A Review on Filtered-X LMS Algorithm

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***Abstract*—In this review paper, a summary of study on previous research on Filtered-X LMS (FxLMS) algorithm is presented. The review is done on the basis of two parameters- first one is based on improvement in FxLMS algorithm and the second one is applications based review. The main advantage of using FxLMS algorithm is that it is computationally simple like the most commonly used Least Mean Square (LMS) algorithm. In addition it includes secondary path effects. To make the FxLMS algorithm more effective, the secondary path estimation should be more precise and accurate. The main drawback of FxLMS algorithm is its slow convergence speed. The convergence speed of FxLMS algorithm is slower than that of conventional LMS algorithm. For active noise control (ANC) applications, FxLMS algorithm based system is usually preferred since the RLS (recursive least square) algorithm based systems (FxRLS) has more computational requirement and lower convergence speed than FxLMS.**

***Index Terms*—FxLMS, secondary path effect, convergence speed, ANC**

I. Introduction

Noise has always been an never ending undesired problem and can results in corruption of desired signals. Hence it's reduction to a tolerance level or complete elimination becomes important. So passive noise control system was introduced but it was effective only for high frequency noise but at lower frequency noise, it becomes bulky and expensive. So for low frequencies noise attenuation, ANC was introduced. Active Noise Control (ANC) is based on the simple principle of superposition theorem. It generates anti noise signal and when this is added with noisy input signal, it results in noise free signal. The basic idea was proposed in 1936 (Lueg, 1936). With the advancement of the adaptive signal processing algorithms and their digital signal processors (DSPs) implementations, the applications of ANC have been developed in different areas. The most popular adaptive algorithm used for ANC applications is the filtered-x least mean square (FxLMS) algorithm (Kuo & Morgan, 1996) which is a modified version of the LMS (least mean square) algorithm (Widrow & Stearns, 1985). LMS is based on steepest descent method, but do not include secondary path effects, so precise anti noise signal cannot be generated. The FxLMS algorithm is computationally simple where secondary path effects are also includes, but its convergence speed is slow. Different ANC algorithms, with improved convergence properties, has been proposed like: Frequency domain ANC systems (Kuo & Tahernezhadi, 1997); Recursive Least Squares (RLS) based algorithms called filtered-x RLS (FxRLS) (Kuo & Morgan, 1996) and Filtered-x fast transversal filter (FxFTF) (Bouchard & Quednau, 2000); Lattice ANC systems (Park & Sommerfeldt, 1996) and Infinite impulse response (IIR) filter based LMS algorithms called filtered-u recursive LMS (FuRLMS) (L. J. Eriksson and Allie, 1987), and filtered-v algorithms (Crawford & Stewart, 1997). The basic problems in the above approaches are inherent stability problems in IIR-based structures, increment in the computational requirement and numerical instability problems in RLS based ANC systems. These reasons make FxLMS still a good choice for ANC applications.

