Design of Higher Order Digital IIR Low Pass Filter Using Hybrid Differential Evolution

D. S. Sidhu
Department of Electronics and Communication Engineering, Giani Zail Singh Punjab Technical University Campus, Bathinda-151001, India
Email: ds.sidhu@yahoo.com

J. S. Dhillon
Department of Electrical and Instrumentation Engineering, Sant Longowal Institute of Engineering and Technology, Longowal-148106, India
Email: jsdhillonp@yahoo.com

Dalveer Kaur
Department of Electronics and Communication Engineering, Punjab Institute of Technology, Kapurthala-144601, India
Email: dn_dogra@rediffmail.com

***Abstract*—In this paper, hybrid technique with differential evolution (DE) method as a global search technique and binary successive approximation (BSA) based evolutionary search method as a local search technique is proposed. Evolutionary search is used for the exploration and pattern search method for exploitation. To initiate with good population, the opposition based learning strategy is applied. Migration strategy is applied to maintain the diversity in the population. The above proposed hybrid technique has been applied effectively to solve the multi-parameter optimization problem of higher order low-pass stable digital IIR filter design. The obtained results prove that the proposed technique is better than or comparable to other algorithms used by other researchers and can be applied to design HP, BP and BS IIR filters.**

***Index Terms*—digital IIR filter, hybrid heuristic search technique, Differential Evolution (DE) method, Binary Successive Approximation (BSA), multi-parameter optimization**

I. INTRODUCTION

Signals are classified into two categories, namely, continuous-time and discrete-time signals. Digital signal processing is a numerical manipulation of discrete-time signal data. With the advent of digital circuit technology, simple, cheaper and faster digital computers with huge memory storage capability are able to perform complex real time digital signal processing. On the other hand, there is need of complex analog processors to perform analog signal processing. Digital signal processors are easy to program and digital signal transfer function can also be modified easily. Error detection, correction and signal compression is also possible in digital signal processing. Filtering is an essential part of signal processing for different applications such as telecommunications, consumer electronics, speech processing, image processing, industrial applications, military electronics and biomedical systems. Filters are used to remove frequencies in certain range of the spectrum of the signal. Digital filters are reliable, flexible, and versatile as compared to analog filters. The sampled values of input signal and the coefficients of transfer function of the digital filter can easily stored in the memory [12], [21].

Based upon the unit impulse response of the system, digital filter are classified as infinite impulse response (IIR) filter and finite impulse response (FIR) filter. IIR filter provides better response than FIR filters having same number of coefficients. But IIR filter may become unstable during optimization process due to its multimodal error surface. During optimization process, the stability of the filter can be obtained by limiting the parameter space in a suitable range. Gradient based algorithm may stick in local minima and may not converge to global minima due to multimodal error surface of IIR filter [20]. In order to overcome this problem and to find the global minima, global optimization techniques like genetic algorithm (GA) [3], differential evolution (DE) [7], particle swarm optimization (PSO) [5] artificial bee colony (ABC), predator prey optimization (PPO) etc. have been used by the researchers [4,10,11,16,19,20,22,23] to design stable digital IIR filters. In this paper, a hybrid technique with differential evolution (DE) method as a global search technique and binary successive approximation (BSA) based evolutionary search method as a local search technique is proposed. The opposition based learning strategy is applied to have a good initial population. To

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maintain the diversity in the population, migration strategy is also applied.

This paper is divided into five sections. In section II, digital IIR filter problem is described. The differential evolution (DE) algorithm and binary successive approximation (BSA) based evolutionary search method for optimal IIR filter design are explained in section III. In section IV, the performance of proposed hybrid heuristic search technique has been evaluated and obtained results are compared with results of other authors. The final conclusion and discussions are outlined in section V.

II. IIR Filter Design Problem

Design of digital IIR filter involves the determination of filter coefficients which meet various specification of the filter such as pass-band width and gain, stop-band width and attenuation, edge frequencies of pass-band and stop-band, peak ripples in pass-band and stop-band.

