Bayesian Multiple Estimation in Flat Rician Fading MIMO Channels

H. Nooralizadeh
Department of Electrical Engineering, Islamshahr Branch, Islamic Azad University, Islamshahr, Iran
Email: nooralizadeh@iiau.ac.ir

***Abstract*—In this paper, the performance of the single-estimation (SE) and multiple-estimation (ME) is investigated in multiple-input multiple-output (MIMO) Rician flat fading channels using the traditional least squares (LS) estimator and the Bayesian minimum mean square error (MMSE) estimator. The closed form equations are obtained for mean square error (MSE) of the estimators in SE and ME cases under optimal training. In ME case, the optimal weight coefficients are achieved for both estimators. Analytical and numerical results show that the LS estimator has lower error in the case of ME than SE. Moreover, it is seen that the performance of MMSE channel estimator in the ME case is better than SE particularly at high signal to noise ratios (SNRs). Furthermore, it is shown that this estimator is more appropriate for the channels with weak line of sight (LOS) propagation paths and/or the low correlations.**

***Index Terms*—rician fading, multiple estimation, least squares, minimum mean square error, multiple-input multiple-output**

I. Introduction

Multiple-input multiple-output (MIMO) system provides substantial benefits in both increasing system capacity and improving its immunity to deep fading in the channel [1], [2]. To take advantage of these benefits, the accurate channel state information (CSI) is required at the receiver and/or transmitter.

Due to low complexity and better performance, training-based channel estimation (TBCE) is widely used in practice for quasi-static or slow fading channels, e.g., indoor MIMO channels [3]-[8]. However, in outdoor MIMO channels where channels are under fast fading, the channel tracking and estimating algorithms as the Kalman filter [9], [10] are used.

In [3], the performance of the least squares (LS), scaled LS (SLS), minimum mean square error (MMSE), and relaxed MMSE (RMMSE) estimators is studied in the Rayleigh fading MIMO channel using TBCE scheme. The MMSE channel estimator has the best performance among the estimators, because it employs more a-priori knowledge about the channel.

In [4]-[6], it is assumed that the MIMO channel has Rician distribution. For MIMO Rician flat fading channels, the new shifted scaled least squares (SSLS) channel estimator is presented in [6]. It is seen that this estimator has the best performance among the LS-based estimators in Rician channel model. Nevertheless, the MMSE channel estimator has lower error than that of SSLS in Rician fading channel model especially at high signal to noise ratios (SNRs) and high spatial correlations [5].

In [7], the performances of the time-multiplexed (TM) and superimposed (SI) schemes have been compared in MIMO channel estimation. It is shown that in fast fading channels and/or for many receiver antennas, the SI scheme is better than TM but in other cases this scheme suffers from a higher estimation error. In part II of this paper [8], to improve the performance of the SI scheme a decision directed approach is applied.

In order to perform the individual channel estimation at the destination, in [11], the SI training strategy is applied into the MIMO amplify-and-forward (AF) one-way relay network (OWRN). The discussion is restricted to the case of a slow, frequency-flat block fading model. A specific suboptimal channel estimation algorithm is applied in [11] using the optimal training sequences and to verify the Bayesian Cramér-Rao lower bound (CRLB) results the normalized mean square error (MSE) performance for the estimation is provided.

In this paper, TBCE method is studied in the flat Rician fading MIMO channels. First, the single-estimation (SE) is considered and the minimum MSE is obtained for LS and MMSE estimators under optimal training. Then, multiple-estimation (ME) is investigated in these estimators. In ME case, the multiple estimates of the channel during received N sub-blocks are combined optimally. The optimal weight coefficients are achieved for both estimators. Furthermore, the minimum MSE under optimal training is obtained for aforementioned estimators.

Simulation results show that both estimators have better performance in the ME case than SE case especially at high SNRs. In addition, it is seen that the multiple estimates of the MMSE channel estimator obtain better performance in the channels with low correlations and/or weak line of sight (LOS) propagation paths.

