Generalization of Some CFAR Detectors for MIMO Radars

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***Abstract*—In this paper we generalize the GOSCA-CFAR, the OSGO-CFAR and the OSSO-CFAR detectors for the MIMO (Multi Input Multi Output) radars. We derive close-form expressions of the probability of false alarm (Pfa) and the probability of detection (Pd) in homogeneous environment. The comparison of these detectors for a non-homogeneous clutter environment showed that the OSSO-CFAR has better performance when the number of interfering is high.**

***Index Terms*—MIMO radars, GOSCA-CFAR, OSGO-CFAR, OSSO-CFAR**

I. Introduction

MIMO radar is characterized by using multiple antennas to simultaneously transmit diverse (possibly linearly independent) waveforms and by utilizing multiple antennas to receive the reflected signals [1].

This concept has the ability to improve radar performance, in terms of false alarm rate and detection, by exploiting radar cross section (RCS) diversity [2]. Based on antennas spatial diversity, two types of MIMO radars have been broadly discussed in literature; the coherent MIMO radar "co-located" and the statistical one "widely separated", as illustrated in Fig. 1.

One important difference between the two configurations is the signal model: Widely Separated antennas take advantage of the spatial properties of extended targets while the target is modeled as a point with no spatial property for co-located antennas.

In other words, Widely Separated antennas see different independent aspects of the target while the co-located antennas see the same target RCS up to some known relative delay due to the geometries of the antennas [3].

In radar automatic detection system, adaptive threshold is a necessary unit to keep the false alarm rate constant under the noise level variation and interferences.

Cell-Averaging (CA-CFAR) gives the optimum estimation for independent and identical exponential distributed cell samples in homogenous environments.

The presence of interfering targets is a case of the non-homogeneous background which can occur in a real situation. In this situation the performance of the CA-CFAR degrades seriously. In order to improve the performance detection in this situation, several detectors are proposed based on the order statistic technique: the OS-CFAR [4], the GOSCA-CFAR [5], the OSGO-CFAR and the OSSO-CFAR [6].

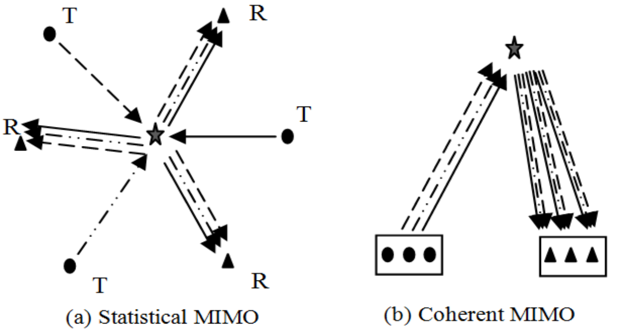

Figure 1. MIMO radar concept.

Target detection in MIMO radars has the interest of several works in the Neyman-Pearson sense [7], [8] or using the Generalized Likelihood Ratio [9].

Recently, Janatian [10] has generalized the CA-CFAR, the SO-CFAR, the OS-CFAR and the ACMLD (Automatic Censored Mean-Level Detector) for the Widely Separated MIMO radars in homogeneous and non-homogeneous clutter (presence of interfering targets).

In this paper, we generalize the GOSCA-CFAR, OSGO-CFAR and the OSSO-CFAR for Widely Separated MIMO radars in the Neyman-Pearson sense. Close form expressions for Pfa and Pd of these detectors in a homogeneous background are derived, assuming a white Gaussian noise model.

The paper is organized as follow: signal model in MIMO radar is developed in section 2, in section 3 we define mathematical models for MIMO radars of the proposed detectors in a homogeneous background. Results and discussions for the performance of the generalized detectors in homogeneous and in the presence of interfering targets are presented in section 4. Conclusions are drawn in section 5.