Cascaded digital IIR filter has several first order and second order sections together [6, 8]. Mathematically, the structure of cascading type IIR filter is stated below [4]:

$$H(\omega, x) = A\left(\prod_{i=1}^{M} \frac{1 + a_i e^{-j\omega}}{1 + b_i e^{-j\omega}}\right)$$

$$\times \left(\prod_{k=1}^{N} \frac{1 + p_k e^{-j\omega} + q_k e^{-2j\omega}}{1 + r_k e^{-j\omega} + s_k e^{-2j\omega}}\right) \quad (1)$$

where

$$X = [A, a_1, b_1, \ldots a_M, b_M, p_1, q_1, r_1, s_1, \ldots, p_N, q_N, r_N, s_N] \quad (2)$$

Vector X denotes the filter coefficients dimensions $N_V \times 1$ with $N_V = 2M + 4N + 1$

The coefficients of the transfer function $H(\omega, x)$ are approximated to design the desired filter. The transfer function is compared with the desired transfer function $H_d(\omega, x)$. From these two values, the magnitude error function $e_1(x)$ is derived and is given below:

$$e_1(x) = \sum_{i}^{k} \left| H_d(\omega_i) - |H(\omega_i, x)| \right| \quad (3)$$

$$\text{where } H_d(\omega_i) = \begin{cases} 1, & for\ \omega_i \in passband \\ 0, & for\ \omega_i \in stopband \end{cases} \quad (4)$$

The ripple magnitude of pass band $\delta_1(x)$ and of stop band $\delta_2(x)$ [2] are defined as below

$$\delta_1(x) = \max_{\omega_i}\{|H(\omega_i, x)|\} - \min_{\omega_i}\{|H(\omega_i, x)|\} \quad for\ \omega_i \in passband \quad (5)$$

And

$$\delta_2(x) = \max_{\omega_i}\{|H(\omega_i, x)|\} \quad for\ \omega_i \in stopband \quad (6)$$

Considering all these objectives, subject to the stability criterion obtained by Jury method [1], the multi criterion constrained optimization problem is stated below:

$$\text{Minimize } f_1(x) = e_1(x) \quad (7a)$$

$$\text{Minimize } f_2(x) = \delta_1(x) \quad (7b)$$

$$\text{Minimize } f_3(x) = \delta_2(x) \quad (7c)$$

Subject to the following stability constraints

$$1 + b_i \geq 0 \quad (i = 1, 2, \ldots, M) \quad (8a)$$

$$1 - b_i \geq 0 \quad (i = 1, 2, \ldots, M) \quad (8b)$$

$$1 - s_k \geq 0 \quad (k = 1, 2, \ldots, N) \quad (8c)$$

$$1 + r_k + s_k \geq 0 \quad (k = 1, 2, \ldots, N) \quad (8d)$$

$$1 - r_k + s_k \geq 0 \quad (k = 1, 2, \ldots, N) \quad (8e)$$

The above multi criterion constrained optimization problem is converted into a scalar constrained optimization problem by using weighted sum of above objective functions as below

Minimize

$$f(x) = \sum_{i=1}^{3} \left(w_i \times f_i(x)\right) + R[g(x)] \quad (8f)$$

Subject to the stability constraints stated by Eq.(8a) to Eq.(8e). w_i are assigned weights to i^{th} objective function.

Exterior penalty method has been applied to handle stability constraints.

$$f(x) = \sum_{i=1}^{3} \left(w_i \times f_i(x)\right) + R \sum_{k=1}^{N} < 1 - s_k >^2$$
$$+ R \sum_{i=1}^{M} [< 1 - b_i >^2 + < 1 + b_i >^2]$$
$$+ R \sum_{k=1}^{N} [< 1 + r_k + s_k >^2 + < 1 - r_k + s_k >^2] \quad (9)$$

where R is large value of penalty parameter.

III. Optimization Technique

Differential evolution (DE) algorithm is a population based algorithm. It is a parallel direct search method which utilizes N_P parameter vectors of N_V-dimensions. N_P remains same during optimization process. Initial vector population is chosen randomly and covers the entire search space. The DE algorithm follows steps like mutation, crossover, selection and migration etc. [7]. The steps are explained below:

A. Parameter Setting

Parameters such as population size (N_P), boundary constraints of each dimension of population vector (N_V), mutation factor f_m, crossover (CR) and maximum number of iterations (IT) are initially selected. Stopping criterion will be the maximum number of iterations. The set of real digital IIR filter coefficients is represented as a population. There are N_P members in the population. The numbers of real IIR filter coefficients are N_V.