Notation: $(\cdot)^H$ is reserved for Hermitian, $(\cdot)^*$ for the complex conjugate, $(\cdot)^{-1}$ for the matrix inverse, $tr\{\cdot\}$ for the trace of a matrix. $E\{\cdot\}$ is the mathematical expectation, I_m denotes the $m \times m$ identity matrix, $\|\cdot\|_F$ denotes the Frobenius norm.

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II. System Model

It is considered a MIMO system with t transmitter and r receiver antennas. For MIMO channel, a flat block fading model is assumed. It means that the channel response is fixed within one block and can change from one block to another one randomly. Each transmitted block has N sub-blocks which contain training and data symbols. Training and data symbols are located in the first and end part of the sub-blocks, respectively. In practice, the channel is estimated using training symbols in the training phase. Then, the results are used for data detection. To estimate the MIMO channel in each sub-block, it is required that $n_p \geq t$ training signals are transmitted by each transmitter antenna. The $r \times n_p$ complex received signal matrix can be expressed as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{V} \tag{1}$$

where X and V are the complex t-vector of transmitted sequences on the t transmit antennas and r-vector of additive receiver noise, respectively. The elements of noise matrix V are independently and identically distributed (i.i.d.) complex Gaussian random variables with zero-mean and unit variance. Then, the correlation matrix of V is given by

$$\mathbf{R_V} = E\{\mathbf{V}^H \mathbf{V}\} = r\,\mathbf{I}_{n_p} \tag{2}$$

In MIMO Rician fading channels with K as Rice factor, the $r \times t$ matrix of channel, H, is defined in the following form [12], [13]:

$$\mathbf{H} = \sqrt{1/(\mathrm{K}+1)}\ \mathbf{H}_{\mathrm{Ray}} + \sqrt{\mathrm{K}/(\mathrm{K}+1)}\ \mathbf{H}_{\mathrm{LOS}} \tag{3}$$

The matrix H_{Ray} explains the Rayleigh component of the channel and the matrix H_{LOS} describes the channel mean value or LOS component of the channel.

Using (3), it is straightforward to show that

$$\mathbf{M} = \mathrm{E}\{\mathbf{H}\} = \sqrt{\mathrm{K}/(\mathrm{K}+1)}\ \mathbf{H}_{\mathrm{LOS}} \tag{4}$$

Also, the correlation matrix of the channel H can be calculated as follows:

$$\begin{aligned}\mathbf{R_H} &= \mathrm{E}\{\mathbf{H}^H \mathbf{H}\} \\ &= \frac{1}{1+\mathrm{K}} \mathrm{E}\{\mathbf{H}_{\mathrm{Ray}}^H \mathbf{H}_{\mathrm{Ray}}\} + \frac{\mathrm{K}}{1+\mathrm{K}} \mathbf{H}_{\mathrm{LOS}}^H \mathbf{H}_{\mathrm{LOS}} \\ &= \frac{1}{1+\mathrm{K}} \mathbf{R}_{\mathbf{H}_{\mathrm{Ray}}} + \frac{\mathrm{K}}{1+\mathrm{K}} \mathbf{H}_{\mathrm{LOS}}^H \mathbf{H}_{\mathrm{LOS}}\end{aligned} \tag{5}$$

Then, the co-variance matrix of the channel H will be as:

$$\begin{aligned}\mathbf{C_H} &= \mathbf{R_H} - \mathrm{E}\{\mathbf{H}\}^H\ \mathrm{E}\{\mathbf{H}\} \\ &= \mathbf{R_H} - \mathbf{M}^H \mathbf{M} = \frac{1}{1+\mathrm{K}} \mathbf{R}_{\mathbf{H}_{\mathrm{Ray}}}\end{aligned} \tag{6}$$

III. Single Channel Estimation

In this section, it is supposed that the number of sub-blocks used for channel estimation is N=1. First, the LS channel estimator is studied. Then, the performance of the Bayesian MMSE channel estimators is examined.

A. LS Channel Estimator

For linear model of (1), the LS channel estimator which minimizes $tr\{(\mathrm{Y}-\mathrm{HX})^H (\mathrm{Y}-\mathrm{HX})\}$ is

$$\hat{\mathbf{H}}_{LS} = \mathbf{Y}\mathbf{X}^H (\mathbf{X}\mathbf{X}^H)^{-1} \tag{7}$$

Under optimal training, it is shown that the error of the estimator is minimized as follows [3]

$$(J_{LS})_{\min} = \frac{r\,t^2}{p} \tag{8}$$

where p is a given constant value considered as the total power of training matrix X.