II. Signal Model in MIMO Radars

We consider a MIMO radar system that has M transmit antennas and N receive antennas with all the antennas

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widely separated as shown in Fig. 1.a. It is supposed that the mth transmitter transmits a signal $\sqrt{E/M}\,s_m(t)$, where E is the total transmitted power and $\|s_m(t)\|^2 = 1$. This means that systems with a lower number of nodes (couples Tx-Rx) have an increased available power per node. In other words, each of the M transmitters is provided with a power of $\frac{E}{M}$. The nth received signal is modelled as follows:

$$r_n(t) = \sum_{m-1}^{M} \alpha_{m,n}(\sigma) s_m\left(t - \frac{R_{m,n}}{c}\right) + e_n(t) \qquad (1)$$

where $s_m(t)$ is the mth transmitted signal, e(t) additive thermal noise, $\alpha_{m,n}$ given by the following expression is a complex coefficient including the amplitude and the phase of the received signal.

$$\sqrt{\frac{E}{M}} \sqrt{\frac{G_t G_r \lambda_m^2 \sigma}{(4\pi)^3 R_m^2 R_n^2}} \exp\left(-j\frac{2\pi R_{m,n}}{\lambda_m}\right) \qquad (2)$$

Assuming that orthogonal waveforms are transmitted, such that, they can be separated in each receiver, the received signal after the matched filtering can be expressed as:

$$q_{m,n} = \alpha_{m,n} + n_{m,n} \qquad (3)$$

The MIMO radar detection problem can be formulated in terms of the binary hypothesis test as follow:

$$Q_0 = \begin{cases} n, & H_0 \\ \alpha + n, & H_1 \end{cases}$$

Under hypothesis H_0, target declared absent in the received signal and the signal contains only clutter. Under hypothesis H_1, target declared present in the received signal and the signal contains both target and clutter.

The classical Neyman-Pearson detector which uses the Likelihood ratio test is given by:

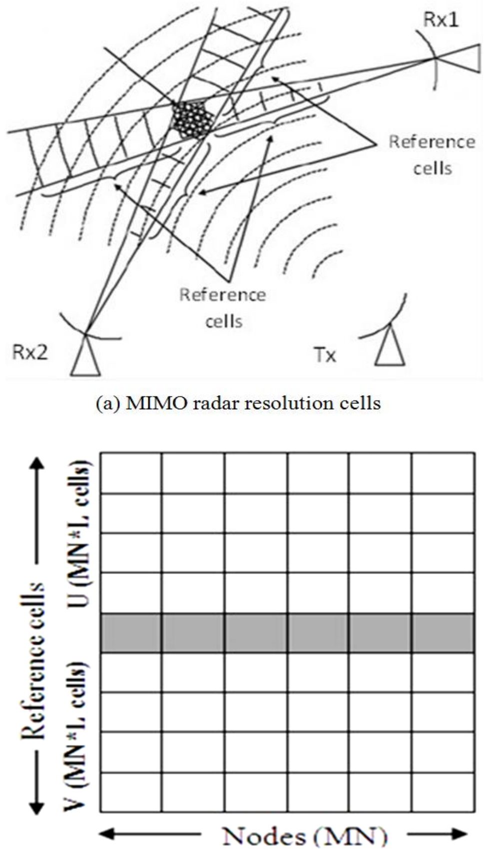
$$T(q_0) = Log\frac{P(q_0/H_1)}{P(q_0/H_0)} \gtrless \gamma \qquad (4)$$

where $P(q_0|\, H_1)$ and $P(q_0|\, H_0)$ are the probability density functions of the observation vector under the hypothesis target present or absent respectively. The threshold γ is determined by tolerated level of false alarms.

This structure is equivalent to the following [2]:

$$\|Q_0\|^2 \gtrless \gamma \qquad (5)$$

where Q_0 is the matched filter outputs for the cell under test and for a CFAR processing $\|Q_0\|^2$ is the sum of the CUT powers and γ=T*Z is a threshold multiplier, where Z is the noise power level estimation and T is the scale parameter.



(a) MIMO radar resolution cells

(b) MIMO radar simplified configuration

Figure 2. MIMO radar concept.

The structure of the received data is shown in Fig. 2, where the range gating method is employed in only one angular-resolution cell Fig. 2.a. Each receiver can sense the echoes of all transmitters, therefore, this structure can be formulated as a matrix with M*N columns and each column contains a CUT (grey cells) plus 2*L reference cells as shown in Fig. 2.b.

