A Novel Approach for Improving Performance of LMS Beamformer

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***Abstract*—In this paper a novel approach derived from the Mapped Real Transform (MRT) is proposed for improving the convergence rate and reducing the computational complexity of the Least Mean Square (LMS) Beamformer. In contrast to the conventional LMS beamformer which applies the LMS algorithm directly on the data received from the sensors, the proposed method utilizes a transformed version of the received data for the LMS adaptation. This transformed version is obtained by applying a One Dimensional Reduced Mapped Real Transform algorithm to the received data. Simulations show that the computational complexity is reduced and the convergence rate is improved when compared with the conventional LMS beamformers. The One Dimensional Reduced Mapped Real Transform based LMS algorithm can be used in radar, seismology, sonar, biomedical applications etc. As a future scope the One Dimensional Reduced Mapped Real Transform can be applied to other adaptive beamformers like Normalized Least Mean Square beamformer and Recursive Least Square beamformer.**

***Index Terms*—LMS beamformer, mapped real transform, one dimensional reduced mapped real transform, convergence rate, computational complexity**

I. Introduction

Beamforming is the process of combining the signals from an array of sensors in such a way that there is constructive interference in the direction of the desired user and destructive interference in the direction of undesired users [1]. Beamforming finds applications in Radar, Sonar, Cellular Communication etc. Beamformers can be classified as non-adaptive and adaptive beamformers. In non-adaptive or conventional beamformers the weights are chosen independent of any data received by the array. The weights steer the array response in a specified direction but interference rejection capability is poor [2].

In adaptive beamformers, the spatial signal processing is performed adaptively. The weights are chosen based on the statistics of the received data, to direct the main beam towards the desired user and place nulls in the path of interfering signals[3]. Adaptive beamformers have more degrees of freedom since they have the ability to adapt in real time to the signal environment. The calculation of weights is done adaptively using algorithms. The Least Mean Square (LMS) algorithm is one of the most popular algorithms for adaptive beamforming.

The LMS algorithm offers the advantages of simplicity and robustness but suffers from the disadvantage of slow convergence [4]. This paper presents an approach to achieve faster convergence and reduce computation time by applying the One Dimensional Reduced Mapped Real Transform (1D-R-MRT) algorithm to the LMS algorithm.

The paper is organized as follows: The signal model for adaptive beamforming is described in Section 2. Conventional LMS algorithm is introduced in Section 3. 1D-R-MRT is presented in Section 4. 1D-R-MRT based LMS beamformer is described in Section 5. Simulation results are presented in Section 6. Conclusions are presented in Section 7.

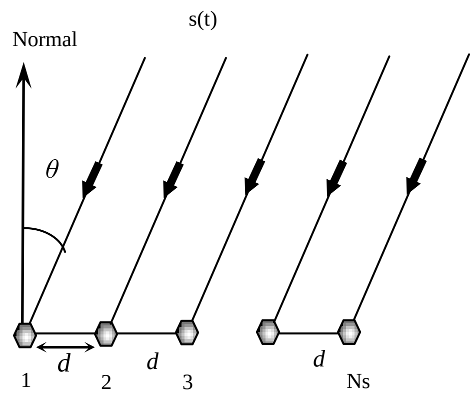

Figure 1. Uniform linear array with signal impinging from direction θ.

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II. SIGNAL MODEL

Assume that K narrowband sources are present in the far field of the array represented as $\{s_k(t), k=1, 2 ...K\}$. Out of these K sources let one source represent the desired signal while the rest denote undesired interferers. The K sources have directions of arrival (DOA) given by $\{\theta_k, k=1, 2 ....K\}$ [5]. Consider that the signal from these sources is received by a uniform linear array of Ns omnidirectional antennas with a spacing d between individual elements (Fig. 1).

