Design and Analysis of Uniform-Band and Octave-Band Tree-Structured Filter Bank

Sandhya Shakya and Jyotsna V. Ogale
Samrat Ashok Technological Institute/Electronics and Communication, Vidisha (M.P.), India
Email: sandhya.cool30@gmail.com, jyoti.ogale@yahoo.com

***Abstract*—This paper presents an optimized design of uniform-band and octave-band tree-structured filter banks (FBs). These filter banks make use of finite impulse response (FIR) low pass prototype filter in both analysis and synthesis sections and provide near-perfect reconstruction by allowing small amount of distortion at the output. The minimum value of distortion is obtained by optimizing the prototype filter coefficients by varying its cut-off frequency. The Blackman window family and with simple optimization algorithm, filter banks are designed with high efficiency. Several design examples are included to show improved performances of designed filter banks over existing methods.**

***Index Terms*—tree-structure, filter bank, uniform-band, octave-band, optimization**

I. INTRODUCTION

Multirate filter banks find wide application in several areas of digital signal processing like speech and image compression, the digital audio industry, sub-band coding, trans-multiplexer, statistical and adaptive signal processing and many other fields [1]-[4]. Based on pass band width filter banks can be classified in two forms i.e. uniform-band filter bank and octave-band filter bank [1], [5]. This paper deals with both types of filter banks. Uniform-band filter bank have many constraints like integer and uniform decimation in each sub-band and limited time frequency resolution. These constraints catalyze the importance of octave-band filter bank. Octave-band filter banks may have any kind of rational decimation in each sub-band, less quantization error, low computational complexity and any extent of time frequency resolution as per requirement of the application. In last few years several methods [6]-[8] have been developed by different authors for designing the multirate filter banks. Several methods have used Blackman window family to design a prototype filter [9], [10].

In this paper tree-structured approach is used to design uniform-band and octave-band filter bank. The parent filter of two-band filter bank is designed by windowing using Blackman window family with simple linear optimization algorithm and comparison is made with earlier reported work [11]-[15]. The proposed design provides near-perfect reconstruction and suffers from distortion at the output. So as to reduce distortion of amplitude, iterative algorithm of linear nature is used. In addition to design methods number of optimization techniques is proposed for uniform and octave-band filter bank [2], [7], [8], [14], [16]. For designing uniform and octave filter banks, two-band filter bank [9] is used as a starting point shown in Fig. 1. This filter bank consist of an analysis section formed by analysis filters $H_L(z)$ and $H_H(z)$ and a synthesis section formed by synthesis filters $G_L(z)$ and $G_H(z)$. In order to reduce the computational workload down-samplers and up-samplers are present between the analysis and synthesis filter bank. This paper deals with, two channel filter bank proposed in [9] as a basic building block where analysis and synthesis filters are FIR filters.

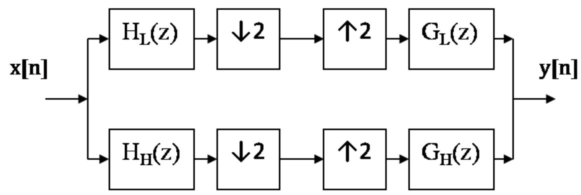

Figure 1. Two channel filter bank

II. PROPOSED UNIFORM-BAND FILTER BANK

The uniform-band tree-structured filter bank is formed by using two-band filter bank shown in Fig. 1. Tree-structured filter bank illustrated in Fig. 2(a) for the eight-band case. The number of bands resulting from this approach is restricted as,

$$M = 2^L, \ L€[1,2, \ldots..] \tag{1}$$

where L denotes the number of levels in the tree-structured filter bank. In each level identical filter banks are used to ensure alias-free and phase distortion free M-band filter banks. It is convenient to make its equivalent representation shown in Fig. 2(b).