III. Mathematical Models for MIMO Radars

A block diagram of the GOSCA, the OSGO and the OSSO CFAR detectors for conventional radars is shown in Fig. 3.

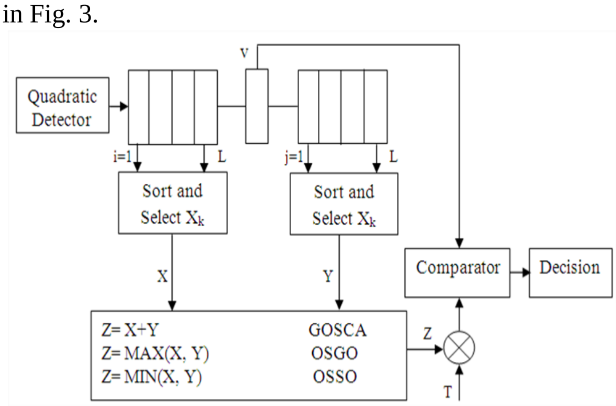


Figure 3. Block diagram of the proposed CFAR detectors in conventional radars.

where the GOSCA uses the SUM, the OSGO uses the MAXIMUM and the OSSO uses the MINIMUM of the two local estimators.

In this section we analyze the proposed CFAR detectors for MIMO radars.

Indeed, target in the test cell of each receiver is assumed to be fluctuating according to the Swerling I model. We assume that the noise in each receiver is white Gaussian noise, and the detection envelope is Rayleigh distributed; hence, test cells and reference cells are exponential distributed, and assumed to be independent and identically distributed. The probability density function (PDF) of the test cells is:

$$f(t)=\frac{1}{\lambda}\exp\left(-\frac{t}{\lambda}\right) \tag{6}$$

where λ= μ when H_0 is assumed and λ= μ*(1+SNR) when H_1 is assumed. SNR indicate the signal to noise power ratio and μ is the background noise average power.

According to the previous assumptions, (5) takes the following expression:

$$q_0=\sum_{j=1}^{MN} q_{0j} \gtrless \gamma \tag{7}$$

where q_{0j} is the CUT in each column which has an exponential distribution, thus, q_0 has a Gamma distribution:

$$f_{q_0}(t)=\frac{1}{\mu\Gamma(MN)}\left(\frac{t}{\mu}\right)^{MN-1}\exp\left(-\frac{t}{\mu}\right) \tag{8}$$

A. Generalization of the GOSCA-CFAR

To generalize this detector for MIMO radars, first, all leading cells U and lagging ones V are ranked separately in ascending order and the two *k*th (k=3MNL/4) largest samples are selected, the noise power level estimation uses the SUM of the two local estimators Z=X+Y. The PDF of Z is given by [11]:

$$f_z(z)=f_x(z)*f_y(z) \tag{9}$$

where $X=U_{(k)}$, $Y=V_{(k)}$

The moment generating function of Z is given by the product of the two moments generating functions of X and Y (X and Y are independent):

$$M_z(s)=M_x(s)*M_y(s) \tag{10}$$

where

$$M_x(s)=\prod_{f=0}^{k-1}\frac{L-f}{L-f+\mu s}$$

And

$$M_y(s)=\prod_{f=0}^{k-1}\frac{L-f}{L-f+\mu s}$$

So we have:

$$M_z(s)=\left(\prod_{f=0}^{k-1}\frac{L-f}{L-f+\mu s}\right)^2 \tag{11}$$

The probabilities of false alarm (Pfa) and detection (Pd) can be obtained by using the contour integral, which can be expressed in terms of the residue theorem [15]:

$$P_{\mathrm{fa}}=-\sum_{i0}\left(\mathrm{Re}\,s\left(\frac{M_{q_0|H_0(s)}}{s}M_z(-Ts)\right),s_{i0}\right). \tag{12}$$

$$P_{\mathrm{d}}=-\sum_{i1}\left(\mathrm{Re}\,s\left(\frac{M_{q_0|H_1(s)}}{s}M_z(-Ts)\right),s_{i1}\right). \tag{13}$$