The signal from a source s_k arriving from a direction θ_k will be received by each array element at different time instants. The resulting phase difference in the received signal between the successive elements of the array is given by

$$\psi_k = \frac{2\pi d \sin\theta_k}{\lambda} \tag{1}$$

where, d is the inter element spacing and λ is the wavelength of the signal. The array steering vector for a signal arriving from a direction θ_k is given by

$$a(\theta_k) = [1 \quad e^{j\psi_k} \ e^{j2\psi_k} \cdots e^{j(Ns-1)\psi_k}]^T \tag{2}$$

The array elements are assumed to be corrupted by Additive White Gaussian Noise represented as $\{n_j(t), j=1, 2.....Ns\}$. The signal received by the array can be represented as [6].

$$X(n) = [a(\theta_1) \quad a(\theta_2) \quad ..... \quad a(\theta_K)] \begin{bmatrix} s_1(n) \\ s_1(n) \\ . \\ . \\ s_K(n) \end{bmatrix} + N(n). \tag{3}$$

The vector representation of the received signals is

$$X(n) = As(n) + N(n) \tag{4}$$

where $\mathbf{X}(n)=[X_1(n) \ X_2(n) \ ......X_{Ns}(n)]^T$ is the column vector of data received by the array, $A=[a(\theta_1) \ a(\theta_2).... \ a(\theta_K)]$ represents the steering matrix, $\mathbf{s}(n)=[s_1(n) \ s_2(n) \ ......s_K(n)]^T$ is the signal column vector generated by the sources, $\mathbf{N}(n)=[n_1(n) \ n_2(n) \ ......n_{Ns}(n)]^T$ is the zero mean spatially uncorrelated Additive Gaussian Noises and $(.)^T$ represents transpose. The sum of the weighted inputs of array elements gives the beamformer output [7] as illustrated in Fig. 2.

$$Y(n) = \mathbf{W}^H \mathbf{X}(n) \tag{5}$$

where *W* is the weight vector.

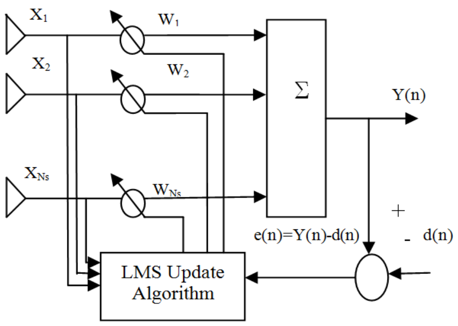

Figure 2. LMS beamformer.

$$\mathbf{W} = [W_1 \quad W_2 \cdots W_{Ns}] \tag{6}$$

and W^H is the Hermitian of the weight vector. The weights are adjusted adaptively using the LMS algorithm.

III. LEAST MEAN SQUARE ALGORITHM

The LMS algorithm is an important stochastic gradient algorithm. A significant feature of LMS algorithm is its simplicity. It does not require measurements of correlation functions and matrix inversions. These factors have made this algorithm very popular.

The LMS algorithm for beamforming consists of two basic processes: 1) an estimation process in which the beamformer output Y(n) is compared with a desired response d(n) and an estimation error e(n) is generated. 2) an adaptation process which automatically adjusts the weights of the beamformer in accordance with the estimation error.

The LMS algorithm changes the weight vector **W** along the direction of the estimated gradient based on the steepest descent method. The weight vector update for LMS algorithm is given by [8].

$$W(n+1) = W(n) + \mu X(n) e^*(n) \tag{7}$$

where μ is the step size, e(n) is the error vector given by

$$e(n) = Y(n) - d(n) \tag{8}$$

e*(n) is the conjugate of the error vector.

The step size (adaptation constant) μ is a positive real valued constant which determines the convergence rate and performance of the algorithm. The upper bound on μ [9] is given by

$$0 < \mu < \frac{2}{\lambda_{\max}} \tag{9}$$

where λ_{max} is largest Eigen value of the autocorrelation matrix $R=E\{X(n)X(n)\}$. The size of the incremental correction applied to the weight vector between successive iterations is controlled by μ. The LMS algorithm also requires knowledge of the transmitted signal. For this purpose, known pilot sequences are transmitted periodically to the receiver.

IV. ONE DIMENSIONAL MAPPED REAL TRANSFORM

The Mapped Real Transform (MRT), formerly called M dimensional Real Transform, performs the frequency domain analysis of one and two dimensional signals in terms of additions only [10]. The periodicity and symmetry of exponential terms in the Discrete Fourier Transform relation is exploited and related data is grouped. The MRT for one dimensional signal is utilized in this paper.

The One Dimensional Mapped Real Transform (1D-MRT) as explained by Meenakshi, Roy and Gopikakumari [11] is as follows

Let x_n, $0 \le n \le N-1$ be a 1-D sequence of size [1xN] and let Y_k, $0 \le k \le N-1$ of size [1xN] be its DFT. The DFT is given by

$$Y_k = \sum_{n=0}^{N-1} x_n W_N^{nk}, \quad 0 \le k \le N-1 \tag{10}$$

where

$$W_N^q = e^{\frac{-j2\pi q}{N}} \tag{11}$$

and k is the frequency index .