The analysis and synthesis filters transfer functions can be expressed as,

$$H_K(z) = \prod_{i=0}^{L-1} H_{il}\left(z^{2^l}\right), \ G_K(z) = \prod_{i=0}^{L-1} G_{il}\left(z^{2^l}\right) \tag{2}$$

where $i=i(l, k)$ is either 0 or 1, l denoting the levels with $l = 0$ being outer most level. Fig. 2 shows eight-band case i.e. M=8 and L=3, where each $H_k(z)[G_k(z)]$ have unique combination of sub-filters $H_{il}(z)[G_{il}(z)]$. The distortion transfer function of the overall design is given as,

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$$V_0(z) = \frac{1}{M}\sum_{k=0}^{M-1} G_k(z) H_k(z) \tag{3}$$

Since two-band filter bank is alias free therefore uniform-band filter bank is also alias free.

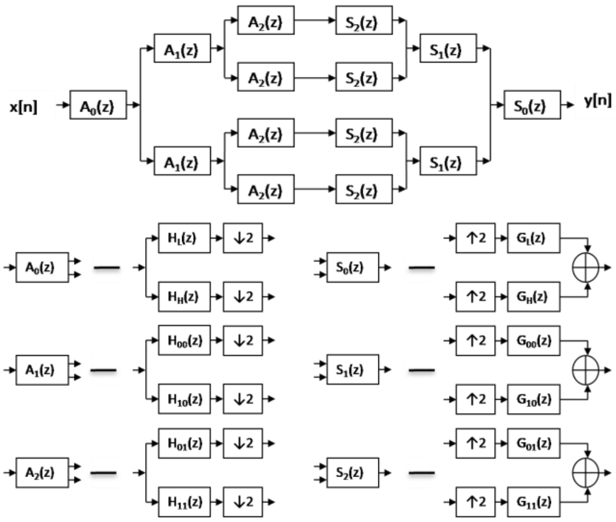


Figure 2(a). Eight-Band uniform tree-structured filter bank

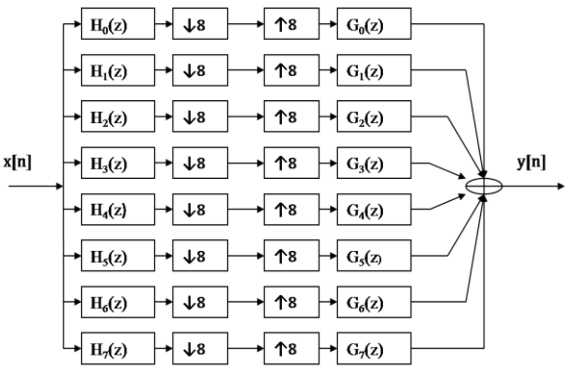


Figure 2(b). An equivalent representation of eight-band uniform tree-structured filter bank

III. Optimization Technique

In this work objective function which is used for optimization is given as,

$$\varphi = max\left[\left|V_0(e^{j\omega})\right| - 1\right] \tag{4}$$

where, M is the number of bands in the uniform filter bank. The step by step process can be given as:

Step 1: Choose the initial value of sampling frequency (F), number of band (M), pass band (ω_p) frequency, stop band (ω_s) frequency.

Step 2: Determine cut off frequency (ω_c) and filter length $(N+1)$.

Step 3: Initialize different optimization pointers like step size (step), search direction (dir), flag, minimum possible value of error (tol) and initial value of error (ierror).

Step 4: Set the While loop with flag=0 and design prototype low pass filter and complementary high pass filter using different window functions and determine the frequency response of other filters.

Step 5: Compare the present value of objective function with initial or previous value (ierror) and minimum possible value (tol).

a) If the present value greater than previous value, step size becomes half and changed the search direction (dir = -dir) go to step 6.
b) If the present value smaller or equals to the minimum possible value (tol.) than flag becomes set to '1'. Control comes out from the loop and go to step 7.
c) If the present value equals to previous value, than flag set to '1' and control comes out from the loop and go to step 7.

Step 6: The value of cutoff frequency is modified as (ω_c = ω_c+ dir ×step) and assign the current value of objective function as previous value and go to step 1.

Step 7: End of the loop and display the value of objective function as optimized value of reconstruction error.

The flow chart for optimization algorithm is given in appendix section and implemented on MATLAB 7.0.