B. Initialization of an Individual Population

Individual population is initialized with random value generated according to a uniform probability distribution in the N_V-dimensional problem space. With the given lower and upper limits of the search space, the population vector is described as below [17]:

$$x_{ij}^t = x_j^{min} + rand(\,)(x_j^{max} - x_j^{min}) \quad (10)$$
$$(j=1, 2, \dots, N_V;\ i=1, 2, \dots, N_P)$$

where *rand( )* is an uniform random number between 0 and 1. x_j^{min} and x_j^{max} are minimum and maximum permissible limits of the j^{th} filter coefficients.

C. Opposition Based Learning

Population based optimization techniques starts with some initial random values. Convergence time and optimized value depends upon the initial guesses of population. Since, there are sufficient (almost 50%) chances that the initial guess is right. So, there is an equal chance that opposite guess may be nearer to the optimal solution [13, 15]. So, starting with the closer of the two guesses as judged by the objective function, better solution is attained with smaller convergence time. The opposite population is obtained using the following expressions.

$$x_{ij}^{t+1} = x_j^U + x_j^L - x_{ij}^t \quad (11)$$
$$(j=1, 2, \dots, N_V;\ i=1, 2, \dots, N_P)$$

where

$$x_j^L = \begin{cases} x_j^{min} \ ; & t=0 \\ min\{x_{ij}^t\ ;\ (i=1,2,\dots,N_P)\} & t>0 \end{cases} \quad (12)$$

$$x_j^U = \begin{cases} x_j^{max} \ ; & t=0 \\ max\{x_{ij}^t\ ;\ (i=1,2,\dots,N_P)\} & t>0 \end{cases} \quad (13)$$
$$(j=1, 2, \dots, N_V)$$

The opposition strategy has also been applied during progressive iterations.

D. Mutation Operator

Mutation is an operator where weighted difference of randomly selected population vectors is added to another vector to generate new vector Z_{ij}. There are ten variations of differential evolution algorithm strategies (MS-1 to MS-10) [17] that can be used for optimization as described below:

$$Z_{ij1}^t = x_{Bj}^t + f_m\left(x_{R_1j}^t - x_{R_2j}^t\right) \quad (14)$$

$$Z_{ij2}^t = x_{R_1j}^t + f_m\left(x_{R_2j}^t - x_{R_3j}^t\right) \quad (15)$$

$$Z_{ij3}^t = x_{ij}^t + f_B\left(x_{Bj}^t - x_{ij}^t\right) + f_m\left(x_{R_1j}^t - x_{R_1j}^t\right) \quad (16)$$

$$Z_{ij4}^t = x_{Bj}^t + f_m\left(x_{R_1j}^t + x_{R_2j}^t - x_{R_3j}^t - x_{R_4j}^t\right) \quad (17)$$

$$Z_{ij5}^t = x_{R_5j}^t + f_m\left(x_{R_1j}^t + x_{R_2j}^t - x_{R_3j}^t - x_{R_4j}^t\right) \quad (18)$$

$$Z_{ij6}^t = x_{Bj}^t + f_m\left(x_{Bj}^t - x_{ij}^t\right) \quad (19)$$

$$Z_{ij7}^t = x_{Bj}^t + f_m\left(x_{Bj}^t - x_{ij}^t - x_{R_1j}^t - x_{R_2j}^t\right) \quad (20)$$

$$Z_{ij8}^t = x_{Bj}^t + f_B\left(x_{Bj}^t - x_{ij}^t\right) + f_m\left(x_{R_1j}^t - x_{R_1j}^t\right) \quad (21)$$

$$Z_{ij9}^t = x_{Bj}^t + f_m(x_{Bj}^t + x_{ij}^t - x_{R_1j}^t - x_{R_2j}^t) \quad (22)$$

$$Z_{ij10}^t = x_{Bj}^t + f_m(x_{Bj}^t - x_{Bj}^{t-1}) \quad (23)$$
$$(j=1,2,\dots\dots,\ N_V;\ i=1,2,\dots\dots,N_P)$$

where R_{ij} is random index of population and f_m is the mutation factor ∈ [0 2].

