now advanced.

## III. An Efficient Decoder Structure for FTN Model

Fig. 2 displays FTN transmission system based on DVB-S2 standards. Let us suppose that the binary digits delivered by source are encoded by a LDPC encoder. The encoded data are reordered by an interleaver, and applied QPSK modulation point with FTN mapper. At the receiver side, an LLR computer is needed to convert the equalizer output, assumed to follow Gaussian distribution, into extrinsic LLRs regarding the code bits, by using a priori LLRs from the previous decoding iteration. This updated set of soft information about the code bits is then deinterleaved and provided as a priori LLRs, for the next decoding iteration.

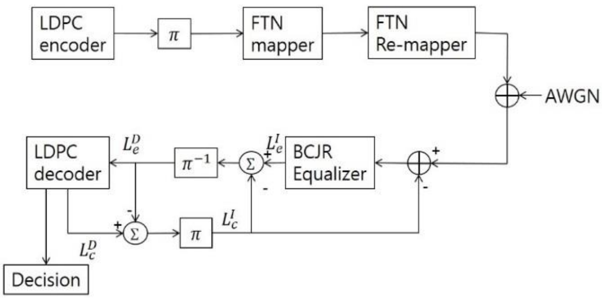



Figure 2. FTN re-mapper

The computation of the LLRs pertaining to the equalizer uses the FTN rate and roll-off values in reconstructing ISI in each carrier. Generally, the decoding method of trellis type as shown in Fig. 2 is BCJR algorithm [9], [10] with soft decision value. BCJR algorithm is a well-known maximum a posteriori probability decoding algorithm which has been proposed earlier for point to point communication applications.

The value of $L_e^D$ after interleaver is computed as $L_e^I - L_c^I$, then input to the turbo decoder. The estimated extrinsic value of $L_c^D$ at decoder output is given by:

$$L_c^D = \log \frac{p(x=+1)}{p(x=-1)} \tag{2}$$

The extrinsic value $L_c^D$ of which calculates the post probability is error correction terms. The re-interleaving of computed value as $L_c^D - L_e^D$ is input to BCJR equalizer, then $L_c^I$ is updated in order to compensate for the errors.

Fig. 3 shows block diagram of FTN mapper. As shown in Equation (1), take into account acceleration factor $\boldsymbol{\tau}$ and M interference signal at time k, quantities may be written in the form:

$$\Gamma_k(t) = \sum_{m=0}^{M-1} \alpha_m h(t - m\tau T) \tag{3}$$

And let us suppose that ISI is limited (L1+L2) symbols. Equation (1) may be written in the form:

$$R_t = \sum_{k=-L_2}^{L_1} \Gamma_k(t) c_{t-k} + b_t \tag{4}$$

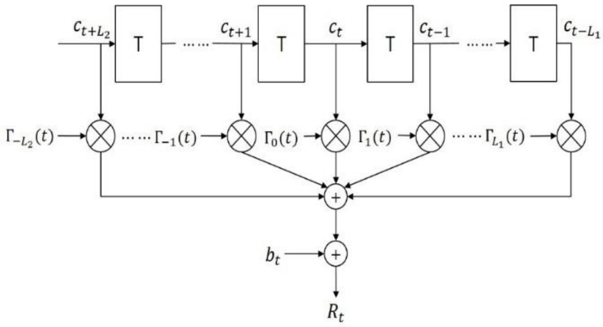


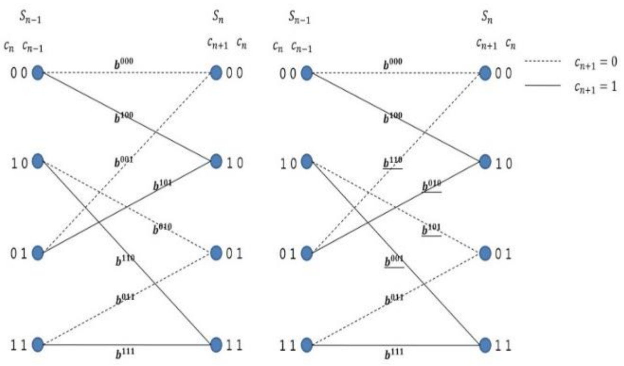

Figure 3. Equivalent discrete-time model of a channel with inter-symbol interference

$b_t$ is zero mean white Gaussian noise with power spectrum density equal to $N_0$. If we denote $S_n = \left(c_{n+L_2}, L, c_{n-L_1+1}\right)$ the state of the equivalent discrete-time channel at time $nT$, sample $R_n$ depends on the channel state $S_{n-1}$ and on the symbol $c_{n+L_2}$. Therefore, the equivalent discrete-time channel can be modelled as a Markov chain and its behaviour can be represented by a trellis diagram (Fig. 2). Where, $b^{xyz}$ is branch metrics for current symbol y in aspect to previous symbol z and next symbol x. Also, $b^{xyz}$ is the same as transmitted FTN signal for y. Based on $\tau$=0.8 and L1=L2=1, $b^{xyz}$ are defined in Table I.

