

# Parameter Identification and Temperature Compensation of Quartz Flexible Accelerometer Based on Total Least Squares

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**Abstract**—In this paper, the static rolling test of quartz flexible accelerometer in gravitational field is studied. We establish its error model and use the method of total least squares for identification. In addition, we study the influence of environment temperature on the accelerometer. The accelerometer output is corrected by compensation the bias and the scale factor. The results of the comprehensive experiments show that the method of total least square can effectively identify the model parameters of accelerometer. After temperature compensation, the temperature-varying the bias and the scale factor are reduced from  $7.201895e-4(g)$ ,  $3.109124e-4(V/g)$  to  $2.070704e-5(g)$ ,  $4.407450e-5(V/g)$ . The stability of the accelerometer output signal has been raised an order of magnitude.

**Index Terms**—quartz flexible accelerometer, total least squares, parameter identification, temperature compensation

## I. INTRODUCTION

Accelerometer is an important component in the inertial navigation system. The factors influencing the precision of accelerometer mainly have two aspects: accelerometer internal factors and external environmental factors of accelerometer work or calibration. Improving precision of model identification of accelerometer is the basic way to reduce the error of inertial navigation system. Therefore, accurately calibration accelerometer error model is very considerable. Accelerometer model parameters identification method, generally using the least squares method, but this method is easily affected by the input error [1], [2]. The total least squares method can be adopted to solve this problem, which can deal with output errors and input errors simultaneously [3], [4]. The common means for reducing the influence of environment temperature on the accelerometer output is the study of the characteristics of the temperature parameters of the accelerometer. Through temperature control box to make the accelerometer work in the multiple specified temperature points, the functional relationship between the accelerometer output and temperature is established, thus the accelerometer output is compensated, and the use accuracy of the accelerometer is augmented [5]-[9].

This paper, the gravitation field test for a long time for quartz flexible accelerometer is performed based on the ambient temperature. The static mathematic model and the temperature model of accelerometer are built. The total least squares method is used to identify more accurate model parameters. The accelerometer stationarity of output signal and the environmental adaptability in different temperatures are enhanced by temperature compensation method.

## II. STATIC MODEL OF ACCELEROMETER

### A. Static Mathematical Model of Accelerometer

The rolling test of accelerometer in gravitational field, usually the input reference quantity is measured by using the projection weight of the gravity acceleration of the laboratory local on the each axis of the accelerometer, with which to test or separate all performance parameters of accelerometer. The calibration bias and scale factor are accurate, however, the others nonlinear coefficients and cross-coupling coefficients are low precision. Accordingly, it is the foundation of a variety of other high g value overload test, and its test method often with single axis and multiple equal test angles of the rolling over accelerometer. When the twelve-position measuring system is adopted and accelerometer is installed at the “gate” state, a static mathematical model equation can be expressed as [10]

$$E = K_0 + \sum_{n=1}^3 K_n \sin^n \theta - K_o \cos \theta - K_{io} \sin \theta \cos \theta \quad (1)$$

where  $E$  is the accelerometer output;  $K_0$  is bias;  $K_1$  is scale factor;  $K_2$  and  $K_3$  are second-order nonlinearity coefficient and third-order nonlinearity coefficient, respectively;  $K_o$  is cross-axis sensitivity coefficient;  $K_{io}$  is cross-coupling coefficient;  $\theta$  is testing angle of accelerometer.

### B. The Static Temperature Model of Accelerometer

The use accuracy of accelerometer is greatly affected by the environmental temperature changes. The reason is that the accelerometer is sensitive to temperature and the surrounding temperature gradient [11]. In the static mathematical model of accelerometer, the model

parameters can be regarded as a function of temperature. To cut down the effects of environmental temperature on the accelerometer output, the polynomial between model parameters and temperature is created as follows

$$K_i(T) = k_{i0} + k_{i1}T + k_{i2}T^2 + \dots + k_{im}T^m \quad (2)$$

where  $K_i$  represents different static temperature model parameters;  $k_{i0}, k_{i1}, \dots, k_{im}$  are the unknown estimated temperature coefficients. The 20°C is selected as standard temperature, that is

$$T = t - 20^\circ\text{C}$$

and  $t$  is environment temperature;  $m$  is degree of polynomial.