E. Crossover Operation

In order to increase the diversity, crossover is introduced. In this operation a trial vector is generated by replacing certain parameters of target vector by the corresponding parameters of a randomly generated donor vector [7].

Trial vector is produced by combining target vector $X_i^t = \left(x_{i1}^t, x_{i2}^t, \dots, x_{iN_V}^t\right)$ with randomly selected vector Z_i^{t+1} as shown below

$$U_i^{t+1} = \left(U_{i1}^{t+1}, U_{i2}^{t+1}, \dots, U_{iN_V}^{t+1}\right) \quad (24)$$

where

$$U_{ij}^{t+1} = \begin{cases} Z_{ij}^{t+1} & if(randb(j) \le CR \ \ or\ j = rnbr(i) \\ x_{ij}^t & if(randb(j) > CR \ \ or\ j \ne rnbr(i) \end{cases} \quad (25)$$

CR is the crossover rate ∈ [0 1]. *randb(j)* is the j^{th} evaluation of a uniform random number generator with outcome ∈ [0 1]. *rnbr(i)* is a randomly chosen index ∈ {1,2, ... N_V } which ensures that U_i^{t+1} gets at least one parameter from Z_i^{t+1}.

F. Selection

To decide whether or not the newly produced offspring should become a member of next generation; the trial vector U_i^{t+1} is compared with the target vector x_i^t using greedy criterion [14]. If f denotes the objective function under minimization, then

$$x_{ij}^{t+1} = \begin{cases} U_{ij}^{t+1} \ \ (j=1,2,\dots,N_V) : \ if\ f\left(U_{ij}^{t+1}\right) < f\left(x_{ij}^t\right) \\ x_{ij}^t \ \ (j=1,2,\dots,N_V) : \ otherwise \end{cases} \quad (26)$$

where *(i=1,2, ... , N_P)*

G. Exploratory Move

In exploratory move, the current point is perturbed in all possible directions for each variable at a time to record the best point. The present point is changed to the best point after each perturbation. At the end of all variable perturbations, if the point found is different from the original point, the exploratory move is a success; otherwise, the exploratory move is a failure.

In evolutionary method for N_V number of filter coefficients, 2^{Nv} feasible solutions are generated. A N_V dimensional hypercube of side Δ is formed around the point. x_i^C represents the coefficients of IIR filter from the current point of hypercube. The better solution is obtained by evaluating the objective function of the IIR filter. Another hypercube is formed around the current better point and the iterative process is continued. All the corners of the hypercube represented in the N_V binary bits code, are explored for better results simultaneously. Table I shows the coefficient pattern for 3-coefficient digital IIR filter where 3 bit binary code is used to represent the 8 corners of the three dimensional hypercube (Fig. 1). The decimal serial numbers of the hypercube are changed into their respective binary codes. The deviation from the current center point is obtained by replacing 0's of the code with -Δ and 1's with +Δ. With the increase of number of coefficients of the IIR filter, the number of hypercube corners increase exponentially and the process becomes time consuming [18].

H. BSA Strategy

To reduce the computational time, binary successive approximation (BSA) strategy is used to explore the optimal solution. BSA strategy to search for the optimal solution is explained in Fig. 2, where solution procedure moves towards the optimal solution by comparing two solutions at a time represented by the two corners of the hypercube [18].