Using the periodicity of the twiddle factor W_N, (10) can be expressed as

$$Y_k = \sum_{n=0}^{N-1} x_n W_N^{((nk))_N} \tag{12}$$

The exponent $((nk))_N$ can have a value p, where p is the phase index with $0 \le p \le N-1$. For a given value of k, by grouping the data that share the same value p for the exponent $((nk))_N$, and also using the relation $W_N^{p+(N/2)} = -W_N^p$, Y_k can be expressed as

$$Y_k = \sum_{p=0}^{M-1} Y_k^{(p)} W_N^p \tag{13}$$

where $Y_k^{(p)}$ is the 1D–MRT of x_n, $0 \le n \le N-1$, defined as

$$Y_k^{(p)} = \sum_{\forall n \Rightarrow ((nk))_N = p} x_n - \sum_{\forall n \Rightarrow ((nk))_N = p+M} x_n \tag{14}$$

where M=N/2.The size of Y_k will be [MxN].

The computation of the DFT coefficients requires the use of complex multiplications. Compared to DFT, the MN coefficients of MRT are computed in terms of additions only. The 1-D MRT maps an array of size (1xN) into a matrix of size (M x N). As the 1D_MRT exploits the relation $W_N^{p+(N/2)} = -W_N^p$ it is valid for all even values of N.

V. One Dimensional Reduced Mapped Real Transform

The [M x N] elements of 1D-MRT are represented as $Y_k^{(p)}$ where $0 \le k \le N-1$ and $0 \le p \le M-1$. Different operations can be performed by selecting only a few elements out of MN elements in the 1D-MRT matrix. Elements of 1D-MRT matrix used for processing are selected according to the application. Exhaustive simulation studies conducted to see if beamforming can be performed using only a few selected elements of 1D-MRT matrix, have shown promising results when one column of 1D-MRT matrix is used. In the proposed One Dimensional Reduced Mapped Real Transform (1D-R-MRT) algorithm for beamforming, only the elements of second column of the [MxN] 1D-MRT matrix, i.e., elements corresponding to k=1, are chosen for processing. The size of data vector from each sensor used for beamforming after this transformation will be reduced from a (1xN) vector to a vector of size (1xN/2). Suitability of the second column (corresponding to k=1) for beamforming is confirmed after exhaustive trials. Coefficients corresponding to frequency index k=1 are obtained by

$$Y_{R\text{-}MRT}[1xN/2] = x_n[1x(1:N/2)] - x_n[1x(N/2+1:N)] \tag{14}$$

where $Y_{R\text{-}MRT}$ is the 1D-R-MRT output vector and x_n is input vector of size (1xN). The data size for processing is reduced by half and the coefficients are obtained without involving any complex multiplication. Since at each sensor, the data to be processed is reduced to half its original size, there is a significant reduction in the size of the data required to be processed by the beamformer.

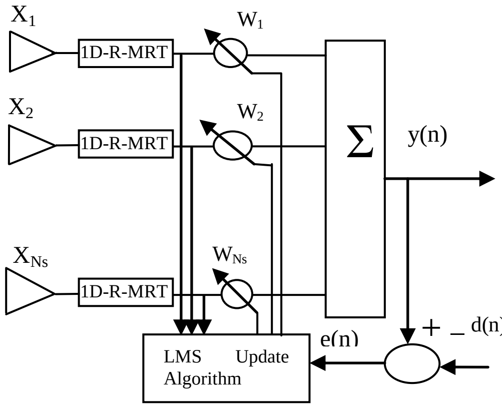

Figure 3. 1D-R-MRT LMS beamformer.

The data received at each sensor is first processed by the 1D-R-MRT algorithm block. After the data is mapped to 1D-R-MRT form, the LMS algorithm is applied as illustrated in Fig. 3.

VI. Simulation Results

For the proposed study, an array of eight sensors with a desired source at 50 degrees and two interferers at 0 degree and -30 degree is considered. The input signal frequency is 1 MHz and is corrupted by Additive White Gaussian Noise. The one dimensional data vector size is [1x256] and SNR is 30dB. After computing the 1D-R-MRT, the size of the vector used for beamforming will be reduced to [1x128]. It is indicated by exhaustive simulation studies that faithful reproduction of the input signal is obtained only if sampling frequencies is 4.655 times the input signal frequency for vector size of [1x256].

