Spread Spectrum Using Chirp Modulated RF Pulses for Incoherent Sampling Compressive Sensing MRI

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***Abstract*—Compressed Sensing Magnetic Resonance Imaging (CS-MRI) has proven to be one of the most promising techniques to reduce the magnetic resonance (MR) data acquisition time. The performance of compressive sensing is highly depending on the level of incoherence between the sparsity transform matrix and the sensing matrix. In conventional MRI, Fourier matrix as a sensing matrix and Wavelet matrix as a sparsifying transform matrix are not optimally incoherent. Moreover, Fourier encoding weakly spreads out the energy and concentrate energy in the center of the "k-space" in a region known as low frequencies region. This imposes restriction on the under-sampling pattern to fully sample the low spatial frequencies and insufficiently sample the high frequencies at high acceleration factors. Such restriction can cause a huge loss in image resolution. This paper investigates the implementation of spread spectrum RF pulses to increase the incoherence and insure the spread of energy. The spread spectrum compressive sensing is achieved experimentally using tailored spatially selective RF pulses and random under-sampling along the phase encodes. Simulation and experimental results suggest that the proposed technique outperforms the conventional Fourier encoding in the framework of CS-MRI and in preserving the image quality at higher acceleration factors.**

***Index Terms*—compressive sensing, magnetic resonance imaging, non-fourier encoding, spread spectrum**

I. Introduction

Magnetic resonance imaging (MRI) is the most demanded non-invasive imaging modalities with great abilities to visualize the internal structure of human anatomy and image its functionality. MRI scan the object and construct an image by acquiring image by Fourier encoding data in frequency domain known as k-space [1], [2]. This acquisition method results in the long scan time of MRI. Compressive Sensing (CS) has shown a great potential to reduce MRI acquisition time and faithfully reconstruct an image from small number of measurements [1]-[5]. In conventional MRI, compressive sensing is possible due to the facts that most MR images are compressible by certain appropriate transforms and acquiring data in Fourier domain (k-space) allows a certain level of incoherent sampling. The image can be fully recovered by using a constrained nonlinear reconstruction method [3].

In MRI, the acquired signal is given by $y = \Phi x$, where x in the actual image and $\Phi \in C^{MxN}$, is the sensing matrix. For an under-sampled data, the signal equation becomes underdetermined which is not possible to recover using linear reconstruction methods. Compressive sensing algorithm promises a full recovery of a sparse signal from under-sampled measurements by providing a solution to the ill-posed nonlinear reconstruction problem:

$$\min_{\hat{x}} \|\Psi\hat{x}\|_{l_1} \ s.t.\ \|y - \Phi\hat{x}\|_{l_2}^2 \le \varepsilon^2 \tag{1}$$

where $\hat{x}$ is the reconstructed signal and $\|\Psi\hat{x}\|$ is the l_1 norm of the sparse signal, ε is the allowed data discrepancy, Φ is the sensing matrix, Ψ is the sparsifying transform matrix and y is the measurement data. The constrained optimization problem (1) is converted to unconstrained problem with regularization penalties for computation purposes:

$$\arg\min \left\{ \|y - \Phi\hat{x}\|_{l_2}^2 + \lambda_1 \|\Psi\hat{x}\|_{l_1} + \lambda_2 \mathrm{TV}(\hat{x}) \right\} \tag{2}$$

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where TV is the total variation of the signal. λ_1, λ_2 are the sparsity and total variation regularization penalties respectively [3].

The performance of compressive sensing depend on the level of incoherence between the sparsifying transform matrix and the sensing matrix shown in (1) and the Restricted Isometry Property (RIP) of sensing matrix.

The RIP is given by:

$$(1-\delta_k)\|x_k\|_{l_2}^2 \le \|\Phi x_k\|_{l_2}^2 \le (1+\delta_k)\|x_k\|_{l_2}^2 \quad (3)$$

where $0<\delta_k<1$ is the RIP constant, Φ is the sensing matrix, and x_k is the k-sparse signal. The RIP constant δ_k is a measure of the energy spread of the signal in the sensing domain and determines performance of CS. Roughly speaking, the smaller δ_k the more evenly energy spread and hence the better the performance of CS. In conventional MRI the sparsifying transform matrix is the Wavelet matrix; the sensing matrix is the Fourier matrix [1], [2], [4], [6]. Fourier encoding concentrates the energy in the low-frequency region. In order to spread the signal energy in the k-space and allow for incoherent sampling, spread spectrum technique is used to modify the sensing matrix. Chirp radio frequency (RF) modulation can be used as controllable spread spectrum technique to achieve energy spread.