A. About ANC

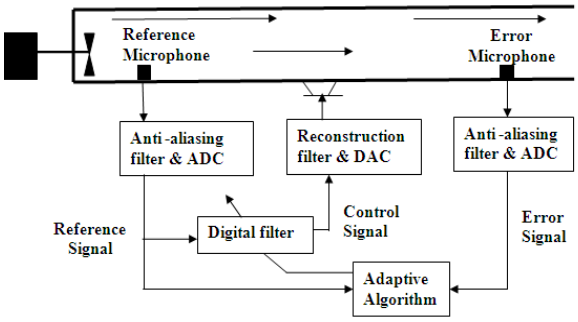

Figure 1a. ANC system based on feed forward approach

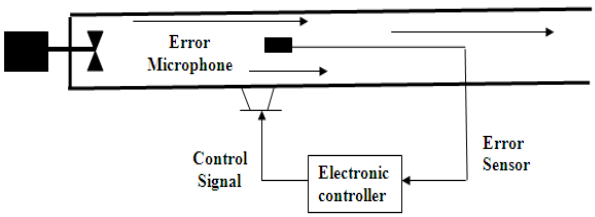

Figure 1b. ANC system based on feedback approach

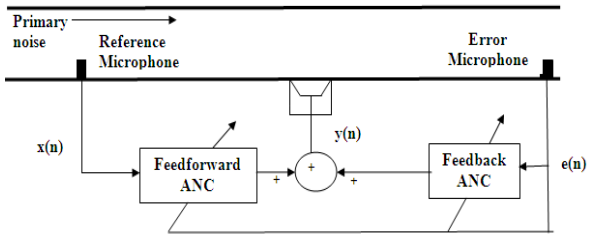

Figure 1c. ANC system based on Hybrid approach

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There are three approaches for implementing ANC-feed forward, feedback and hybrid approach. Fig. 1a shows ANC system based on feed forward approach. It contains basically two types of sensors-reference sensor and error sensor. Here error sensor is not feed backed to controller. Controller and physical system are treated separately. It provides better noise reduction in steady state response. Fig 1b shows basic ANC structure based on feedback approach. It contains mainly error sensor. Here error signal is feed backed to controller. Controller and physical system are considered as single coupled unit. It provides better noise reduction for transient state response. Fig. 1c [1] shows combination of both approaches i.e. Hybrid approach. The advantage of using Hybrid approach is that a lower order filter can be used to achieve the same performance with that obtained by above two approaches. Also ANC structures can be implemented using any of the six ways - Adaptive transversal ANC (which uses adaptive filter, FIR or IIR realizations), Lattice ANC (it includes lattice predictor having two input and two output channel), Frequency domain ANC (it converts all signals in frequency domain using FFT etc. then do computations, this makes system fast), Sub band ANC(used in case of large tap length, it processes signals in sub bands, so have advantage of lower computation burden and faster convergence), RLS based ANC(involves usage of RLS algorithm), Modal ANC (it decomposes the ANC problem, hence reducing computation and increases convergence) [1].

B. *Secondary Path*

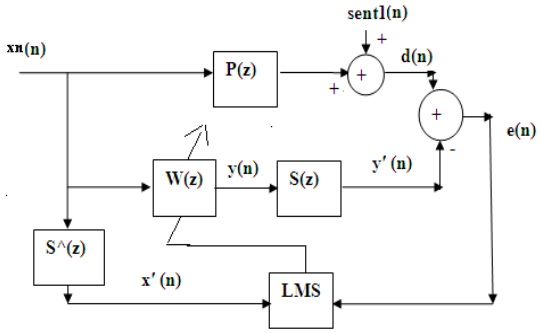

Figure 2. Shows ANC system with FxLMS algorithm

The devices added to the system for implementing ANC system like digital filter, anti-aliasing filter, ADC, DAC etc also adds up to noise. Such forms a secondary path and estimation of this path, so becomes important in order to generate precise anti-noise signal. Precise estimation of this path makes ANC more effective. Secondary path (S(z)) can be estimated offline or online. In case of Offline estimation, Secondary path is estimated before operation of ANC system. In case of Online, Secondary path is estimated while ANC is in operation. This type of estimation is needed when Secondary path is time varying. No doubt, such system becomes much complex as it involves many circuitries for monitoring and detecting Secondary path with regular operation of ANC system. There are basically two ways to compensate for this Secondary path. One is to placing inverse filter, 1/S(z) in series with secondary path S(z). the second way is, placing identical filter (S^(z)) in reference signal path to the weight update of LMS algorithm. which realizes so called FXLMS algorithm. Fig. 2. [1] Shows ANC system with FxLMS algorithm, where S (z) represent secondary path and S^(z) represent reference signal generated to compensate for secondary path [1].

In this paper, Section I contains Introduction, Section II. A contains Review on improvements in FxLMS algorithm, Section II. B contains Application based Review and Section III contains conclusions and paper ends up with References.

II. Review on Improvement in FxLMS Algorithm

There are two basic approaches for ANC system: feed forward and feedback approaches and combination of these two approaches has resulted in a new hybrid approach. Naitik Nakrani, and Niteen Patel in year 2012, compared these two approaches for the two cases: broad band and narrow band noise, keeping filter order and step size fixed for both cases. It was concluded that feedback approach gives good results in narrow band noise and feed forward approach gives good results in broad band noise environment [2]. Study of convergence for multi variable FxLMS algorithm (in context to ANC system) was done by A. Kuo Wang, and Wei Ren [3] in April 1999. Here multi-channel FxLMS for narrowband and broadband stationary noise is studied. Liang Wang, Woon-Seng Gan, Yong-Kim Chong, and Sen M Kuo [4] in 2013, analyzed step size bound for narrowband ANC system, for both perfect and imperfect secondary path estimation which helps in estimating convergence rate in practical operation.

Different forms of FxLMS has been generated, all generated by modifying FxLMS algorithm. Two new variants of MFxLMS (along with its l_2-stability) were developed in year 1995 by Markus Rupp, and Ali H. Sayed [5], i.e. modification to FxLMS algorithm. MFxLMS improves convergence rate as compared to FxLMS algorithm but it requires heavy computation (3M elementry) per time step. So two new variants were developed such that convergence rate is better than FxLMS but computation per time step (2 M) is less than MFxLMS. In year January 2002, Weight Constrained FxLMS Algorithm (CFxLMS) for Feed forward ANC system was introduced by Hui Lan, Ming Zhang, and Wee Ser [6] where weights of filter were given specific upper and lower bounds, so called CFxLMS, such an algorithm have been found effective against error microphone. P. Babu, A.Krishnan, and V. Saravanan [7] in year 2010, for improving performance of ANC system, developed a variable threshold based algorithm. It has modified conventional FxLMS algorithm, by dynamically thresholding secondary path signal by wavelet transform. The basic principle of wavelet thresholding depends on the fact that for many real time signals e.g. noise, limited no. of wavelet coefficients in lower band are enough to regenerate original signal. Hence if we reduce number of wavelet coefficients, lesser than a particular value, say