IV. Design Example for Uniform-Band FB

In this section we use Blackman window family to construct two-band QMF bank which is further used to construct the three-level eight-band uniform filter bank according to the tree-structure of Fig. 2.

Example-1: An eight-band maximally decimated uniform tree-structured filter bank with decimation factor = 8 has been designed with ω_s=0.59π and filter order N=41 using Blackman window family. The obtained values of different performance parameters are summarized in Table I and discussed.

TABLE I. Performance Comparison with Earlier Reported Work for Uniform Eight Band

Work	Window Function	M	N	A_s	RE
P. Vaidyanathan, *et al.* [1]	Kaiser	8	39	35	10.810×10^{-3}
Kha, *et al.* [11]	Kaiser	8	41	35.8	5.50×10^{-3}
J. Ogale, *et al.* [12]	Kaiser	8	43	35.8	4.31×10^{-3}
	Cosh	8	45	35.8	2.00×10^{-3}
Proposed	Blackman	8	41	75	1.7×10^{-3}
	Exact Blackman	8	41	87	2.3×10^{-3}
	Blackman Nuttall	8	41	110	4.2×10^{-3}
	Blackman-Harris 3-term	8	41	80	1.7×10^{-3}
	Blackman-Harris 3-term	8	41	70	2.2×10^{-3}
	Blackman-Harris 4-term	8	41	110	3.8×10^{-3}
	Blackman-Harris 4-term	8	41	85	2.3×10^{-3}

Magnitude responses of the prototype filters designed using Blackman window family is shown in Fig. 3(a).

The magnitude responses of the sub-filters are shown in Fig. 3(b) and amplitude distortions for Blackman window family is shown in Fig. 3(c). Further the above mentioned filter bank has been utilized for sub-band coding of speech signal obtained from MATLAB tool-box of speech processing. The speech signal is of duration 4001 samples sampled at 7418Hz. Table II

Illustrate the quality measures of the speech signal in terms of PRD, MSE, ME and SNR. Original and reconstructed speech signal is shown in Fig. 4.

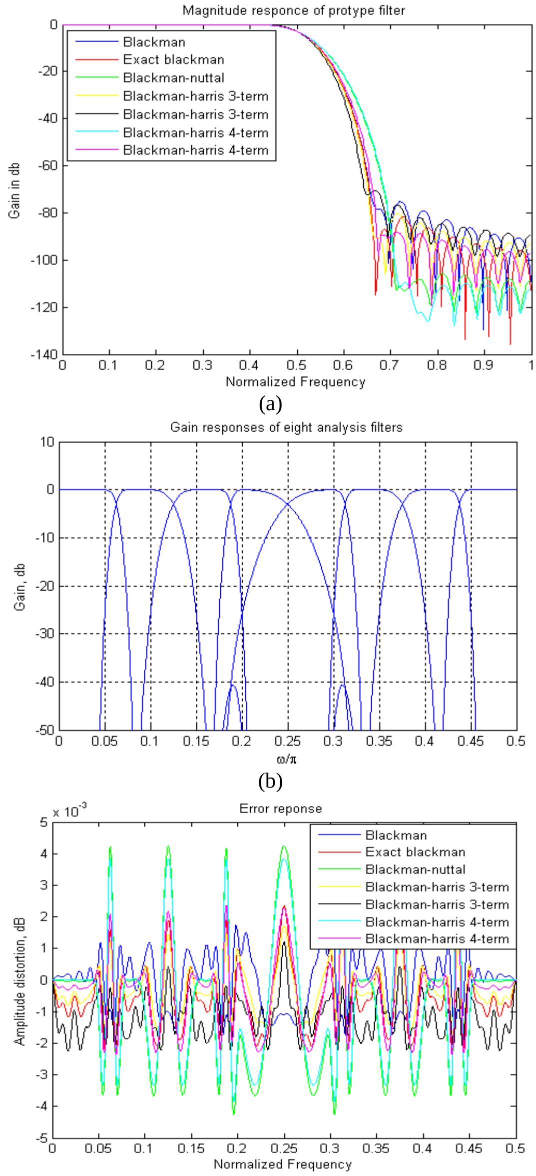


Figure 3. (a) Magnitude responses of prototype filters (b) Magnitude response of analysis filters for Blackman window (c) Amplitude distortion for different windows