TABLE I. COEFFICIENT VECTOR AT THE CORNERS OF Hypercube

Hyper cube corner	Possible combinations of 3-bits	Distance of hypercube from centre point	Possible coefficient pattern of the IIR filter at the corner of hypercube		
	$C_2\,C_1\,C_0$	$x_{i3}^c, x_{i2}^c, x_{i1}^c$			
0	000	$-\Delta_{i3}\ -\Delta_{i2}\ -\Delta_{i1}$	$x_{i3}^c - \Delta_{i3}$	$x_{i2}^c - \Delta_{i2}$	$x_{i1}^c - \Delta_{i1}$
1	001	$-\Delta_{i3}\ -\Delta_{i2}\ +\Delta_{i1}$	$x_{i3}^c - \Delta_{i3}$	$x_{i2}^c - \Delta_{i2}$	$x_{i1}^c + \Delta_{i1}$
2	010	$-\Delta_{i3}\ +\Delta_{i2}\ -\Delta_{i1}$	$x_{i3}^c - \Delta_{i3}$	$x_{i2}^c + \Delta_{i2}$	$x_{i1}^c - \Delta_{i1}$
3	011	$-\Delta_{i3}\ +\Delta_{i2}\ +\Delta_{i1}$	$x_{i3}^c - \Delta_{i3}$	$x_{i2}^c + \Delta_{i2}$	$x_{i1}^c + \Delta_{i1}$
4	100	$+\Delta_{i3}\ -\Delta_{i2}\ -\Delta_{i1}$	$x_{i3}^c + \Delta_{i3}$	$x_{i2}^c - \Delta_{i2}$	$x_{i1}^c - \Delta_{i1}$
5	101	$+\Delta_{i3}\ -\Delta_{i2}\ +\Delta_{i1}$	$x_{i3}^c + \Delta_{i3}$	$x_{i2}^c - \Delta_{i2}$	$x_{i1}^c + \Delta_{i1}$
6	110	$+\Delta_{i3}\ +\Delta_{i2}\ -\Delta_{i1}$	$x_{i3}^c + \Delta_{i3}$	$x_{i2}^c + \Delta_{i2}$	$x_{i1}^c - \Delta_{i1}$
7	111	$+\Delta_{i3}\ +\Delta_{i2}\ +\Delta_{i1}$	$x_{i3}^c + \Delta_{i3}$	$x_{i2}^c + \Delta_{i2}$	$x_{i1}^c + \Delta_{i1}$

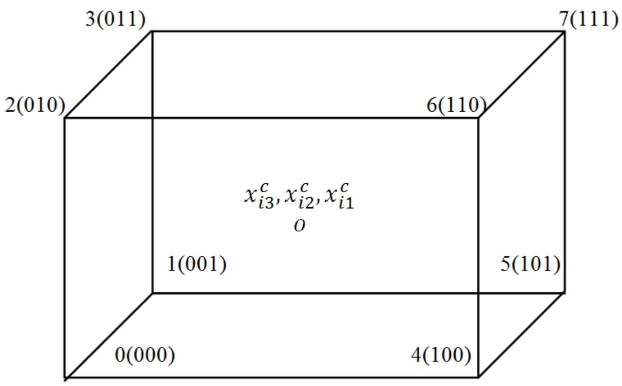

Figure 1. Three dimensional hypercube representing corners in decimals

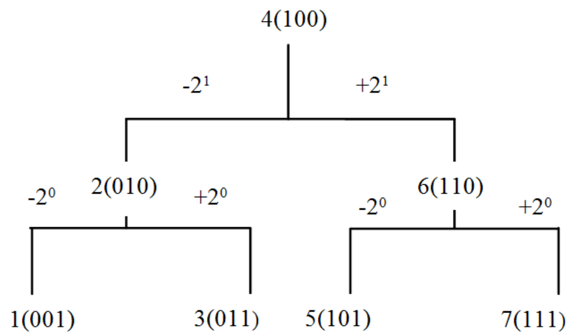

Figure 2. BSA for 3-bits code

The search process is started by initializing the coefficient vector x_j^{Ct} giving objective function. To perform BSA strategy by the iterative process C_j^t is initially selected as below

$$C_{ij}^t = \begin{cases} 1; & for\ (j = 1) \\ 0; & for (j = 2, 3, 4, \dots, N_V) \end{cases} \quad (27)$$

Two corners, with reference to above selected corner, are created for comparison as below

$$C_{i1j}^t = \begin{cases} 1; & for\ i + 1 \\ C_{ij}^t; & for (j = 1,2,, \dots i, (i + 2), \dots, N_V \end{cases} \quad (28)$$

$$C_{i2j}^t = \begin{cases} 0; & for\ i \\ C_{i1j}^t; & for (j = 1,2,, \dots (i - 1), (i + 1), \dots, N_V \end{cases} \quad (29)$$