### III. THE TOTAL LEAST SQUARES METHOD FOR IDENTIFICATION PARAMETERS

For the data matrix  $A \in R_n^{m \times n}$  ( $m > n$ ) which is subjected to the effect of error matrix  $E_A$  and the observation vector  $y \in R_n^{m \times 1}$  is perturbed by error vector  $e_y$ , simultaneously. Both  $E_A$  and  $e_y$  are assumed to independently and identically distribute rows with zero mean and the same variance. The estimated parameters vector is  $\xi \in R_n^{m \times 1}$ . The mathematical model of total least squares (TLS) can be described as [12], [13]

$$y - e_y = (A - E_A)\xi \quad (3)$$

Unknown parameters vector of the total least squares estimation can be obtained

$$\xi = (A^T A - \sigma_{n+1}^2 I)^{-1} A^T y \quad (4)$$

where  $\sigma_{n+1}$  is the singular value of  $A$ .

Accelerometer output value contains variety of errors and the test turntable includes positioning error which is caused by measured angle error, control system positioning accuracy, verticality and rotary precision. The temperature error involves continuous temperature drift and thermometer itself accuracy of manufacturing, etc. Consequently, different static mathematical model parameters  $K_i$  and diverse temperature coefficients  $k_{i0}, k_{i1}, \dots, k_{im}$  can be determined by introducing the TLS method.

### IV. STATIC TEMPERATURE COMPENSATION OF ACCELEROMETER

#### A. The Collection and Preprocessing of Data

Based on the natural environment temperature as the test temperature point of the accelerometer experiment, the output of the accelerometer data is obtained by adopting the LabVIEW data acquisition software and the data acquisition card PCI-6221 which is 16 bit resolution. Through regular test method, the temperature in the range of 10°C to 35°C are gained, the precision of the temperature measuring meter is  $\pm 0.1^\circ\text{C}$ , the sampling

frequency is 1000Hz and the sampling time for 20 seconds of the accelerometer in each temperature.

It is worth mentioning that the de-noising of accelerometer output signal is done by applying the singular value decomposition (SVD) method before utilizing the TLS method identify model parameters. The cause is that the accelerometer output is always affected by all kinds of noise in the gravitational field twelve-position rolling test. The noise is not only evidently shortens the signal-to-noise ratio, but also impacted the analysis accuracy and the reliability of the signal. The concrete method of SVD de-noising pretreatment can be found in [14]-[16].

#### B. The Temperature Compensation Analysis of the Accelerometer

In order to analysis all parameters are influenced by temperature, the  $K_2$ , the  $K_3$ , the  $K_o$  and the  $K_{io}$  along with the variation of temperature are described in Fig. 1, the curve tracing of the  $K_0$  vary with temperature is drawn as Fig. 2, the  $K_1$  changes with temperature is painted as Fig. 3.

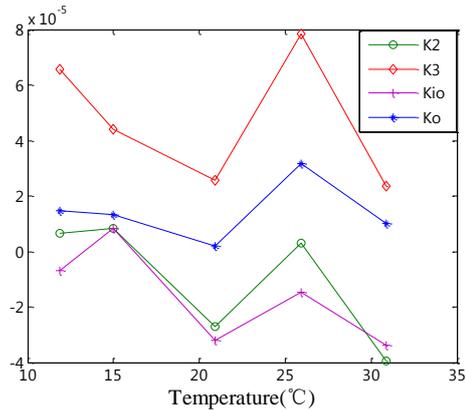


Figure 1. The curves of static mathematical model parameters the  $K_2$ , the  $K_3$ , the  $K_o$  and the  $K_{io}$  along with temperature variation.

It can be seen from Fig. 1, Fig. 2 and Fig. 3 that the  $K_2$ , the  $K_3$ , the  $K_o$  and the  $K_{io}$  do not change significantly with temperature, whereas the  $K_0$  and the  $K_1$  are altered more obvious with temperature. In fact, the influence that the bias and the scale factor are impacted by temperature is larger than other parameters when accelerometer works under different temperatures [5]-[9]. As a result, the  $K_2$ , the  $K_3$ , the  $K_o$  and the  $K_{io}$  can be treated as a quantity that has nothing to do with temperature. Due to the  $K_0$  and the  $K_1$  are strong correlation with temperature, compensating for the  $K_0$  and the  $K_1$  with respect to temperature will distinctly increase the measurement accuracy of accelerometers.