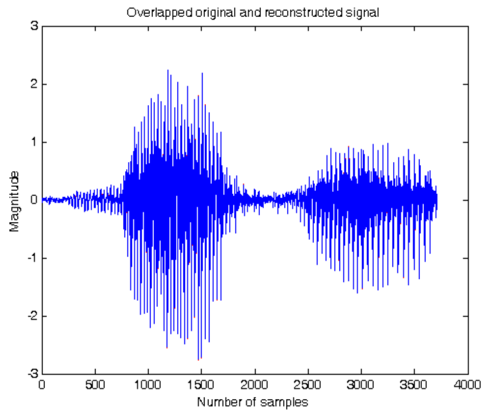


Figure 4. Overlapped original and reconstructed speech signal

It is evident from Table I that among all fixed windows with moderate side lobe fall of ratio of Blackman family, Blackman window offered the lowest reconstruction error *(RE)* of 1.70×10^{-3}. Further the Blackman-Harries (>67dB) window also competed well and offered *RE* of 1.70×10^{-3}. From Table II it may be concluded that the error magnitude between original and reconstructed speech signal is quite low.

The maximum error (*ME*) falls within the range of 0.11-0.12 indicated local distortion of very few samples. The *MSE*, in range of 1.960e-007 (Blackman window), suggests good conformity of reconstructed signal with the original signal. Further the reconstructed signals maintained a close similarity with the original signal. Blackman–Harris 4 term and Blackman–Nuttall provided superior stop band attenuation with respect to frequency.

It is write mentioning that the PRD obtained for speech signal by Blackman window falls within the acceptable range i.e. 2-10% in practice. Other parameters (MSE, ME and SNR) have also been improved and acceptable.

TABLE II. QUALITY ASSESSMENT OF SPEECH SIGNAL FOR EIGHT-BAND UNIFORM FB

Window Function	*MSE*	*ME*	*PRD*	*SNR*
Blackman	1.960e-007	0.0023	0.10	60
Exact Blackman	2.786e-007	0.0028	0.12	58
Blackman Nuttall	2.247e-006	0.0080	0.35	49
Blackman-Harris 3-term	8.216e-007	0.0052	0.21	53
Blackman-Harris 3-term	1.066e-006	0.0055	0.24	52
Blackman-Harris 4-term	2.101e-006	0.0077	0.34	49
Blackman-Harris 4-term	1.119e-006	0.0056	0.24	52

V. PROPOSED OCTAVE-BAND FILTER BANK

This filter bank also realized using two-band filter bank. At first level of decomposition either upper or lower band of two-channel filter bank is decomposed into another two-band with half of pass band and double of decimation factor. Decomposition process for three band octave filter bank with (4, 4, 2) decimation factors are shown in Fig 5(a). By cascading equivalent structure is obtained shown in Fig. 5(b). For M-band octave-band filter bank decimation factors $M_0, M_1, M_2, \ldots\ldots\ldots\, M_{m-1}$ for each band must satisfy the following condition [15]

$$\sum_{k=0}^{M-1} \frac{1}{M_k} = 1 \tag{5}$$

The z-transform of analysis and synthesis filters for three- band octave filter bank is given as:

$$H_0(z) = H_L(z)H_{10}(z),\ G_0(z) = G_L(z)G_{10}(z) \tag{6}$$

$$H_1(z) = H_L(z)H_{11}(z),\ G_0(z) = G_L(z)G_{11}(z) \tag{7}$$

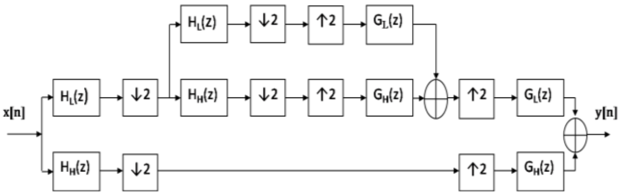
$$H_2(z) = H_H(z),\ G_2(z) = G_H(z) \tag{8}$$