In reference to these two corners, coefficient vectors are generated as shown in Table 1. Mathematically, it is represented in the generalized form

$$x_{imj}^t = x_{ij}^{Ct} + \Delta_{imj}^t \quad (30)$$

$$(m=1,2;\ j = 1, 2, \dots, N_V;\ i = 1, 2, \dots, N_P)$$

where

$$\Delta_{imj}^t = \begin{cases} +\Delta_{ij} & if\ C_{imj}^t = 1 \\ -\Delta_{ij} & if\ C_{imj}^t = 0 \end{cases} \quad (31)$$

$$(m=1,2;\ j = 1, 2, \dots, N_V;\ i = 1, 2, \dots, N_P)$$

The initial increment to coefficients is decided by

$$\Delta_j = \frac{x_j^{max} - x_j^{min}}{\delta} \quad (32)$$

Objective functions at x_{1j}^k and x_{2j}^k are evaluated as follows

$$F_{im}^t = f\left(x_{imj}^t\right) \qquad (m = 1, 2) \quad (33)$$

The minimum value of these two is selected to be computed with rest of the corners, generated subsequently and the selected corner for the generation of the next two corner is

$$C_{ij}^t = \begin{cases} C_{i1j}^t & if\ F_{i1}^t < F_{i2}^t \\ C_{i2j}^t & if\ F_{i1}^t > F_{i2}^t \end{cases} \quad (j = 1, 2, \dots, N_V) \quad (34)$$

This process is repeated till all the corners of the hypercube are explored and the overall minimum is selected to find the new centre point for the next iteration. When the last element of C_{ij}^{t} vector contains one of the last branch of BSA tree is reached, which ensures that all corners of hypercube are explored, the procedure is terminated. In BSA method, the number of computations is reduced by large amount. As elaborated in Table II.

TABLE II. COMPARISON OF NUMBER OF FUNCTION EVALUATIONS

Value of M and N	Number of committed coefficients N_V	Number of corners of hypercube (2^{N_V})	Number of comparisons by BSA method ($2 \times N_V$)
1, 1	7	128	14
2, 2	13	8192	26
3, 3	19	524,288	38
4, 4	25	33,554,432	50
5, 5	31	2,147,483,648	62

I. Pattern Move:

The pattern move is designed to use the information obtained in the exploratory move. It finds the minimization of the function by moving in the established pattern direction. A new point is calculated by jumping from the current best point x_i^t along the direction connecting the previous best point x_i^{t-1} and is explained below [18]:

$$x_j^{t+1} = x_j^t + \eta\left(x_j^t - x_j^{t-1}\right) \quad (j = 1, 2, ..., N_V) \qquad (35)$$

where η is an accelerating factor and is a random number lies between 0.5 and 2.0. If the pattern move does not improve the solution, the pattern move is not accepted and the extent of exploratory move is reduced.

J. Migration

With the progress of iterations, the population diversity decreases rapidly, which causes the decrease in exploration of search space. Due to this, clustered individuals are unable to reproduce newly better solution by mutation and crossover. To increase the exploration of search space, the migration operation is applied. It also decreases the selection pressure for a small population. The j^{th} gene of i^{th} individual is randomly regenerated as below [9]:

$$x_{ij}^{t+1} = \begin{cases} x_{bj}^{t+1} + R_i\left(x_j^{min} - x_{bj}^{t+1}\right) & if\ \delta < \dfrac{x_{bj}^{t+1} - x_j^{min}}{x_j^{max} - x_j^{min}} \\ x_{bj}^{t+1} + R_i\left(x_j^{max} - x_{bj}^{t+1}\right) & if\ otherwise \end{cases} \qquad (36)$$

where x_{bj}^{t+1} is the best individual. R_i and δ are uniform random numbers.

IV. DESIGN OF IIR LP FILTER

All the ten different mutation strategies (MS-1 to MS-10) have been used to design cascaded form of digital IIR low-pass filter with higher order. The filter order is varied by varying the values of M and N in Eq.(1). The values of M and N have been varied from 2 to 7. So the low-pass filter has been designed for order 6 to 21. Frequency range of pass-band and stop-band has been taken as $0 \le \omega \le 0.2\pi$ and $0.3\pi \le \omega \le \pi$ respectively.

To design digital IIR filter, 200 evenly distributed points are set in the frequency span [0, π], so that the number of discrete frequency points in Eq. (3) are 182 for LP filters in the prescribed pass-band and stop-band range. In the proposed approach for designing LP digital IIR filters, the population has been taken as 50. The rate of opposition has been kept at 0.6. The mutation factor f_m and crossover rate has been taken as 0.85 and 0.1 respectively. The exploratory move has been repeated 20 times. The maximum number of iterations has been taken as 100.