where Res[.] denotes the residue, s_{i0} and s_{i1} are the poles of the moments generating functions $M_{q_0|H_0(s)}$ and $M_{q_0|H_1(s)}$ which are the MGF of the cell under test q_0 under the hypothesis H_0 and H_1 respectively, lying in the left half *S*-plane. $M_z(-Ts)$ is the MGF of the noise power level estimated at z=-*T*s. The MGFs $M_{q_0|H_0(s)}$ and $M_{q_0|H_1(s)}$ are given by:

$$M_{q_0|H_0(s)}=\frac{1}{(1+\mu s)^{MN}} \tag{14}$$

$$M_{q_0|H_1(s)}=\frac{1}{\left(1+\mu\left(1+\frac{SNR}{M}\right)s\right)^{MN}} \tag{15}$$

where there is only one pole of order *MN* at $s_0=-1/\mu$ under hypothesis H_0 and $s_0=-1\Big/\mu\left(1+\frac{SNR}{M}\right)$ under hypothesis H_1 and as [26] the residue at a pole of order *n* is given by:

$$\mathrm{Re}\,s(f(s),s_0)=\frac{1}{(n-1)!}\lim_{s\to s_0}\left\{\frac{d^{n-1}}{ds^{n-1}}\left[f(s)*(s-s_0)^n\right]\right\} \tag{16}$$

The probability of false alarm and detection become:

$$P_{\mathrm{fa}}=-\frac{1}{(MN-1)!}\frac{d^{MN-1}}{dz^{MN-1}}\left(\frac{1}{z}\left(\prod_{f=0}^{k-1}\frac{L-f}{L-f-Tz}\right)^2\right)\Big|_{z=-1} \tag{17}$$

$$P_{\mathrm{d}}=-\frac{1}{(MN-1)!}\frac{d^{MN-1}}{dz^{MN-1}}\left(\frac{1}{z}\left(\prod_{f=0}^{k-1}\frac{L-f}{L-f-\frac{Tz}{1+\frac{SNR}{M}}}\right)^2\right)\Big|_{z=-1} \tag{18}$$

B. Generalization of the OSGO-CFAR

The same scheme of the GOSCA-CFAR is used to generalize the OSGO-CFAR for MIMO radars where the MAXIMUM operator is employed in place of the SUM:

$$Z=\max\left(U_{(k)},V_{(k)}\right) \tag{19}$$

where $U_{(k)}$ and $V_{(k)}$ are the *k*th (k=3MNL/4) largest samples in the leading and lagging parts U and V, the PDF of Z is given by:

$$f_z^{OSGO}(z) = 2 f_{U_{(k)}}(z) F_{U_{(k)}}(z) \tag{20}$$

where

$$f_{U_{(k)}}(z) = \frac{k}{\mu}\binom{MNL}{k}\left(1-\exp\left(-\frac{z}{\mu}\right)\right)^{k-1}\exp\left(-\frac{z}{\mu}\right)^{MNL-k+1} \tag{21}$$

$$F_{U_{(k)}}(z) = \sum_{i=k}^{MNL}\binom{MNL}{i}\left(1-\exp\left(-\frac{z}{\mu}\right)\right)^{i}\exp\left(-\frac{z}{\mu}\right)^{MNL-i} \tag{22}$$

$$P_{\text{fa}} = \int_0^\infty f_z(z)\Pr(q_0 \ge Tz|H_0)dz\,. \tag{23}$$

$$P_{\text{d}} = \int_0^\infty f_z(z)\Pr(q_0 \ge Tz|H_1)dz\,. \tag{24}$$

After some calculations, the probability of false alarm of the OSGO is found to be:

$$P_{\text{fa}}^{\text{OSGO}} = 2k\binom{MNL}{k}\sum_{i=k}^{MNL}\binom{MNL}{i}\sum_{j=0}^{k+i-1}\binom{k+i-1}{j}(-1)^j \times \sum_{f=0}^{MN-1}\frac{1}{f!}T^f\left[\frac{1}{2MNL-k-i+j+T+1}\right]^{f+1} \tag{25}$$

Replacing T with $T\Big/\left(1+\frac{SNR}{M}\right)$ we find the probability of detection of the OSGO –CFAR.