The optimum sampling frequency for different sampling sizes are given in Table I.

TABLE I. Optimum Sampling Frequency for Various Sampling Sizes.

One Dimensional sample vector size	Sampling frequency
256	4.655 f
512	4.096 f
1024	4.0475 f
2048	4.521 f
4096	4.1f
8192	4.2249f

The normalized beam patterns for the conventional LMS beamformer and the 1D-R-MRT LMS beamformer are compared in Fig. 4.

It is observed from Fig. 4 that the normalized beam pattern of the 1D-R-MRT LMS beamformer is comparable to that of the conventional LMS beamformer.

Also in comparison with the conventional LMS beamformer, the 1D-R-MRT LMS beamformer exhibits deeper nulls in direction of interfering signals i.e, at the interfering DOA of 0 degree and -30 degree, showing that the interference rejection capability of the 1D-R-MRT LMS beamformer is better than that of the conventional LMS beamformer.

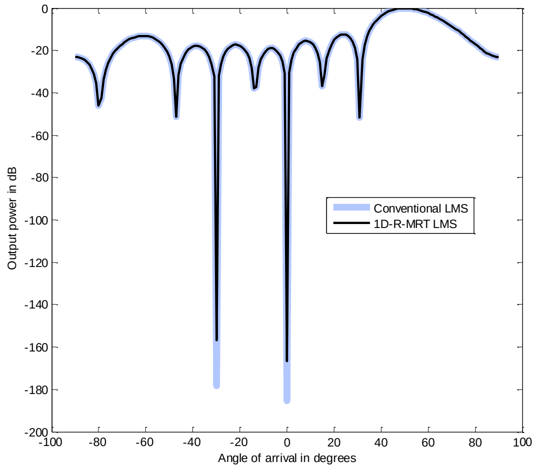

Figure 4. Comparison of normalized beam patterns. [No. of sensors = 8, DOA of desired signal = 50degrees, DOA of Interference1 = 0degree, DOA of Interference2 = -30degrees, SNR=30dB, No. of samples=256, μ = 0.01]

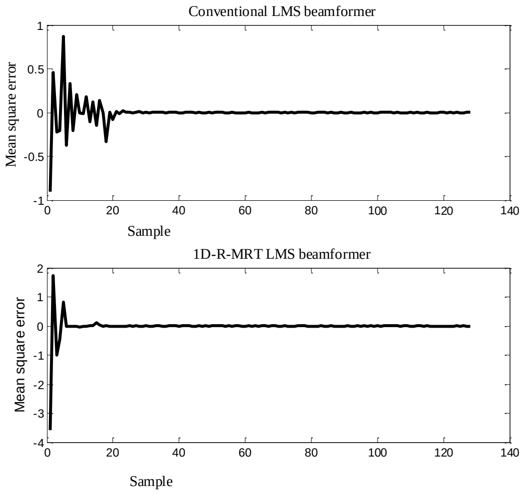

Figure 5. Comparison of mean square error. [No. of sensors = 8, DOA of desired signal =50degrees, DOA of Interference1 = 0degree, DOA of Interference2 = -30degrees, SNR=30dB, No. of samples = 256, μ = 0.01].

Next the performance of the conventional LMS beamformer and the 1D-R-MRT LMS beamformer on the basis of mean square error is compared as indicated in Fig. 5. It can be observed from Fig. 5 that the mean square error is initially large in the case of 1D-R-MRT LMS beamformer compared to conventional LMS beamformer. But the mean square error converges faster for the 1D-R-MRT LMS beamformer.

The convergence of the weights of the conventional LMS beamformer and the 1D-R-MRT LMS beamformer are compared in Fig. 6. It is observed from Fig. 6 that the weights of 1D-R-MRT LMS beamformer converge much faster than the weights of conventional LMS algorithm. The 1D-R-MRT LMS beamformer thus has faster adaptation to the input data compared to the conventional LMS beamformer.