The design of octave band filter bank posses the linear phase property [9], [13]. An analysis/synthesis octave bank retains the reconstruction properties of the building-block analysis/synthesis two-channel filter bank. Optimization technique described in Section III is also used for this filter bank with objective function,

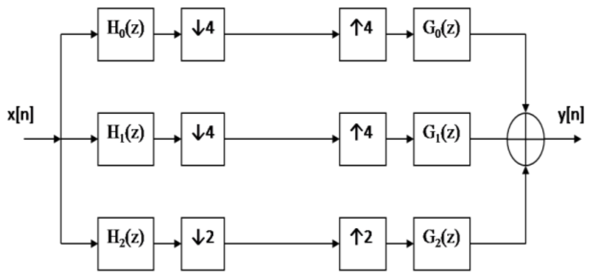
$$\varphi = max\left(abs\left(\left(\sum_{n=0}^{M-1}\left(abs\left(H_n(e^{j\omega})\right)\right)^2\right)-1\right)\right) \quad (9)$$

$$0 < \omega < \frac{\pi}{M}$$

where M is the number of bands.



(a)



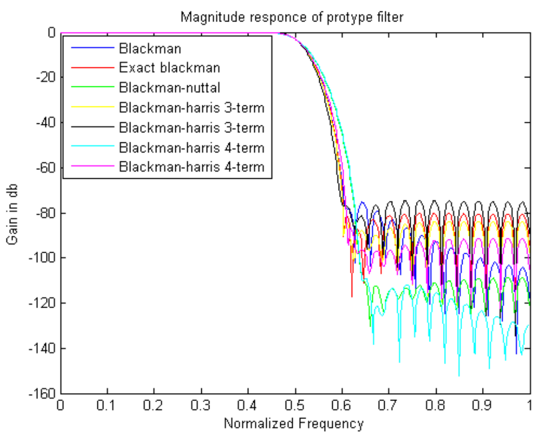
(b)

Figure 5. (a) Three-Band octave filter bank (b) Equivalent structure of three-band octave filter bank

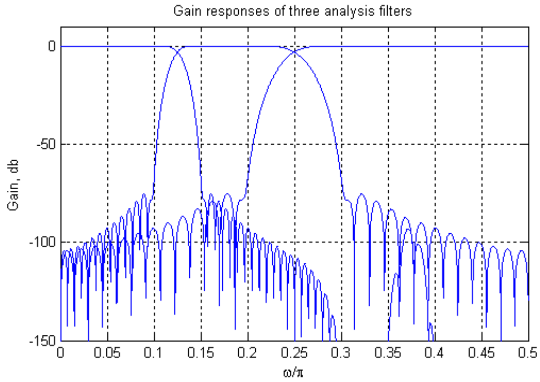
VI. Design Example for Octave-Band FB

In this section examples are illustrated and comparisons have been made with earlier reported work [14]-[18].

Example 1: A three-band and a five-band octave-band filter banks are proposed with (4, 4, 2) and (16, 16, 8, 4, 2) decimation factors. The prototype filter specifications are N=63, ω_p=0.5π and ω_s=0.563π. The obtained values of performance parameters for different windows are summarized in Table III and Table IV. Fig. 6(a), Fig. 6(b) and Fig. 6(c) shows magnitude responses of prototype filters, magnitude responses of analysis filters and amplitude distortion functions respectively for three-band. Similarly for five-band these are shown in Fig. 7(a), Fig. 7(b) and Fig. 7(c) respectively.



(a)