#### C. Compensation Process and Results of the Accelerometer

Choosing the degree of polynomial is 3 in (2), the temperature coefficients of the  $K_0$  and the  $K_1$  are obtained according to the TLS method identification and are

shown in Table I. Through the temperature coefficient values in Table I and the corresponding temperature, the

fitted values of the  $K_0$  and the  $K_1$  are acquired. The fitting results are shown in Table II.

TABLE I. TEMPERATURE COEFFICIENTS OF THE  $K_0$  AND THE  $K_1$

Temperature coefficient	$k_{i0}$	$k_{i1}$	$k_{i2}$	$k_{i3}$
$i = 0$	3.361150e-4	-4.833246e-5	-6.272767e-7	1.278691e-7
$i = 1$	0.0295172	9.001510e-6	-3.537117e-7	8.927932e-8

TABLE II. THE FITTED VALUES OF THE  $K_0$  AND THE  $K_1$

Model parameter	Temperature (°C)	Original data	Fitted value	Fitted residue
$K_0$ (g)	11.9	6.225287e-4	6.184974e-4	4.031345e-6
	15.0	5.366733e-4	5.461117e-4	-9.438402e-6
	20.9	3.034695e-4	2.922009e-4	1.126863e-5
	25.9	0.475373e-4	0.553796e-4	-7.842260e-6
	30.9	-0.976607e-4	-0.996414e-4	1.980683e-6
$K_1$ (V/g)	11.9	0.029382	0.029374	8.580642e-6
	15.0	0.029432	0.029452	-2.008946e-5
	20.9	0.029549	0.029525	2.398503e-5
	25.9	0.029560	0.029576	-1.669210e-5
	30.9	0.029693	0.029689	4.215846e-6

The Table II indicates that there is a good fitting result for the  $K_0$  and the  $K_1$ , the maximum absolute values of the fitted residual are 1.126863e-5(g) and 2.398503e-5(V/g), respectively. Hence, with the temperature coefficients of the Table I, the  $K_0$  and the  $K_1$  are compensated. Its compensation can be represented as

$$\Delta K_0 = 4.833246 \times 10^{-5} T + 6.272767 \times 10^{-5} T^2 - 1.278691 \times 10^{-7} T^3 \quad (5)$$

$$\Delta K_1 = -9.001510 \times 10^{-6} T + 3.537117 \times 10^{-7} T^2 - 8.927932 \times 10^{-8} T^3 \quad (6)$$

The curves of the  $K_0$  and the  $K_1$  variation with temperature after the temperature compensation as shown in Fig. 2 and Fig. 3. Before temperature compensation, the alterations of the  $K_0$  and the  $K_1$  are 7.201895e-4(g) and 3.109124e-4(V/g), respectively. Whereas after temperature compensation, the variation of the  $K_0$  is 2.070704e-5(g), and the  $K_1$  is 4.407450e-5(V/g) with temperature.

According to the compensation of the  $K_0$  and the  $K_1$  for temperature, the accelerometer actual output is adjusted. The accelerometer output change with the temperature as shown in Fig. 4. Fig. 4(a) and Fig. 4(b) are the output of the accelerometer which in the position of +1g gravitational field.

The Fig. 4(a) shows that the accelerometer output has an apparently downward trend as the temperature increases further, the standard deviation of original output is 1.984475e-4(V).

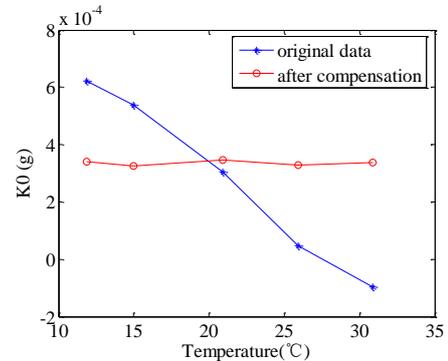


Figure 2. Curves of the variation of the  $K_0$  with temperature before and after compensation.

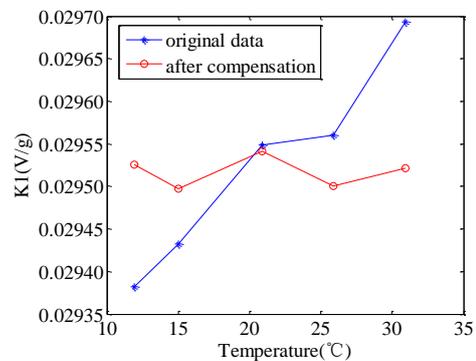


Figure 3. Curves of the variation of the  $K_1$  with temperature before and after compensation.