This estimator achieves the classical CRLB, hence, it is efficient. However, the LS estimator utilizes only received signals and transmitted symbols that are given at the receiver. It has no knowledge about the channel.

B. Bayesian MMSE Channel Estimator

For linear model of (1), the Bayesian MMSE channel estimator of H is given by [5]

$$\hat{\mathbf{H}}_{MMSE} = \mathbf{M} + (\mathbf{Y} - \mathbf{MX})\,\mathbf{A} \tag{9}$$

C. Where

$$\mathbf{A} = (\mathbf{X}^H \mathbf{C_H} \mathbf{X} + r\,\mathbf{I}_{n_p})^{-1} \mathbf{X}^H \mathbf{C_H} \tag{10}$$

The performance of the MMSE channel estimator is measured by the error matrix $\varepsilon = H - \hat{H}_{MMSE}$, whose probability density function (pdf) is Gaussian with zero mean and the following covariance matrix:

$$\mathbf{C}_{\boldsymbol{\varepsilon}} = \mathbf{R}_{\boldsymbol{\varepsilon}} = \mathrm{E}\{\boldsymbol{\varepsilon}^H \boldsymbol{\varepsilon}\} = (\mathbf{C}_{\mathbf{H}}^{-1} + \frac{1}{r}\mathbf{X}\mathbf{X}^H)^{-1} \tag{11}$$

Then, the MMSE estimation error is given by

$$\begin{aligned}J_{MMSE} &= \mathrm{E}\left\{ \left\| \mathbf{H} - \hat{\mathbf{H}}_{MMSE} \right\|_F^2 \right\} \\ &= \mathrm{E}\{tr(\boldsymbol{\varepsilon}^H \boldsymbol{\varepsilon})\} \\ &= tr\{(\mathbf{C}_{\mathbf{H}}^{-1} + \frac{1}{r}\mathbf{X}\mathbf{X}^H)^{-1}\}\end{aligned} \tag{12}$$

To minimize (12) subject to the transmitted power constraint $tr\{\mathrm{XX}^H\}=p$, the Lagrange multiplier method is used. The problem can be written as follows:

$$\begin{aligned}L(\mathbf{X}\mathbf{X}^H, \eta) &= tr\{(\mathbf{C}_{\mathbf{H}}^{-1} + \frac{1}{r}\mathbf{X}\mathbf{X}^H)^{-1}\} \\ &+ \eta\,[tr\{\mathbf{X}\mathbf{X}^H\} - p]\end{aligned} \tag{13}$$

where η is the Lagrange multiplier. By differentiating (13) with respect to X and setting the result equal to zero, it is obtained that the optimal training matrix should satisfy

$$\mathbf{X}\mathbf{X}^H = \frac{p + r\,tr\{\mathbf{C}_{\mathbf{H}}^{-1}\}}{t}\mathbf{I}_t - r\,\mathbf{C}_{\mathbf{H}}^{-1} \quad (14)$$

Substituting (14) back into (12), the MSE will be minimized as

$$(J_{MMSE})_{\min} = \frac{r\,t^2}{p + r\,tr\{\mathbf{C}_{\mathbf{H}}^{-1}\}} \quad (15)$$

IV. Multiple Channel Estimation

In order to improve the performance of the estimators, the multiple estimates of the channel during received N sub-blocks are combined. In this section, it is assumed that the channel response is fixed within N sub-blocks. In other words, the coherent time of the channel is enough to use N sub-blocks for channel estimation. Suppose that N estimates $\hat{\mathbf{H}}_1, ..., \hat{\mathbf{H}}_N$ of the MIMO channel are obtained based on the training matrices $\mathbf{X}_1, ..., \mathbf{X}_N$, respectively. The results are combined in the following linear method:

$$\hat{\mathbf{H}}_{\mathrm{ME}} = \sum_{n=1}^{N} a_n \hat{\mathbf{H}}_n \quad (16)$$

where the optimal weight coefficients $a_1, ..., a_N$ are obtained so that the MSE (17) is minimized subject to $\sum_{n=1}^{N} a_n = 1$.