threshold value, it can still generate the original signal, (here noise). Drawback of thresholding parameters is that they are estimated offline and cannot be updated during online operation of ANC systems. Miguel Ferrer [8] in January 2013, introduced a new concept of convex combination to for FxLMS algorithm for improving performance of ANC system. It is especially effective for system identification (when compared to existing algorithms). Convex combination approach means combining the two filters with complementary capabilities such that the effective performance of the filter is better than the individual performance of each filter. Though the overall convergence rate and MSE has improved but only drawback with this type of combination is heavy computation. Such a combination has worked well in both stationary and non stationary noise environment. Boyan Huang, and Yegui Xiao [9] in February 2013, added a Variable Step Size algorithm to conventional FxLMS algorithm. It has developed VSS-FxLMS algorithm for narrowband ANC system. This algorithm has convergence rate higher than FxLMS (nearly equal to FxRLS) and it works good in both stationary and non stationary noise environments. Also its cost and computation complexity lies between FxLMS and FxRLS. Dah Chung Chang, and Fei-Tao Chu [10] in November 2013,further improved the VSS FxLMS algorithm by introducing and including Variable Tap Length to it ie. Both tap length and step size are allowed to vary as per the application requirement. Normally as tap length is increased so reduces the convergence rate. But this algorithm is especially useful in such cases as it improves the convergence rate significantly. Convergence rate can also be improved by data reusability. Data-reusing (DR) type adaptive algorithm was introduced by Muhammad Tahir Akhtar [11] in year 2013 for ANC with impulse noise. It normalizes the step size for an impulse noise source, this will freeze the adaptation i.e. make weight constant for very large impulse.

FxLMS algorithm analysis can be based on Stochastic (based on many assumptions, so is not preferred when reference signal is time periodic) or deterministic approach. Stochastic Analysis of FxLMS Algorithm along with nonlinear elements in Secondary Paths was done in year June 2002 by Márcio H. Costa, José Carlos, M. Bermudez, and Neil J. Bershad [12]. This is important for cancellation of acoustic noise when a saturation nonlinearity models the nonlinear distortion which is due to amplifiers, transducers etc. Here Deterministic nonlinear recursions are generated for Gaussian inputs for the transient mean weight, mean square error (MSE), and cross covariance matrix of the adaptive weight vector at different times. Stochastic Analysis of FxLMS for Narrowband ANC system was done by Yegui Xiao, Akira Ikuta, Liying Ma, and K. Khorasani [13] in year July 2008. Here, considering the dynamics of the system, various difference equations are generated, all in terms of convergence of the mean and mean squared errors for the Discrete Fourier Coefficients (DFCs) of the secondary source. It has also generated Steady state expressions for DFC estimation of MSE as well as residual noise power in closed forms and stability bound for the FXLMS in MSE. Also in year March 2012, Shing Chow Chan, and Yijing Chu [14] analyzed and constructed FxLMS algorithm for Broadband ANC System which included Online Secondary Path estimation. Here new difference equations were generated for convergence rate and estimating secondary path for the system and upon analyzing these equations, stability, steady state EMSEs and the secondary path were determined. Seiji Miyoshi [15] in year 2013, generated learning curves for FxLMS algorithm using statistical mechanical analysis. It does not include any independence assumptions, small step size or limited tap size assumptions except the assumption that tapped delay line should be long enough. It has considered unknown system, angles between coefficient vectors (adaptive filter) and it's shifted filters as macroscopic variables. By calculating correlations between coefficient vectors and past tap input vector, it has developed set of differential equations providing deterministic results, for macroscopic variables. Upon solving these equations, it has been found that curves so generated matches with those generated after simulations. Tuomas Haarnoja [16] in January 2014, presented exact LTP (Linear Time Periodic) representation for FxLMS algorithm, based on deterministic analysis of FxLMS algorithm with assumption that only reference signal is synchronously sampled. It covers multi channel topology for various parallel filters. This representation is LTI (Linear Time Invariant) if length of filter is selected properly.