C. Generalization of the OSSO-CFAR

As the generalization scheme of the GOSCA-CFAR and the OSGO-CFAR the OSSO-CFAR uses the MINIMUM operator as follow:

$$Z = \min\left(U_{(k)}, V_{(k)}\right) \tag{26}$$

where $U_{(k)}$ and $V_{(k)}$ are the *k*th (k=3MNL/4) largest samples in the leading and lagging parts U and V, the PDF of Z is given by: :

$$f_z^{OSSO}(z) = 2 f_{U_{(k)}}(z) - f_z^{OSGO}(z) \tag{27}$$

After some calculations, the probability of false alarm of the OSGO is found to be:

$$P_{\text{fa}}^{\text{OSSO}} = k\binom{MNL}{k}\sum_{i=0}^{k-1}\binom{k-1}{i}(-1)^i\sum_{f=0}^{MN-1}\frac{1}{f!}T^f \times \sum_{f=0}^{MN-1}\frac{1}{f!}T^f\left[\frac{1}{MNL-k-i+T+1}\right]^{f+1} - P_{\text{fa}}^{\text{OSGO}} \tag{28}$$

Replacing T with $T\Big/\left(1+\frac{SNR}{M}\right)$ we find the probability of detection of the OSSO –CFAR.

IV. Results and Discussions

In this section, we present the performance of the CFAR detectors described above in homogenous and non-homogenous situation (interfering targets). Results are given for a design Pfa of 10^{-4}, L=8 and k takes different values that depend on the number of nodes (MN) considered.

For the MIMO concept, we assume that interfering targets have the same radar cross section as the target in the CUT.

Fig. 4 shows the Pd of these detectors versus SNR (dB) in a homogeneous background in the case of one node MN=1 (SISO), two nodes MN=2 (one transmitter M=1 and two receivers N=2) and in the case of four nodes MN=4 (two transmitters M=2 and two receivers N=2). We observe form this Figure that the GOSCA-CFAR detector has the best performance and the OSSO-CFAR detector has the smallest performance, which is well known in comparison of the performance of these detectors for the SISO concept.

We can observe from this Figure also that where the number of nodes increases best performance is obtained. In other word, less values of SNR are needed in MIMO radar to achieve the same detection performance compared to the SISO one.

Fig. 5 shows the Pd for the proposed detectors in a homogeneous background in the case of four nodes (MN=4); in this case, according to the receivers number, two different configurations are presented: one transmitter M=1 and four receivers N=4 or two transmitters M=2 and two receivers N=2.

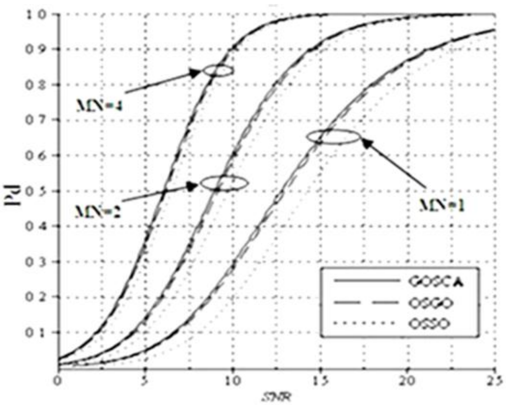


Figure 4. Pd of the different detectors versus SNR(dB) in homogenous background for pfa=10^{-4} , for diferent number of nodes MN=1, 2 and 4 (M=1).

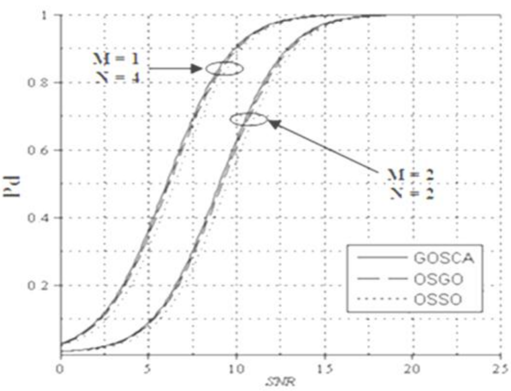


Figure 5. Pd of the different detectors versus SNR(dB) in homogenous background for pfa=10^{-4} , k=24, for 4 nodes in two case (M=1, N=4) and (M=2, N=2).