The algorithm was run for 100 generations for each mutation strategy and for each order of the filter. The best results obtained from the ten mutation strategies are shown in the Table III. Out of these ten implemented mutation strategies, it is observed that MS-7 mutation strategy with M and N equal to 7 each and filter order 21 gives the best result.

The results obtained from MS-7 strategy are better than the results given by Tang *et al.* [8], Tsai *et al.* [14], Kaur *et al.* [22] and Singh *et al.* [23] shown in Table IV. The coefficient of low pass digital IIR filter having order 21 designed with mutation strategy MS-7 are given in Table V.

The magnitude error verses iterations graph is shown in Fig. 3. The frequency response of low-pass filter with order 21 designed by MS-7 mutation strategy is shown in Fig. 4. The pole zero plot of the above filter is depicted in Fig. 5. All poles lie within the unit circle shows that the designed filter is stable.

TABLE III. RESULTS FOR HIGHER ORDER IIR LP FILTER

Mutation Strategy	Order	Magnitude Error	Pass-Band Performance	Stop-Band performance
MS-1	13	0.1210	$0.9971 \le \lvert H(e^{j\omega})\rvert \le 1.0030$ (0.0059)	$\lvert H(e^{j\omega})\rvert \le 0.0189$ (0.0189)
MS-2	15	0.1419	$0.9895 \le \lvert H(e^{j\omega})\rvert \le 1.0027$ (0.0132)	$\lvert H(e^{j\omega})\rvert \le 0.0069$ (0.0069)
MS-3	20	0.2308	$0.9759 \le \lvert H(e^{j\omega})\rvert \le 1.0035$ (0.0276)	$\lvert H(e^{j\omega})\rvert \le 0.0234$ (0.0234)
MS-4	21	0.2095	$0.9891 \le \lvert H(e^{j\omega})\rvert \le 1.0042$ (0.0151)	$\lvert H(e^{j\omega})\rvert \le 0.0030$ (0.0030)
MS-5	18	0.2370	$0.9826 \le \lvert H(e^{j\omega})\rvert \le 1.0062$ (0.0236)	$\lvert H(e^{j\omega})\rvert \le 0.0287$ (0.0287)
MS-6	16	0.1223	$0.9756 \le \lvert H(e^{j\omega})\rvert \le 1.0036$ (0.0280)	$\lvert H(e^{j\omega})\rvert \le 0.0066$ (0.0066)

MS-7	21	0.1024	0.9961≤ $\|H(e^{j\omega})\|$ ≤1.0032 (0.0071)	$\|H(e^{j\omega})\|$ ≤0.0094 (0.0094)
MS-8	21	0.1914	0.9789≤ $\|H(e^{j\omega})\|$ ≤1.0040 (0.0251)	$\|H(e^{j\omega})\|$ ≤0.0204 (0.0204)
MS-9	18	0.1323	0.9953≤ $\|H(e^{j\omega})\|$ ≤1.0017 (0.0064)	$\|H(e^{j\omega})\|$ ≤0.0080 (0.0080)
MS-10	11	0.1127	0.9968≤ $\|H(e^{j\omega})\|$ ≤1.0021 (0.0053)	$\|H(e^{j\omega})\|$ ≤0.0101 (0.0101)

TABLE IV. RESULTS FOR IIR LP FILTER BY OTHER RESEARCHERS.

Method	Order	Magnitude Error	Pass-Band Performance	Stop-Band Performance
PPO [23]	3	3.6611	0.9178≤ $\|H(e^{j\omega})\|$ ≤1.0176 (0.0998)	$\|H(e^{j\omega})\|$ ≤0.1611 (0.1611)
RCGA [22]	3	4.0095	0.9335≤ $\|H(e^{j\omega})\|$ ≤1.0160 (0.0825)	$\|H(e^{j\omega})\|$ ≤0.1510 (0.1510)
TIA [14]	3	3.8157	0.9012≤ $\|H(e^{j\omega})\|$ ≤1.0000 (0.0988)	$\|H(e^{j\omega})\|$ ≤0.1243 (0.1243)
HGA [8]	3	4.3395	0.8870≤ $\|H(e^{j\omega})\|$ ≤1.0000 (0.1139)	$\|H(e^{j\omega})\|$ ≤0.1802 (0.1802)