$$J_{ME} = \mathrm{E}\left\{ \left\| \mathbf{H} - \sum_{n=1}^{N} a_n \hat{\mathbf{H}}_n \right\|_F^2 \right\} \quad (17)$$

Then, the optimization problem is

$$\min_{a_1,...,a_N} \mathrm{E}\left\{ \left\| \mathbf{H} - \sum_{n=1}^{N} a_n \hat{\mathbf{H}}_n \right\|_F^2 \right\} \quad S.T \quad \sum_{n=1}^{N} a_n = 1 \quad (18)$$

In this section, the problem (18) will be solved considering the LS and the Bayesian MMSE channel estimators.

A. Multiple LS Estimation

Using (1), the LS estimator (7) can be rewritten as

$$\hat{\mathbf{H}}_{LS} = \mathbf{H} + \mathbf{V}\mathbf{X}^H (\mathbf{X}\mathbf{X}^H)^{-1} \quad (19)$$

Using (19), the error of the multiple LS estimation will be written as

$$\begin{aligned} J_{Multiple\,LS} &= \mathrm{E}\left\{ \left\| \mathbf{H} - \sum_{n=1}^{N} a_n \hat{\mathbf{H}}_n \right\|_F^2 \right\} \\ &= \mathrm{E}\left\{ \left\| \mathbf{H} - \sum_{n=1}^{N} a_n (\mathbf{H} + \mathbf{V}_n \mathbf{X}_n^H (\mathbf{X}_n \mathbf{X}_n^H)^{-1}) \right\|_F^2 \right\} \end{aligned} \quad (20)$$

Using the constraint $\sum_{n=1}^{N} a_n = 1$ and with some calculations, the result is

$$\begin{aligned} J_{Multiple\,LS} &= \mathrm{E}\left\{ \left\| \sum_{n=1}^{N} a_n \mathbf{V}_n \mathbf{X}_n^H (\mathbf{X}_n \mathbf{X}_n^H)^{-1} \right\|_F^2 \right\} \\ &= \mathrm{E}\{tr\{(\sum_{n=1}^{N} a_n \mathbf{V}_n \mathbf{X}_n^H \mathbf{E}_n)^H (\sum_{m=1}^{N} a_m \mathbf{V}_m \mathbf{X}_m^H \mathbf{E}_m)\}\} \\ &= tr\{\sum_{n=1}^{N}\sum_{m=1}^{N} a_n^* a_m \mathbf{E}_n \mathbf{X}_n \mathrm{E}\{\mathbf{V}_n^H \mathbf{V}_m\} \mathbf{X}_m^H \mathbf{E}_m\} \\ &= r\,tr\{\sum_{n=1}^{N} |a_n|^2 \mathbf{E}_n\} \end{aligned} \quad (21)$$

where $\mathbf{E}_n = (\mathbf{X}_n \mathbf{X}_n^H)^{-1}$, and the latter one is obtained using the following equation:

$$\mathrm{E}\{\mathbf{V}_n^H \mathbf{V}_m\} = \begin{cases} r\,\mathbf{I}_{n_p} & ; \quad n = m \\ \mathbf{0} & ; \quad n \neq m \end{cases} \quad (22)$$

For multiple LS estimation, the problem (18) can be written as

$$\min_{a_1,...,a_N} tr\left\{ \sum_{n=1}^{N} |a_n|^2 \mathbf{E}_n \right\} \quad S.T \quad \sum_{n=1}^{N} a_n = 1 \quad (23)$$

The LS estimator is unbiased. The constraint in (23) guarantees that the multiple LS estimation is also unbiased. To solve (23), the Lagrange multiplier method is used. The problem can be written as

$$L(a_1,...,a_N,\eta) = tr\left\{ \sum_{n=1}^{N} |a_n|^2 \mathbf{E}_n \right\} + \eta \left\{ \sum_{n=1}^{N} a_n - 1 \right\} \quad (24)$$