TABLE I. $b^{xyz}$ Values (T=0.8, L1=1, L2=1)

| x y z | $b^{xyz}$ |
|---|---|
| 0 0 0 | -1.390080 |
| 0 0 1 | -1.095634 |
| 0 1 0 | 0.801187 |
| 0 1 1 | 1.095634 |
| 1 0 0 | -1.095634 |
| 1 0 1 | -0.81187 |
| 1 1 0 | 1.095634 |
| 1 1 1 | 1.390080 |

Based on Fig. 4(a) and Table I, the Euclidean distance between branch metrics at each node is very small. In BCJR equalizer, the performance depends on Euclidean distance of branch metrics between possible transitions at each node, in order to maximize Euclidean distances, we proposed FTN re-mapper by changing the trellis diagram. In $b^{xyz}$, Euclidean distance is depended on y symbols. Therefore, FTN re-mapper reconstruct trellis diagram as shown in Fig. 4(b). To maximize Euclidean distance, we change




(a) FTN mapper (b) FTN re-mapper

Figure 4. Trellis diagram

A set of triples (previous state, channel output, next state) uniquely defines a finite state machine on which the BCJR operates. As an illustration a trellis is shown in Fig. 3(c). It has 4 states $(s_0, s_1, L, s_3)$, and each state is given by a different 2-bit pattern. The states in vertical columns represent all possible states that the channel (or FSM) can take at a given time instant, while the labeled edges represent possible transitions. Neighboring columns thus represent consecutive time instants.

Given $s'$ the previous state, $s=(c_{j-m}, c_{j-m+1}, L, c_j, c_{j+1}, L, c_{j+m})$ - the present state, $u=(u_1, u_2, L, u_n)$ - the transmitted code word, and $y=(y_1, y_2, L, y_n)$ - the received sequence, the log-likelihood ratio (LLR) (denoting the bit reliability) of $u_j(j=1,2,...,n)$, is calculated as:

$$L(u_j) = \max^*\left[\alpha_{j-1}(s') + \gamma_j(s',s) + \beta_j(s)\right] \quad s', s \quad u_j = 1$$
$$-\max^*\left[\alpha_{j-1}(s') + \gamma_j(s',s) + \beta_j(s)\right] \quad s', s \quad u_j = 1 \quad (5)$$

The forward metric, $\alpha_j(s) = \log p(s_j = s, y_1^j)$ is given by:

$$\alpha_j(s) = \max^*\left[\alpha_{j-1}(s') + \gamma_j(s',s)\right] \quad (6)$$

The backward metric, $\beta_j(s) = logp(y_{j+1}^n \mid s' = s)$ as:

$$\beta_{j-1}(s') = \max^*\left[\beta_j(s) + \gamma_j(s',s)\right] \quad (7)$$

And the branch metric $\gamma_j(s',s)$ is given by:

$$\gamma_j(s',s) = \log p(s_j = s, y_j \mid s_{j-1} = s') \quad (8)$$
$$= \log p(y_j \mid u_j)\; p(u_j)$$

The $\max^*$ - operator is defined as:

$$\max^*(x,y) = \max(x,y) + \log\left(1 + e^{|x-y|}\right) \quad (9)$$

LDPC decoder and BCJR equalizer are connected through the interleaving and de-interleaving that update each other's information repeatedly. The inner coded bits are then subtracted from the input and interleaved. The interleaved output is cancelled a posteriori from the proceeding received signal. Interleaving helps receiver convergence.

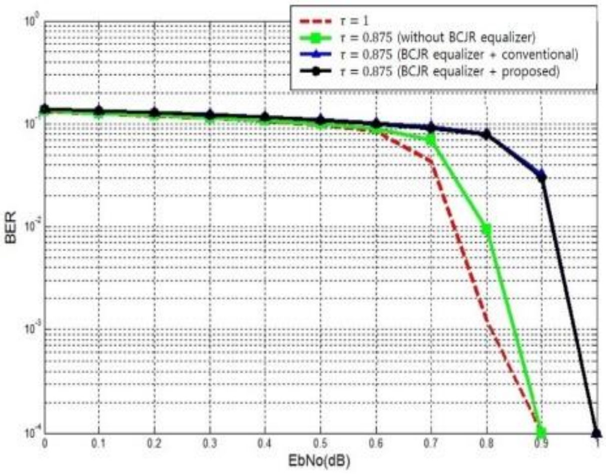
## IV. Simulation Results

Computer simulations are used to study the performance evaluation. For the comparison purpose, Table II listed parameters for simulation.