Recently, non-Fourier encoding for CS-MRI has been investigated for energy spread and incoherent sampling. Gaussian random encoding was proposed as an alternative to Fourier encoding in CS-MRI. However it is very difficult to implement and requires impractical computation power and storage memory for reconstruction [7]-[9]. Toeplitz random matrices were also used as encoding matrix. In simulation, Toeplitz random encoding showed good spread of energy and computational advantages over the Gaussian random encoding, but experimentally it is undesirable due to its long RF pulses. Ref [10] proposed a new spread spectrum technique using Chirp modulation. This technique uses second-order shim coils to modulate the RF pulse. The implementation of such method is not conventional and can be impractical due to its limitation on the intensity of energy spread. Despite the notable spread of energy that the above encoding methods have achieved, there is still no experimental superiority for using these methods over Fourier encoding

In this paper, we propose a practical method and implementation technique of the non-Fourier encoding. In this method, we use the Chirp modulated RF pulses to spread the energy along the phase encoding direction by exciting the scanned object along the phase encoding direction with different profile at each excitation. Using the small tip angel approximation, the RF pulse is the Fourier transform (FFT) of the desired profile [1], hence, modulating the spatial profile with Chirp is equivalent to a time shift of the previous RF waveform to generate the next RF waveform.

In this work, we demonstrate that encoding Chirp modulated RF pulses achieves desirable energy spread and outperforms Fourier encoding in preserving resolution when using CS-MRI at high acceleration factor. This method outperforms other non-Fourier encoding in terms of implementation, computational burden and RF pulse duration.

II. Proposed Method

In this paper, we propose a spread spectrum compressive sensing MRI technique that uses phase modulated RF pulses for encoding. The method exploits the phase modulated RF pulses along the "phase encoding" direction, while keeping conventional Fourier encoding along the readout directions. Compressive sensing image reconstruction algorithm is then performed on the under-sampled acquired data to reconstruct the desired image. In conventional MRI, with Fourier encoding, the acquired signal is given by $y = Fx$, where x in the actual image and F is the Fourier sensing matrix. Fourier matrix is not optimally incoherent with the Wavelet sparsifying transform matrix [10], [11]. In compressive sensing, the higher the incoherence between the sensing matrix and the sparsifying transform matrix, the higher the performance in recovering the original signal [10]-[12]. In order to increase the sampling incoherence, Chirp modulated RF pulses can be used as encoding basis to spread the energy, increase the incoherency and hence, reduce the reconstruction error for randomly sampled signal. Chirp encoded MRI signal can be written as $y = F\Theta x$, where Θ is a diagonal matrix that is used to modulate the Fourier basis with its diagonal entry given as:

$$\rho_{(r,r)} = diag[e^{-i(\Delta C r^2 + \Delta C)}] \quad (4)$$

ΔC is the intensity of the Chirp modulation, and r is the position of the encoded point along the direction of modulation [13] .

Since Chirp modulation allows the energy to be spread along the phase encoding direction, we propose to randomly under-sample the data along the phase encoding direction with a uniform probability as shown in Fig. 1. For comparison, Fig. 1 gives the variable density random under-sampling pattern commonly used in the literature to under-sampling Fourier encoding where the center of the k-space is always fully sampled due to the concentration of energy at the low frequencies region.

A. Spread Spectrum CS-MRI Pulse Sequence Design

Fig. 2 shows the proposed pulse sequence diagram. The conventional pulse sequence was modified and the original sinc RF pulse was replaced with Chirp modulated RF pulses. In this proposed pulse sequence, different Chirp modulated RF pulse is applied at each excitation with a constant phase encoding gradient.

Phase encoding gradient: The constant phase encoding gradient is calculated using (5) where, G_y is the gradient strength in the phase encoding direction, γ is the gyromagnetic ratio, and Δt_{RF} is the dwell time (the duration of a single point of the RF waveform), FOV_y is the size of the field of view in mm along phase encoding direction.

$$FOV_y = \frac{1}{\gamma G_y \Delta t_{RF}} \tag{5}$$

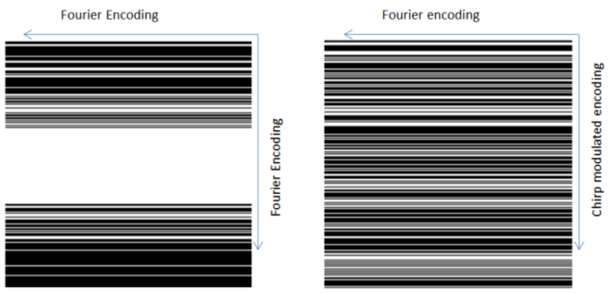

Figure 1. Variable density random sampling pattern for the conventional Fourier encoding generated using Gaussian density function (left) and uniformly random sampling pattern for Chirp encoding (right) the white lines represent the sampled region and the black lines represent un-sampled/ discarded data.