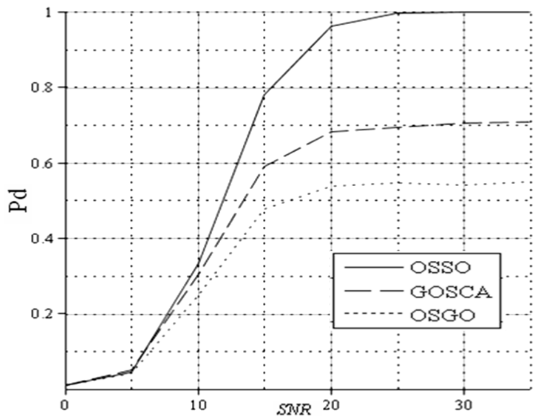


Figure 6. Pd of the different detectors versus SNR(dB) in the presence of five interfering targets for pfa=10^{-4} , k=24 and MN=2 (M=1, N=2).

It is observed from this Figure that the number of receivers influence on the detection performance. So, when the number of nodes is kept constant, the configuration which has a high number of receivers gives best performance.

In presence of interfering targets, Fig. 6 shows the performance detection comparison of the different detectors in the presence of five (5) interfering targets in the case of two nodes (M=1, N=2). Where we observe that when the number of interfering targets increases, the OSSO-CFAR has best performance and the performance of the GOSCA and the OSGO-CFAR degrade seriously.

V. CONCLUSION

In this paper, we have analyzed the detection performance of the GOSCA, OSGO and the OSSO-CFAR detectors for MIMO radars in homogeneous and non-homogeneous environment. The obtained results have shown that the OSSO-CFAR has best performance in the case of a high number of interferences.

REFERENCES

[1] J. Li and P. Stoica, *MIMO Radar Signal Processing*, New Jersey: Wiley, 2009.

[2] E. Fishler, *et al*., "Spatial diversity in radars- models and detection performance," *IEEE Transaction on Signal Processing*, vol. 54, no. 3, pp. 823-838, 2006.

[3] C. Y. Chong, "Signal processing for MIMO radars: Detection under gaussian and non-gaussian environments and application to STAP," Ph.D. thesis, Supelec, France, 2011.

[4] H. Rohling, "Radar CFAR thresholding in clutter and multiple target situations," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 19, no. 4, pp. 608-621, 1983.

[5] H. You, "Performance of some generalised modified order statistics CFAR detectors with automatic censoring technique in multiple target situations," *IEE Proceedings, Radar, Sonar and Navigation*, vol. 141, no. 4, pp. 205-212, 1994.

[6] A. R. Elias-Fuste, M. G. DE Mercado, and E. R. Davo, "Analysis of some modified order statistics CFAR: OSGO and OSSO CFAR," *IEEE Transaction on Aerospace and Electronic Systems*, vol. 26, no. 1, pp. 197-202, 1990.

[7] C. Y. Chong, F. Pascal, J.P. Ovarlez, and M. Lesturgie, "MIMO Radar Detection in Non-Gaussian and Heterogeneous Clutter," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 1, pp. 115-126, 2010.

[8] G. Cui, L. Kong, and X. Yang, "Multiple-input multiple-output radar detectors design in non-Gaussian clutter," *IET Radar, Sonar and Navigation*, vol. 4, no. 5, pp. 724-732, 2010.

[9] J. Liu, Z. J. Zhang, Y. Cao, and S. Yang, "A closed-form expression for false alarm rate of adaptive MIMO-GLRT detector with distributed MIMO radar," *Signal Processing*, vol. 93, pp. 2771-2776, 2013.

[10] N. Janatian, M. Modarres-Hashemi, and A. Sheikhi, "CFAR detectors for MIMO Radars," *Circuits Systems and Signal Processing*, vol. 32, no. 3, pp. 1389-1418, 2013.

[11] A. Papoulis and A. B. Smith, *Probability, Random Variables and Stochastic Processes*, McGraw-Hill, Asia, 2002.