The Fig. 4(b) indicates that the accelerometer output maintains stable basically, which standard deviation is

6.603851e-5(V), the temperature compensation effect is prominent.

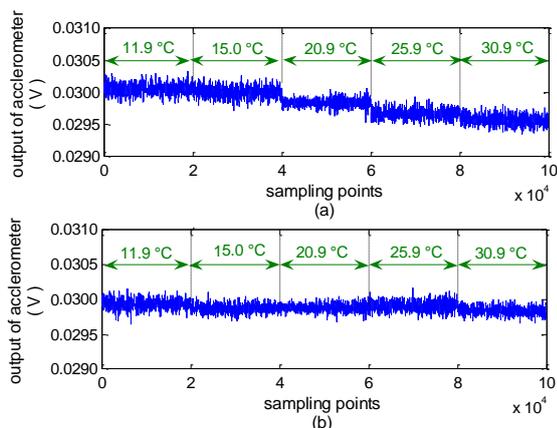


Figure 1. (a) Output of the accelerometer before temperature compensation in the position of +1g gravitational field; (b) Output of the accelerometer after temperature compensation in the position of +1g gravitational field.

In order to test whether the above temperature model and temperature coefficients can be applied to other temperatures to perfect the output of the accelerometer, the output of the accelerometer at 18.8°C, 28.8°C and 32.1°C are compensated with the temperature coefficients of Table I. The accelerometer output before temperature compensation in the position of +1g gravitational field is as shown in Fig. 5(a), and after temperature compensation is as shown in Fig. 5(b). The standard deviations are 1.974324e-4(V) and 8.743841e-5(V) before and after temperature compensation, respectively. It implies that the accelerometer output of other temperatures could be compensated predictably with the temperature coefficients of Table I.

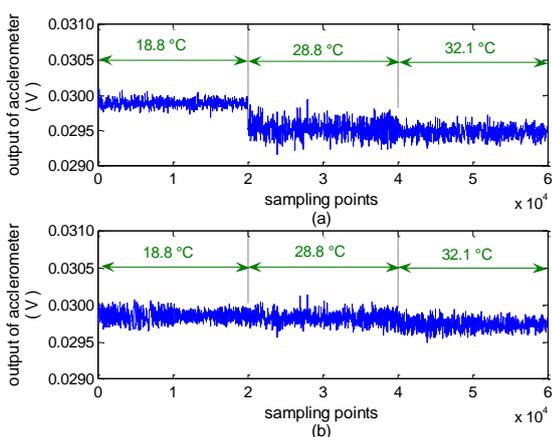


Figure 2. (a) The original output of accelerometer under other temperatures in the position of +1g gravitational field; (b) The compensatory output of the accelerometer under other temperatures in the position of +1g gravitational field.

By the above results can be seen, for compensating the  $K_0$  and the  $K_1$  on temperature can realize the accelerometer output compensation, and the temperature model and temperature coefficients can also be used for

other temperatures to make good the accelerometer output. So the temperature model and the compensation method are correct and feasible. The compensation of the accelerometer output has something to do with the temperature coefficients of the compensation model, the selection standard environment temperature and the degree of the fitting polynomial. If setting up a suitable standard temperature and a degree of the fitting polynomial in all the temperature ranges of the accelerometer working, a better output of the accelerometer can be achieved after temperature compensation.

## V. CONCLUSION AND FUTURE RESEARCH

This paper, the TLS method is utilized to identify the static mathematical model and the temperature model of the quartz flexible accelerometer in gravitational field by twelve-position rolling test. The ambient temperature can serve as the experiment temperature of the accelerometer, a series of experiments are carried out as time goes on. Through the method of temperature compensation, the accelerometer output can be compensated. The results support that the TLS method to identify parameters of high precision, the effect of the accelerometer temperature compensation is obvious and the stability of the accelerometer output signal is improved.

In order to further improve the accuracy of accelerometer, more factors affecting the output of accelerometer, such as the output drifting caused by time, etc. can be taken into account in the future work.

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