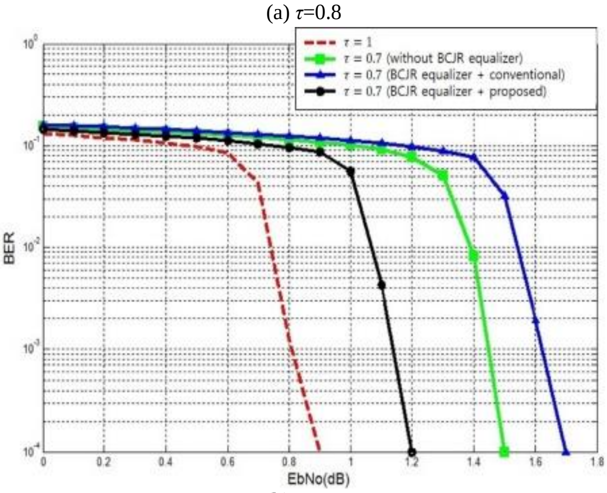
TABLE II. Type Sizes for Camera-Ready Papers

| | |
|---|---|
| Channel coding | LDPC |
| Coding rate | 1/2 (N = 64800) |
| (L1, L2) | (1, 1) |
| $\tau$ | 0.8, 0.7, 0.5 |
| Modulation | QPSK |
| Inner iteration | 60 |
| Outer iteration | 3 |

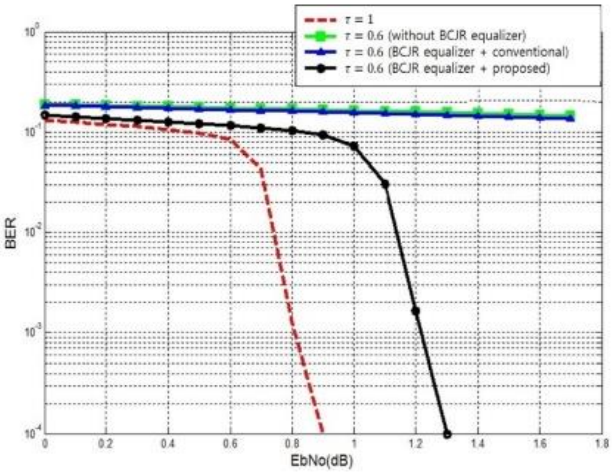
In the Table II, inner iteration means iteration number of LDPC decoder itself and outer iteration means total iteration number between LDPC decoder and BCJR equalizer.




(a) $\tau$=0.8




(b) $\tau$=0.7




(c) $\tau$=0.6

Figure 5. BER performance in according to $\tau$

Fig. 5 shows bit error performance according to $\tau$. On Gaussian channel, compared to original signal with $\tau$=1, the performance of FTN signal is degraded than by in the range of 0.1dB to 0.3dB. In case of $\tau$ is 0.8, performance of employing the BCJR equalizer is worse than on LDPC decoder, this can be explained by the fact that BCJR equalizer is not well operated induced from small Euclidean distance of branch metrics at node. This is the reason $\tau$ is lager, Euclidean distance is smaller. Otherwise, in case of $\tau$ is 0.7 and $\tau$ is 0.6, the performance with BCJR equalizer is better than that of only LDPC decoder. In aspect to performance of proposed algorithm which using the FTN re-mapper in order to maximize the Euclidean distance, coding gain of 0.5dB can be achieved compared to conventional algorithm with using only FTN mapper. Furthermore, conventional algorithm can't correct errors at all in case of $\tau$ is 0.8, however the proposed algorithm is doing well operated. Based on simulation results, the proposed algorithm is very effective at low value of $\tau$.

## V. CONCLUSION

This paper presented efficient decoder structure for FTN signalling which transmitting information at a rate higher than the allowed Nyquist limit. A new structure combating detrimental effects of inter symbol interference on FTN signalling has been presented. In order to maximize the Euclidean distance, we employed FTN re-mapper which reordering the branch matrices on trellis diagram.

As shown in simulation results, in case of small interference, that is large value of $\tau$, performance of employing the BCJR equalizer is worse than on LDPC decoder, this can be explained by the fact that BCJR equalizer is not well operated induced from small Euclidean distance of branch metrics at node. Otherwise, small value of $\tau$, the performance with BCJR equalizer is better than that of only LDPC decoder.

In aspect to performance of proposed algorithm which using the FTN re-mapper in order to maximize the Euclidean distance, coding gain of 0.5dB can be achieved compared to conventional algorithm with using only FTN mapper.

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