To find $a_1, ..., a_N$, the partial derivatives of (24) with respect to a_i $(i = 1, 2, ..., N)$ are computed. Then, the results are set equal to zero. Finally, the optimal weight coefficients in the multiple LS estimation are obtained from:

$$a_n = \frac{1}{tr\{\mathbf{E}_n\} \sum_{l=1}^{N} 1/tr\{\mathbf{E}_l\}} \quad ; \quad n = 1, ..., N \quad (25)$$

It is straightforward to show that under optimal training for LS estimator

$$tr\{\mathbf{E}_n\} = tr\{(\mathbf{X}_n \mathbf{X}_n^H)^{-1}\} = \frac{t^2}{P_n} \quad (26)$$

where p_n is the total power of training matrix X_n which is used during the training phase in the sub-block *n*. Suppose that $p_n = k_n p$ is the transmitted power during the *n*-th training period and $p_{tot} = \sum_{n=1}^{N} p_n = N \times p$ is the total transmitted power during the N training periods. Then $\sum_{n=1}^{N} k_n = N$ and using (26), the optimal weight coefficients (25) can be rewritten as

$$a_n = \frac{1}{(t^2 / k_n p)\sum_{l=1}^{N}(p_l / t^2)} = \frac{k_n p}{\sum_{l=1}^{N} p_l} = \frac{k_n}{N} \quad (27)$$

Using (26) and (27), under optimal training, the error (21) is minimized as follows

$$J_{Multiple\,LS\,(\min)} = \frac{r t^2}{p N^2}\sum_{n=1}^{N} k_n = \frac{r t^2}{N\, p} \quad (28)$$

Comparing (28) and (8), it is seen that in the multiple LS estimation, the error reduces by the number of sub-blocks N which is used for channel estimation. It is notable that the error (28) is independent of p_n, the transmitted power during the n-th training period. It means that for uniform training powers and non-uniform training powers during N training periods, the error is the same.

B. Multiple Bayesian MMSE Estimation

Using (1), the MMSE channel estimator (9) can be rewritten as

$$\hat{\mathbf{H}}_{MMSE} = \mathbf{M} + (\mathbf{H} - \mathbf{M})\mathbf{XA} + \mathbf{VA} \quad (29)$$

Using (17) and (29), the MSE of multiple Bayesian MMSE channel estimator is expressed as

$$\begin{aligned} J_{Multiple\,MMSE} &= \mathrm{E}\left\{ \left\| \mathbf{H} - \sum_{n=1}^{N} a_n \hat{\mathbf{H}}_n \right\|_F^2 \right\} \\ &= \mathrm{E}\left\{ \left\| \mathbf{H} - \sum_{n=1}^{N} a_n \left(\mathbf{M} + (\mathbf{H}-\mathbf{M})\,\mathbf{X}_n \mathbf{A}_n + \mathbf{V}_n \mathbf{A}_n\right) \right\|_F^2 \right\} \\ &= \mathrm{E}\left\{ \left\| (\mathbf{H}-\mathbf{M})(\mathbf{I}_t - \sum_{n=1}^{N} a_n \mathbf{X}_n \mathbf{A}_n) - \sum_{n=1}^{N} a_n \mathbf{V}_n \mathbf{A}_n \right\|_F^2 \right\} \\ &= tr\{(\mathbf{I}_t - \sum_{n=1}^{N} a_n^* \mathbf{A}_n^H \mathbf{X}_n^H)\mathbf{C_H}(\mathbf{I}_t - \sum_{m=1}^{N} a_m \mathbf{X}_m \mathbf{A}_m) \\ &\quad + \sum_{n=1}^{N}\sum_{m=1}^{N} a_n^* a_m \mathbf{A}_n^H\, \mathrm{E}\{\mathbf{V}_n^H \mathbf{V}_m\}\mathbf{A}_m\} \end{aligned} \quad (30)$$