ANC system can also be generated without using exact secondary path estimation. Cheng Yuan Chang, and Deng-Rui Chen [17] in year September 2010, introduced a new algorithm called adaptive genetic algorithm (AGA), which has replaced FxLMS. The advantage of AGA is that it does not require calculation of secondary path also it avoids local minima problem which is usually occurring in FxLMS. Iman Tabatabaei Ardekani, and Waleed H. Abdulla [18] in September 2012, developed a new approach as per which , if we deliberately estimate an incorrect secondary path (ie. developing a reference secondary path signal which is not identical to secondary path signal), it can give better convergence rate (when compared to system which has generated exact secondary path) but with some reduction in stability bound and performance of steady state, which is all under tolerance limit, so resulting in no harm to system performance. Jian Liu, Jinwei Sun, and Yegui Xiao [19] in November 2012, analyzed FxLMS in frequency mismatch (FM) in mean sense for a narrowband ANC system. For this, various difference equations were developed for convergence rate, guaranteeing stability conditions. Liang Vincent Wang Woon-Seng Gan, Andy W. H. Khong, and Sen M. Kuo [20] in November 2013, investigated convergence behavior for feedback narrow band ANC system with imperfect secondary path estimation. Up till now, no approach has included reference signal synthesis error due to feedback nature, but here it is modeled using secondary path estimation error.

III. Application Based Review

TABLE I. Summary on Review of FxLMS Algorithm

S. No	Algorithm	Findings	Citations
1	FxLMS	Simple computation and Convergence slower than conventional LMS	[5]
2	MFxLMS	Convergence better than FxLMS but involves heavy computation.	[5]
3	MFxLMS1 And MFxLMS2	Convergence is better than FxLMS (equal to MFxLMS) but computation is lesser than MFxLMS	[3]
4	CFxLMS	Developed for broad band ANC. Here weights of filter are given specific upper and lower bound. Convergence rate has increased.	[6]
5	Variable threshold based FxLMS	Convergence rate has increased with little increase in computation	[7]
6	Convex combinat-ion based FxLMS	Convergence rate is high but with heavy computation, since it involves parallel combinations	[8]
7	VSS FxLMS	Developed for Narrow Band ANC, convergence rate has increased (nearly equal to FxRLS) with little computations. Good for both stationary and non stationary noise envir-onment. Cost and computation complexity lies between FxLMS and FxRLS	[9]
8	Data reusability based FxLMS	Used for impulse noise, it normalizes step size and so improves convergence rate.	[11]
9	VSS FxLMS with variable tap length	Maintains good convergence rate even in case of long tap length applications	[10]
10	FxWLMS & FxLMLS	Developed for ANC application to hearing aid and found effecti-ve in feedback cancellation in presence of outliers.	[23]

Applications of FxLMS are widely used in ANC system. MRI (magnetic resonance image for medical diagnosis) noise reduction was proposed by Nokhaeng Lee, and Youngjin Park [21] in October 2013. It is used for reducing low frequency noise in MRI with high SPL (sound pressure level). It uses feedback ANC approach. It finds characteristics of MRI noise and then finds a method of open loop control based on ensemble averages of noise and also including adaptive control for minimizing the residue error which is generated during operation.ANC system can also be used in headphones. Markus Guldenschuh [22] in October 2013, proposed Secondary Path Modeling techniques for ANC headphones. Secondary path estimation becomes must, since sudden lift or abrupt put on can result in more then 90 ° phase change and so the calculated secondary path can go wrong, resulting in divergence in FxLMS. He has proposed three methods of avoiding divergence in FxLMS. ANC system can also be used in hearing machine but FxLMS does not generate satisfactory results. Vasundhara, Ganapati Panda, and N.B. Puha [23] in year 2014, developed two new algorithms which are the improved version of FxLMS: Filtered-X Wilcoxon LMS (FXWLMS) and Filtered-X least mean log square (FXLMLS). They are found to be effective in feedback cancellation in the presence of outliers. FxLMS based ANC is also applicable for snoring noise reduction, Chakravarthy S. R, Kuo, and S. M [24] in year 2006 developed a system for reducing snore noise. It generated quite zone using three different FxLMS algorithms for getting good results. FxLMS with ANC can also be used in mine tunnel. Fan Jing [25] in year 2010. Mine tunnel noise environment can be assumed as duct noise system. It analyzes noise in central water pump house and tries to reduce noise. ANC can also be used in engine noise reduction. Ali, M.E.H.R, Attari, and M.A [26] in year 2011, implemented ANC system on DSP kit using combination of FxLMS and LMS algorithm. In IT industries, workers are at times highly prone to severe server's noise, such an exposure to noise for long duration can lead to health problems, so ANC can also be used in such environment. This concept was given in 2014 by M.K Sharma and R. Vig [27]. Development on FxLMS algorithm is summarized in a tabular form in Table I.

IV. Conclusion

A review on FxLMS algorithm and their applications has been studied and presented. The improvement based review is done in terms of convergence speed, complexity etc. Also various applications of FxLMS algorithm are discussed in the review.

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