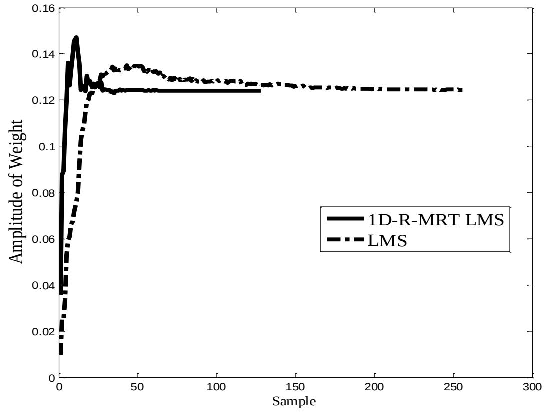

Figure 6. Comparison of convergence of weights. [No. of sensors = 8, DOA of desired signal =50degrees, DOA of Interference1 = 0degree, DOA of Interference2 = -30degrees, SNR=30dB, No. of samples = 256, μ = 0.01].

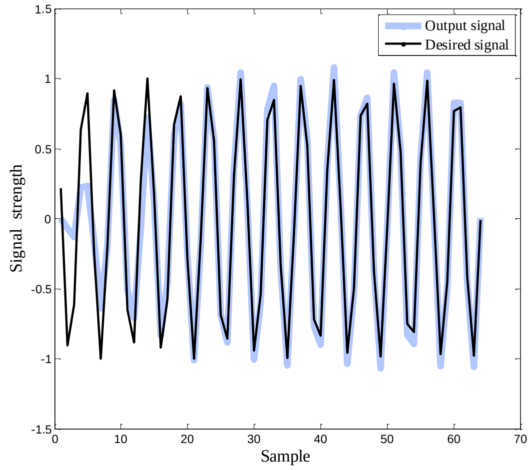

Figure 7. Comparison of 1D-R-MRT LMS beamformer output with the desired signal. [No. of sensors = 8, DOA of desired signal =50 degrees, DOA of Interference1 = 0 degree, DOA of Interference2 = -30 degrees, SNR=30dB, No. of samples =256, μ = 0.01].

The beamformer output of the 1D-R-MRT LMS beamformer closely follows the desired signal after the weight stabilizes as can be seen from Fig. 7.

TABLE II. COMPARISON OF COMPUTATION TIMES.

No: of samples	Computation time in seconds	
	Conventional LMS beamformer	1D-R-MRT LMS beamformer
256	0.0066	0.0046
512	0.0113	0.0067
1024	0.0239	0.0110
2048	0.0713	0.0291
4096	0.2102	0.0713
8192	0.9251	0.1997

[No. of sensors = 8, DOA of desired signal =50degrees, DOA of Interference1 = 0degree, DOA of Interference2 = -30degrees, SNR=30dB, μ = 0.01].

The 1D-R-MRT beamformer requires lesser computation time compared to conventional LMS beamformer. The computation times required by the conventional LMS and 1D-R-MRT LMS beamformers are compared in Table II.

It can be observed from Table II that the 1D-R-MRT LMS beamformer exhibits a considerable reduction in computation time.

The simulations were performed using MATLAB 8.1 on Intel Core I3 CPU processor.

VII. Conclusion

In this paper, a novel method for improving the performance of the conventional LMS beamformer is proposed. In the proposed 1D-R-MRT LMS beamformer, the coefficients of the MRT matrix have been analysed and a few coefficients suitable for beamforming have been identified. The optimum sampling frequencies, required for faithfully reproducing the original signal, corresponding to various sample sizes have also been studied. The 1D-R-MRT processing does not involve any complex multiplications and at the same time reduces the data size for beamforming. The simulation results show that the performance of the 1D-R-MRT LMS beamformer is improved compared to the conventional LMS algorithm. The beampatterns of the 1D-R-MRT LMS beamformer and conventional LMS algorithm are comparable.

The interference rejection capability of the 1D-R-MRT LMS beamformer is higher than the conventional LMS beamformer as it exhibits deeper nulls in the direction of interfering signals. The computation time and complexity of the 1D-R-MRT LMS beamformer is lesser than that of the conventional LMS beamformer as the data output from 1D-R-MRT block is half the size of the input signal. The weights and mean square error of the 1D-R-MRT LMS beamformer converges faster than the conventional LMS beamformer. Thus the proposed 1D-R-MRT LMS beamformer exhibits improved performance compared to the conventional LMS beamformer as it provides faster convergence, better interference rejection and lower computation time.

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