Using (23), (10), and with some calculations, the MSE (30) can be expressed as

$$\begin{aligned} J_{Multiple\,MMSE} &= tr\{\mathbf{C_H}\} - \sum_{n=1}^{N} a_n\, tr\{\mathbf{C_H}\mathbf{X}_n \mathbf{A}_n\} \\ &\quad + \sum_{n=1}^{N} (|a_n|^2 - a_n^*)\, tr\{\mathbf{A}_n^H \mathbf{X}_n^H \mathbf{C_H}\} \\ &\quad + \sum_{m=1}^{N}\sum_{\substack{n=1 \\ n\neq m}}^{N} a_n^* a_m\, tr\{\mathbf{A}_n^H \mathbf{X}_n^H \mathbf{C_H}\mathbf{X}_m \mathbf{A}_m\} \end{aligned} \quad (31)$$

The optimization problem is

$$\min_{a_1,\ldots,a_N} J_{Multiple\,MMSE} \quad S.T \quad \sum_{n=1}^{N} a_n = 1 \quad (32)$$

The MMSE estimator is biased. The constraint in (32) results in that the multiple MMSE estimation is also biased. The Lagrange multiplier method is used as

$$L(a_1,\ldots,a_N,\eta) = J_{Multiple\,MMSE} + \eta\left\{\sum_{n=1}^{N} a_n - 1\right\} \quad (33)$$

The partial derivatives of (33) are obtained with respect to a_i $(i = 1,2,\ldots,N)$, then, the result is set equal to zero as

$$\begin{aligned} \frac{\partial L}{\partial a_i} &= -tr\{\mathbf{C_H}\mathbf{X}_i \mathbf{A}_i\} + a_i^*\, tr\{\mathbf{A}_i^H \mathbf{X}_i^H \mathbf{C_H}\} \\ &\quad + \sum_{\substack{n=1 \\ n\neq i}}^{N} a_n^*\, tr\{\mathbf{A}_n^H \mathbf{X}_n^H \mathbf{C_H}\mathbf{X}_i \mathbf{A}_i\} + \eta = 0 \end{aligned} \quad (34)$$

Using the equation $\mathbf{X}_i \mathbf{X}_i^H = ((p + r\,tr\{\mathbf{C_H^{-1}}\})/t)\,\mathbf{I}_t - r\,\mathbf{C_H^{-1}}$ as the optimal training condition in MMSE channel estimator and with some calculations, (34) reduces to

$$r t^2 \left(\frac{r\,tr\{\mathbf{C_H^{-1}}\}}{p_i + r\,tr\{\mathbf{C_H^{-1}}\}} - 1\right)\sum_{\substack{n=1 \\ n\neq i}}^{N} \frac{a_n^*}{p_i + r\,tr\{\mathbf{C_H^{-1}}\}} + \eta = 0 \quad (35)$$

Using $p_1 = \ldots = p_N = p_{tot}/N = p$, as the uniform power allocation, and $\sum_{n=1}^{N} a_n = 1$, (35) reduces to

$$\frac{r t^2 p}{(p + r\,tr\{\mathbf{C_H^{-1}}\})^2}\,(a_i^* - 1) + \eta = 0 \quad (36)$$

Using (36) and $\sum_{n=1}^{N} a_n = 1$, the Lagrange multiplier can be obtained as

$$\eta = \left(\frac{N-1}{N}\right)\frac{r t^2 p}{(p + r\,tr\{\mathbf{C_H^{-1}}\})^2} \quad (37)$$

Substituting (37) back into (36), it is shown that in the uniform power allocation a_n is

$$a_n = \frac{1}{N} \;;\quad n = 1,\ldots,N \quad (38)$$

Using (38) and under optimal training, the MSE (31) is minimized in the uniform power allocation as

$$\begin{aligned} J_{Multiple\,MMSE\,(\min)} &= \frac{r t^2}{p + r\,tr\{\mathbf{C_H^{-1}}\}} \\ &\quad \times \left(\frac{1}{N} + \frac{N-1}{N}\,\frac{r\,tr\{\mathbf{C_H^{-1}}\}}{p + r\,tr\{\mathbf{C_H^{-1}}\}}\right) \end{aligned} \quad (39)$$

When N=1, (39) reduces to the special case of (15) for single channel estimation with the MMSE estimator. According to (39), it is seen that the error decreases when the number of sub-blocks N increases.