Excitation RF pulses: using the small tip analysis [1], RF pulses can be calculated by taking the Fourier transform of the desired profile. At each excitation, Chirp modulated RF pulse will generate a unique profile along the y direction. Number of excitation will determine the acceleration factor. For example, for a 256 x 256 image, 128 excitations will result in an acceleration factor of 2.

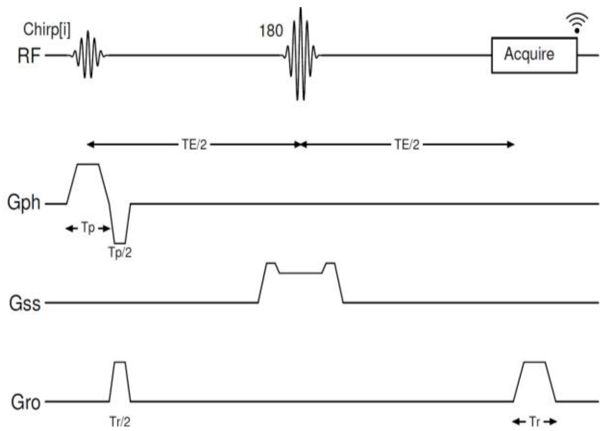

Figure 2. Timing diagram for the implementation of the Chirp modulated RF pulse sequence in 2D imaging, where G_{ph} is the gradient in phase encoding (y) direction, G_{ss} is the gradient in slice (z) direction and G_{ro} is the gradient in readout (x) direction. The RF pulse Chirp[i] is the RF pulse that excites the *i*-th profile along y-direction. The 180 is the refocusing pulse used to select the slice in z direction. TE is echo time.

A 180 degree RF pulse is used to select a slice along the z-direction.

B. *RIP Analysis of Encoding Matrix*

As stated before, the performance of CS is highly dependent on RIP. Fig. 3 compares the mean and the standard deviation of the maximum singular values of sensing matrices for the proposed and Fourier encoding. The distance from the singular value to the value 1 gives approximately the RIP constant δ_k in (3). This analysis indicates that the proposed encoding should outperform the conventional Fourier encoding in recovering the image.

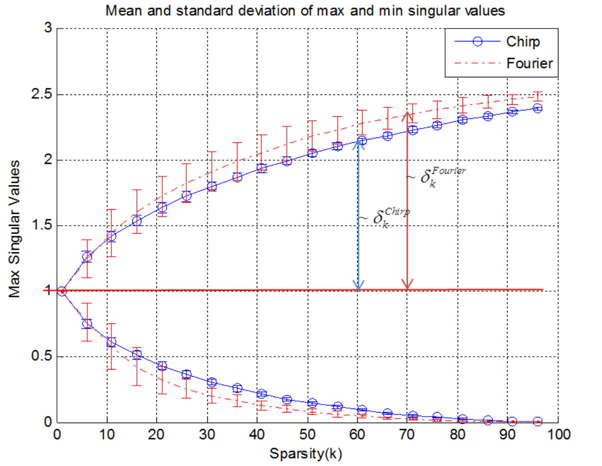

Figure 3. RIP constant approximation for Fourier matrix and Chirp modulated matrix using mean and standard deviation of maximum singular value verses the sparsity k of the signal to be recovered.

III. RESULTS

The results were compared with the conventional Fourier CS-MRI results. For experimental validation, pulse sequences were designed and implemented on a 3T Siemens Skyra MRI scanner (Siemens Healthcare, Erlangen, Germany). A structural phantom was scanned twice: once using the standard "SpinEcho" sequence which uses Fourier encoding, and then using the proposed sequence which implements the Chirp modulated RF pulses as shown in Fig. 2. The duration of the RF pulses was set to be equal to that of Fourier encoding.

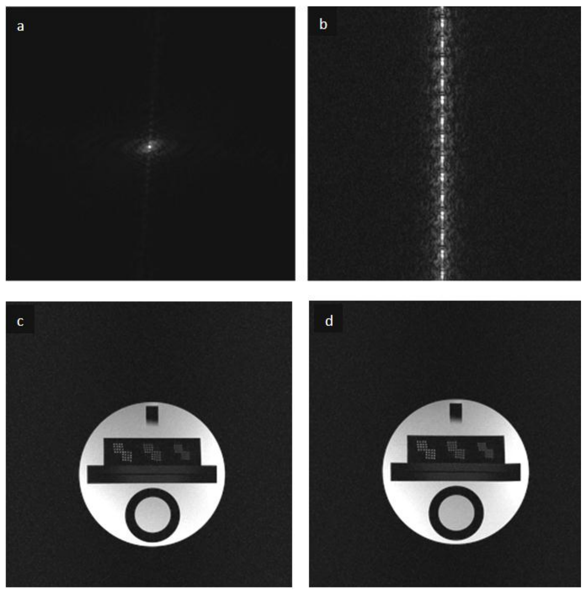

Figure 4. K-space spread spectrum; a the energy spread on the k-space when using conventional Fourier encoding, b energy spread on the k-space (along phase encoding) when using the proposed Chirp modulated RF pulses (proposed Chirp encoding). c,d reconstructed image from fully sampled data using Fourier and Chirp encoding respectively.