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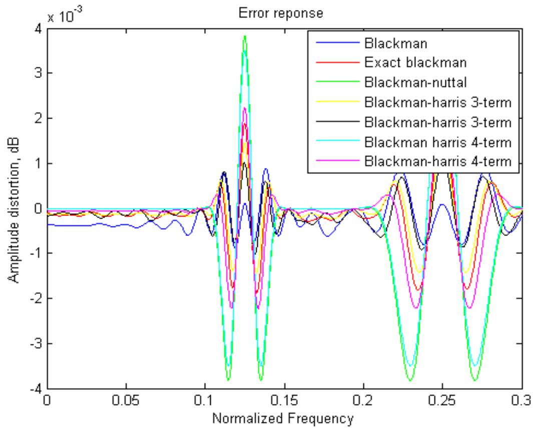
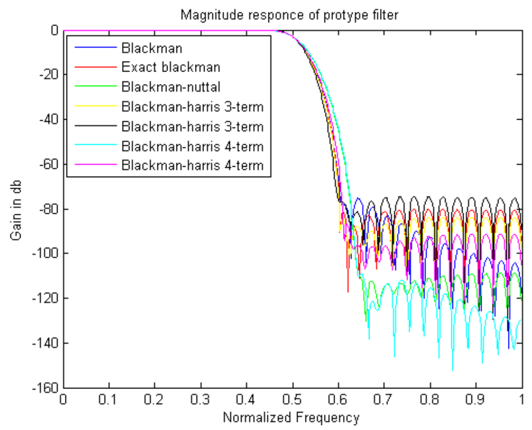
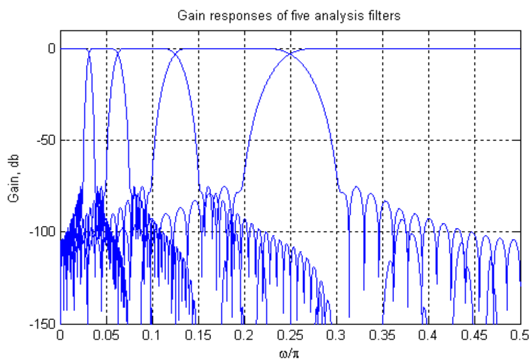


(c)

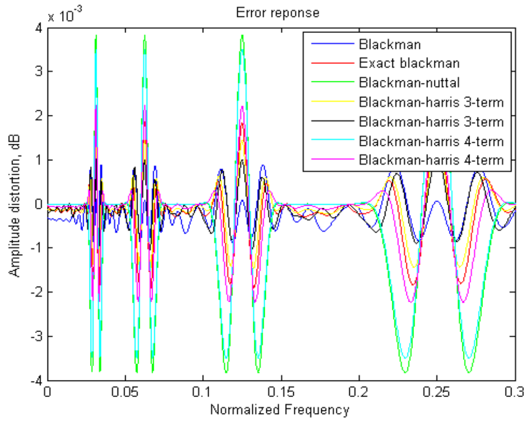
Figure 6. (a) Magnitude responses of prototype filters (b) Magnitude response of three analysis filters for Blackman window (c) Amplitude distortion for different window functions for three-band



(a)



(b)



(c)

Figure 7. (a) Magnitude responses of prototype filters (b) Magnitude response of five analysis filters for Blackman window (c) Amplitude distortion for different window functions for five-band

It is evident from Table III and Table IV that among all fixed windows Blackman window offered the lowest reconstruction error of 8.82×10^{-4} and 8.86×10^{-4} respectively for three-band and five-band FBs. Further the Blackman-Harris 3-term window also competed well and offered reconstruction error of 1.0×10^{-3} for both three and five band FBs. The superior performances provided by Blackman-Harris 4-term and Nuttall respectively due to high stop band attenuation.

TABLE III. PERFORMANCE COMPARISONS WITH EARLIER REPORTED WORK FOR THREE-BAND OCTAVE FB

Work	Band *(M)*	Technique/ Windows	*N*	A_s (dB)	*RE*
Xie, *et al.* [14]	Three (4 ,4, 2)	PM Method	63	-110	7.803×10^{-3}
Li, *et al.* [15]	Three (4, 4, 2)	PM Method	64	-60	7.803×10^{-3}
Soni, *et al.* [16]	Three (4, 4, 2)	Blackman Blackman Nuttall Blackman-Harris 3-term	63 63 63	-75 -110 -70	8.6×10^{-4} 3.85×10^{-3} 1.1×10^{-3}
Ogale, *et al.* [17]	Three (4, 4, 2)	Blackman Blackman Nuttall Blackman-Harris 3-term	63 63 63	-75 -110 -83	9.69×10^{-4} 3.80×10^{-3} 1.57×10^{-3}
Kumar, *et al.* [18]	Three (4, 4, 2)	Modified Method	63	-78	2.82×10^{-3}
Proposed	Three (4, 4, 2)	Blackman Exact Blackman Blackman Nuttall Blackman-Harris 3-term Blackman-Harris 3-term Blackman-Harris 4-term Blackman-Harris 4-term	63 63 63 63 63 63 63	-75 -80 -110 -85 -80 -120 -87	8.82×10^{-4} 1.8×10^{-3} 3.8×10^{-3} 1.4×10^{-3} 1.0×10^{-3} 3.5×10^{-3} 2.2×10^{-3}