TABLE V. LOW-PASS HIGHER ORDER FILTER COEFFICIENTS

A=0.000449		a_1= 1.274889	b_1= 0.81176
a_2= -0.32605	b_2= -0.43228	a_3= 0.586009	b_3= 0.41511
a_4= -1.50260	b_4= -0.36690	a_5= -1.10887	b_5= 0.29999
a_6= 0.966069	b_6= -0.60639	a_7= 0.093035	b_7= -0.20393
p_1= -0.59637	q_1= 0.730059	r_1= -0.80454	s_1= 0.40789
p_2= 0.314458	q_2= 0.656849	r_2= -1.30106	s_2= 0.376008
p_3= -0.46209	q_3= 1.143897	r_3= 0.488833	s_3= -0.40059
p_4= -0.32759	q_4= 0.927278	r_4= -1.41990	s_4= 0.877123
p_5= -0.43667	q_5= -0.79457	r_5= -0.52263	s_5= -0.3518
p_6= 0.014299	q_6= -0.48939	r_6= -1.27992	s_6= 0.627376
p_7= -0.74028	q_7= 0.956699	r_7= -0.69231	s_7= 0.000611

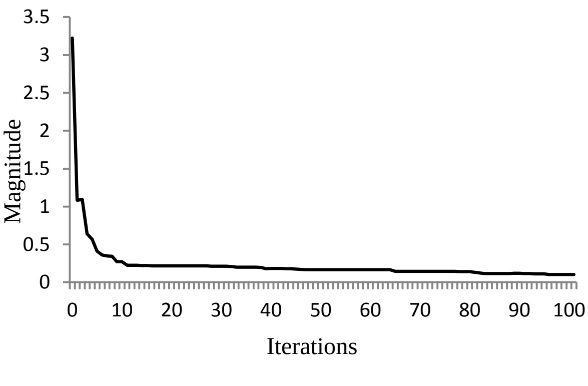

Figure 3. Magnitude verses iterations graph for LP filter using MS-7

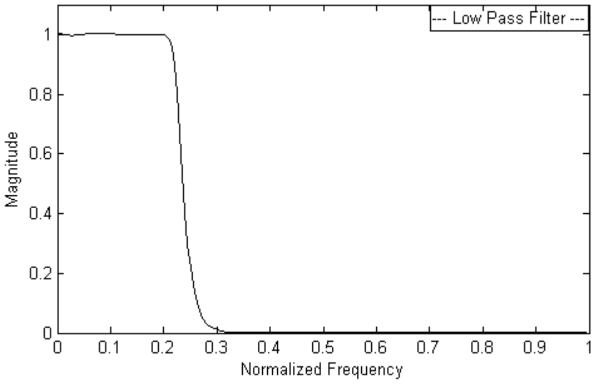

Figure 4. Frequency response of LP filter using MS-7

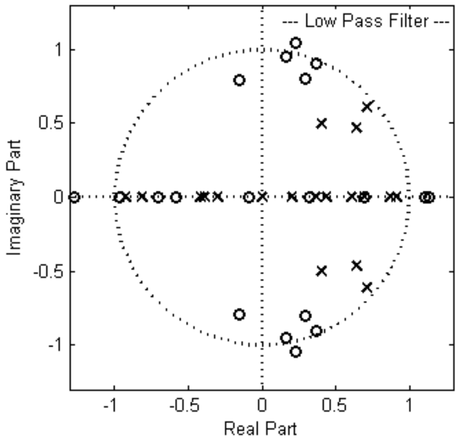

Figure 5. Pole-Zero plot of LP filter using MS-7

V. CONCLUSION

This paper proposes the design of digital IIR filter by using hybrid heuristic search technique having opposition based differential evolution algorithm for global search optimization and binary successive approximation algorithm for local search. As shown through results, the proposed method works well with arbitrary random initialization. On the basis of the results obtained by this method, it can be concluded that the proposed hybrid heuristic search technique gives better performance in terms of magnitude approximation as compared to the PPO and GA methods. So the proposed hybrid heuristic search technique, a unique combination of global search method and exploratory search method yields a powerful option for design of digital IIR filter.

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