V. NUMERICAL RESULTS

In this section, the performance of the LS and MMSE estimators is numerically examined in the case of SE and

ME. As a performance measure, it is considered that the channel MSE is normalized by the average channel energy as

$$NMSE = \frac{E\{\|\mathbf{H}-\hat{\mathbf{H}}\|_F^2\}}{E\{\|\mathbf{H}\|_F^2\}} \quad (40)$$

Same as [6], the elements of the covariance matrix of the channel are defined as follows

$$[\mathbf{C_H}]_{k,l} = \frac{r}{1+\mathrm{K}}\rho^{|k-l|} \ , \ 0 \le |\rho| \le 1 \ , \ k,l = 1,\ldots,t \quad (41)$$

Fig. 1 shows Normalized MSE (NMSE) of the LS channel estimator with optimal training versus SNR in the case of SE and ME. According to this figure, increasing the number of the sub-blocks N results in a lower error of the estimation. In other words, the performance of the LS estimator in ME case is better than SE case. Clearly, the performance of the LS estimator is independent of the channel Rice factor K and the correlation coefficients ρ [6].

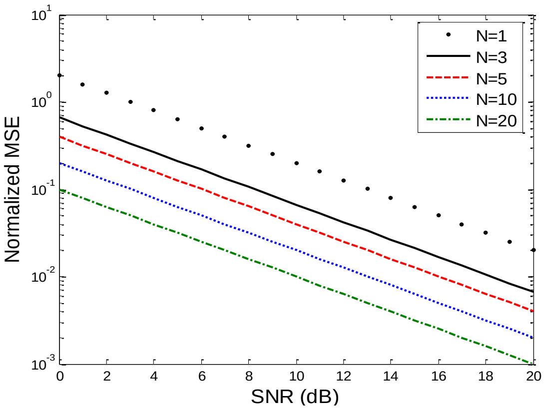

Figure 1. Normalized MSE of the LS estimator in the case of SE (N=1) and ME (N=3, 5, 10, 20) for r=t=2.

Fig. 2, Fig. 3, Fig. 4, and Fig. 5 indicate the NMSE of the MMSE channel estimator in the case of SE and ME. As depicted in these figures, the MMSE estimator has better performance in ME case than SE especially at high SNRs. However, at low SNRs, the NMSEs of the estimator for various numbers of sub-blocks N are analogous particularly for high values of K and/or ρ.

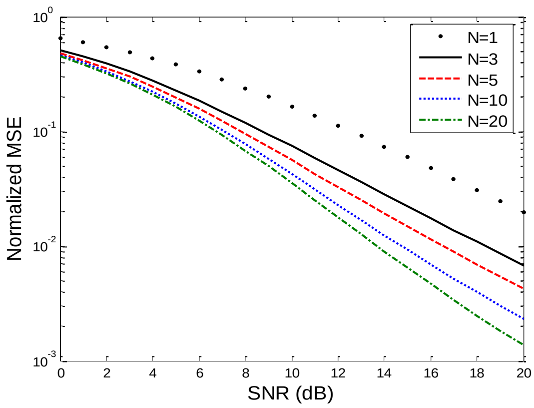

Figure 2. Normalized MSE of the MMSE estimator in the case of SE (N=1) and ME (N=3, 5, 10, 20) for K=0 (r=t=2, ρ=0.2).

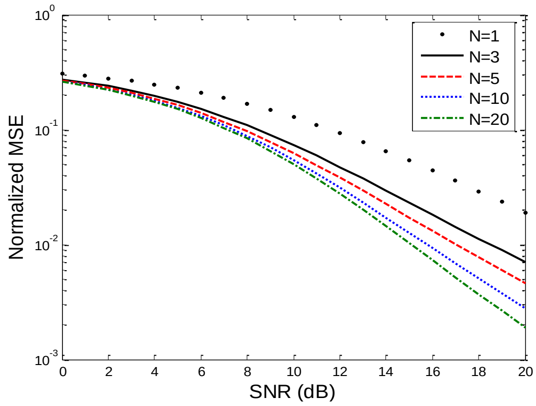

Figure 3. Normalized MSE of the MMSE estimator in the case of SE (N=1) and ME (N=3, 5, 10, 20) for K=0 (r=t=2, ρ=0.8).