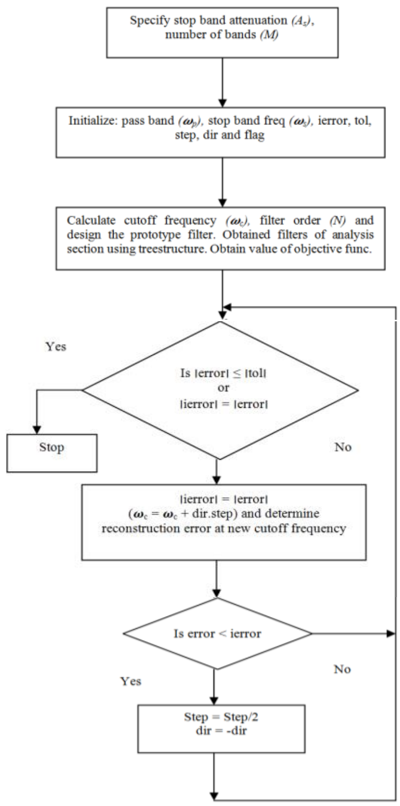
TABLE IV. PERFORMANCE COMPARISONS WITH EARLIER REPORTED WORK FOR FIVE-BAND OCTAVE FB

Work	Band *(M)*	Technique/ Windows	*N*	A_s (dB)	*RE*
Kumar, *et al.* [18]	Five (16, 16, 8, 4, 2)	Modified Method	63	-78	3.01×10^{-3}
Proposed	Five (16, 16, 8, 4, 2)	Blackman Exact Blackman Blackman Nuttall Blackman-Harris 3-term Blackman-Harris 3-term Blackman-Harris 4-term Blackman-Harris 4-term	63 63 63 63 63 63 63	-75 -80 -110 -85 -80 -120 -85	8.86×10^{-4} 1.9×10^{-3} 3.8×10^{-3} 1.4×10^{-3} 1.0×10^{-3} 3.5×10^{-3} 2.2×10^{-3}

VII. CONCLUSION

Simple and efficient designs of uniform and octave-band filter banks are presented using fixed window of Blackman family. The performance comparison of proposed work with earlier reported work shows that proposed designs provide better performance than the earlier reported work with sufficiently high stop band attenuation at the same computational cost in both cases. Therefore proposed designs can be used in wide variety of applications where high stop band attenuation with smaller amplitude distortion is desirable like broadband signal processing and image coding. The main advantage of these designs is its simplicity with minimum distortion parameters.

APPENDIX FLOW CHART FOR OPTIMIZATION ALGORITHM



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Sandhya Shakya was born in Vidisha, Madhya Pradesh, in 1990. She persuaded the B.E degree in Electronics and Communication from the Samrat Ashok Technological Institue, Vidisha (M.P) India in 2012 and pursuing M.Tech from Samrat Ashok Technological Institute, Vidisha (M.P), India. Her research interests include filter and multirate system.

Jyotsna V. Ogale was born in Jabalpur, Madhya Pradesh, in 1972. She persuaded BE and ME degrees in Electronics and Telecommunication and Communication System from the Rani Durgawati Vishwavidhyalaya, Jabalpur (M.P) India in 1995 and 1998 respectively. She joined the department of Electronics in Samrat Ashok Technological Institute, Vidisha (M.P) in 1999 and is currently Associate Professor in EC Department and pursuing Doctorate from Rajiv Gandhi Technical University (M.P), India.