To validate the ability of the proposed method in spreading the energy in the k-space, we have observed the acquired k-space of the proposed encoding method and compared it with the acquired k-space data of the conventional Fourier encoding scheme. Fig. 4 shows the achieved spread of energy of the Chirp encoding along the phase direction (y-direction) compared to the concentration of energy in the center of the k-space when using Fourier encoding.

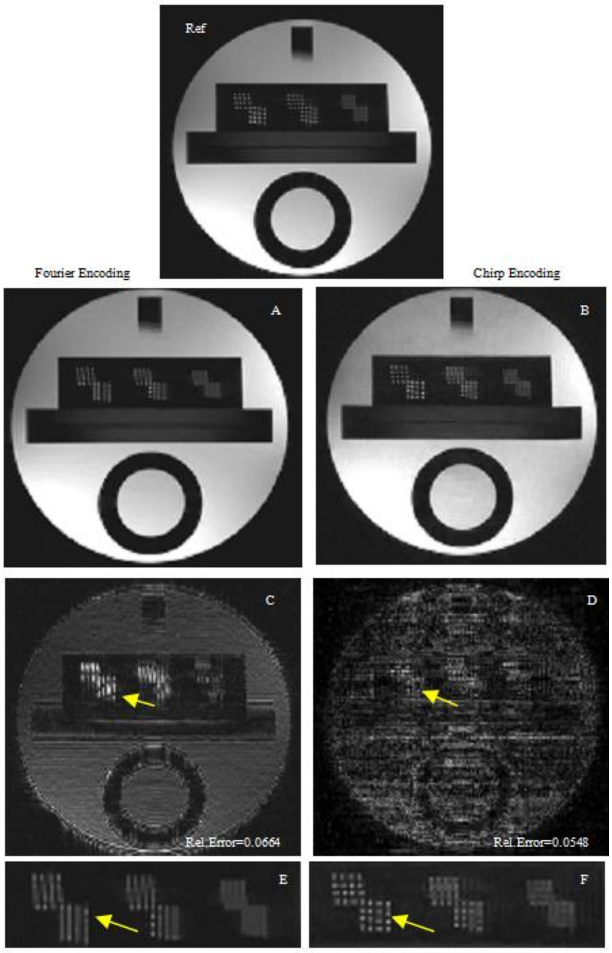

Figure 5. Image of structure phantom scanned for demonstration, Ref is the 256 x 256 fully sampled image acquired using conventional Fourier based method; A-B show images acquired and reconstructed using Fourier and Chirp techniques respectively with acceleration factor of 2.5 (i.e.40% sampled data) . C-G show the relative error between the reconstructed images and Ref. E-F are zoomed in images from the under-sampled reconstructed image A and B respectively.

Fig. 5 Ref shows the fully sampled reference image that is used to compare the performance of the proposed method. We have acquired the full k-space data for both method and then performed under-sampling offline according to the masks proposed in Fig. 1. The proposed method is compared with Fourier encoding with acceleration factor of 2.5 (i.e. 40% sampled data). Fig. 5 A-B are the reconstructed images for the acceleration factor 2.4 using compressive sensing with Fourier and Chirp encoding respectively. Fig. 5 C-D shows the relative error between the reference and Fourier and Chirp reconstructed images respectively. Fig. 5 E is the zoomed portion of Fourier encoded image. Fig. 5 G is the zoomed portion of Chirp encoded image. The above results show that at acceleration factor of 2.5, Chirp encoded image has lower relative error (relative error = 0.0548) compared to Fourier encoded image (relative error = 0.0664). In Fourier encoded image the edges and the doted region (i.e. high frequencies) suffer the most due to the variable density sampling where high frequency region is insufficiently sampled.

IV. Conclusion

The proposed implementation of spread spectrum CS-MRI has been achieved using Chirp encoding which uses modulated RF pulses to spread the energy along the phase direction. The technique has been implemented by modifying the conventional Fourier pulse sequence and introducing Chirp modulated RF pulses which controls the spread of energy and increases the sampling incoherency. Uniform under-sampling along the phase direction has been performed after acquiring the full k-space data. The proposed method allows easy implementation of spread spectrum CS-MRI, uniformly random sampling and shorter RF pulses than that of other non-Fourier encoding. The reconstructed images from the under-sampled scan data demonstrate that the proposed spread spectrum CS-MRI with modulated Fourier sensing matrix outperforms the conventional CS-MRI. The proposed technique also outperforms other non-conventional non-Fourier encoding scheme in feasible and practical implementation.

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