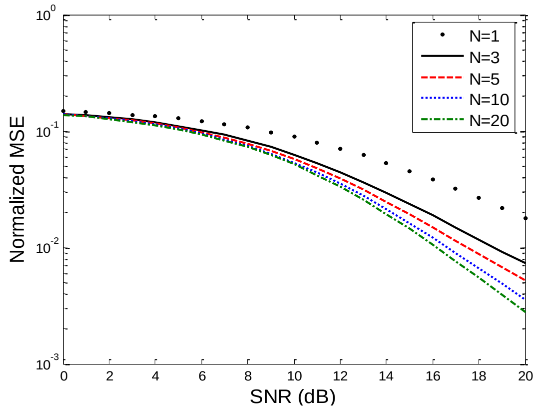

Figure 4. Normalized MSE of the MMSE estimator in the case of SE (N=1) and ME (N=3, 5, 10, 20) for K=5 (r=t=2, ρ=0.2).

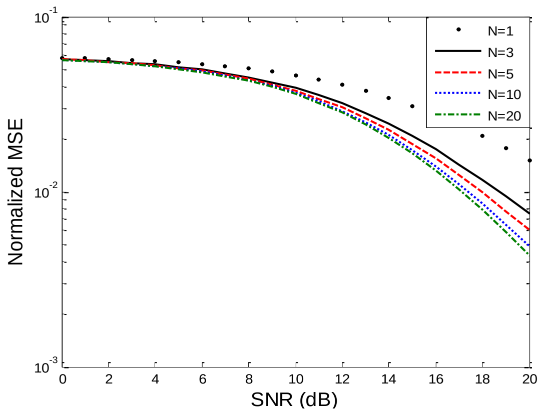

Figure 5. Normalized MSE of the MMSE estimator in the case of SE (N=1) and ME (N=3, 5, 10, 20) for K=5 (r=t=2, ρ=0.8).

Comparing these figures, it is seen that improving the performance of MMSE channel estimator with increasing the number of sub-blocks N depends on the channel fading and the correlations. In the case of low correlations (ρ=0.2) and Rayleigh fading (K=0), ME with MMSE channel estimator results in a better performance than the case of high correlations (ρ=0.8) and Rician fading (K=5). In other words, the superiority of the multiple estimates of MMSE channel estimator appears in the channels with low correlations and Rayleigh fading.

VI. CONCLUSIONS

The performance of LS and MMSE estimators in the case of SE and ME has been probed in the Rician flat fading MIMO channels. In the both SE and ME cases, the channel estimation errors have been obtained under optimal training. In the case of ME, the optimal weight coefficients and MSE are achieved for aforementioned estimators.

Analytical and numerical results show that the MSE of both estimators decreases when the number of sub-blocks *N* is increased. It is shown that ME with MMSE estimator is proper for both Rayleigh and Rician fading channels and/or for both low correlations and high correlations. However, it is more suitable for the channel with low correlations and/ or Rayleigh fading.

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Hamid Nooralizadeh was born in Sarab, Iran, on 10 May, 1968. He received the B.S. degree in Electrical Engineering from Tehran Science and Industry University, Tehran, Iran, in 1992 and the M.S. degree in Electrical Engineering from Tehran South Branch, Islamic Azad University, Tehran, Iran, in 1996. In September 1999, he joined the Department of Electrical and Electronics Engineering, Tehran South Branch, Islamic Azad University, as a Faculty member. In 2005, he also joined the Department of Electrical Engineering, Tehran Science and Research Branch, Islamic Azad University, as a Ph.D. Candidate. He received the Ph.D. degree in Electrical Engineering from Tehran Science and Research Branch, Islamic Azad University, Tehran, Iran, in 2010. In October 2006, he joined the Department of Electrical and Electronics Engineering, Islamshahr Branch, Islamic Azad University, as a Faculty member and now he is with them. His research interests include channel estimation, MIMO, signal processing, image processing and wireless communications

