

Determination of MIMO Channel Capacity and Enhancement of MIMO System Performance Using Tomlinson Harashima Precoding

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Abstract—Multiple input multiple output (MIMO) systems using multiple transmit and receive antennas are widely recognized as the vital breakthrough that will allow future wireless systems to achieve higher data rates with limited bandwidth and power resources. The capacity of MIMO systems depends strongly on whether the channel state information (CSI) is available at the transmitter or not. The main aim of this paper is to carry out investigations of MIMO system capacity under different assumptions about the underlying channel. The analysis of MIMO channel capacity has been carried out for two different cases based on the quality of channel state information at the transmitter side assuming that the receiver has full channel information. When the CSI is known to transmitter, the MIMO channel capacity has been determined using waterfilling algorithm. Also, analysis over the capacity of the MIMO channel systems has been considered when CSI is considered imperfect at the transmitter. In this paper we will also study about precoding techniques used to enhance the performance of MIMO system. By precoding we mean all methods applied at the transmitter that facilitates detection at the receiver. Precoding has been used in single user (SU) MIMO to improve the SNR at the receiver. In multiuser (MU) MIMO systems precoding is essential to eliminate or minimize Multiuser Interference (MUI).

Index Terms—multiple input multiple output (MIMO) systems, channel state information (CSI), channel capacity, precoding

I. INTRODUCTION

Wireless systems continue to strive for ever higher data rates. This goal is particularly challenging for systems that are power, bandwidth, and complexity limited. However, another domain can be exploited to significantly increase channel capacity: the use of multiple transmit and receive antennas. The large spectral efficiencies associated with multiple MIMO (multiple input and multiple output) channels are based on the premise that a rich scattering environment provides independent transmission paths from each transmit antenna to each receive antenna. Therefore, for single-user systems, a transmission and reception strategy that exploits this structure achieves capacity on approximately

min (M, N) separate channels, where M is the number of transmit antennas and N is number of receive antenna. Much subsequent work has been aimed at characterizing MIMO channel capacity under more realistic assumptions about the underlying channel model and the channel estimates available at the transmitter and receiver. In this paper, we examine MIMO wireless capacity with CSI (channel state information) known at transmitter and CSI not known at transmitter. When the channel is time-varying in nature and has multiple definitions, depending on what is known about the channel state or its distribution at the transmitter and/or receiver and whether capacity is measured based on averaging the rate over all channel states/distributions or maintaining a constant fixed or minimum rate. Channel capacity for deterministic and random MIMO channel where the CSI is perfectly known to receiver but unknown to the transmitter side is presented. In this case ergodic capacity defines the rate that can be achieved via this fixed-rate strategy based on receiver averaging over all channel states. Specifically, when the instantaneous channel gain, called the channel state information (CSI), is known perfectly at both transmitter and receiver, the transmitter can adapt its transmission strategy relative to the instantaneous channel state. In this paper we will determine the channel capacity for Rayleigh fading channel using waterfilling algorithm assuming that the transmitter and receiver has full CSI knowledge. In this paper we will study about the precoding techniques used at transmitter to enhance the capacity of MIMO system. The precoding vectors for a single user MIMO system are generally determined based on the downlink channel of one corresponding user. The precoding strategy that maximizes the throughput, called channel capacity, depends on the channel state information available in the system. With precoding techniques employed at the transmit side, the required computational effort for each user's receiver can be reduced and eventually the receiver structure can be simplified. There are mainly two types of precoding techniques, linear and non-linear. Linear precoding is characterized by its simplicity since the data signal is linearly transformed at the transmitter. The nonlinear precoding is named from its nonlinear processing, and a superior performance is achieved compared to the linear precoding algorithms. In this

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paper, Tomlinson-Harashima precoding (THP), a nonlinear pre-equalization technique, is proposed for transmission over multiple-input/multiple-output channels.

II. MIMO CHANNEL MODEL

MIMO technology is a wireless technology that uses multiple antennas at transmitter and receiver side to transfer more data at same time in order to increase the efficiency and performance of the wireless link. Fig. 1 represents the MIMO system.

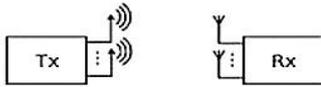


Figure 1. MIMO system

For narrowband single user MIMO system, we have considered that the transmit and receive antenna are omnidirectional. The receiver is assumed to have ideal channel estimates so it can separate and decode the symbols transmitted from each antenna [1]. The MIMO channel model is shown in Fig. 2.

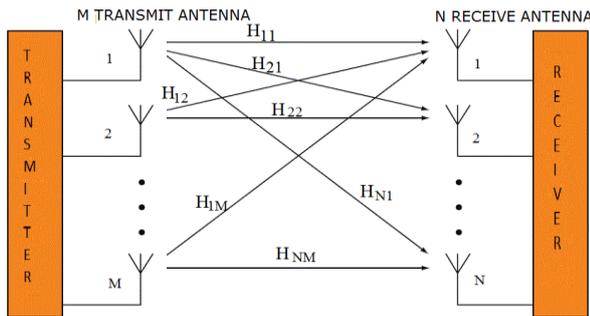


Figure 2. MIMO channel model

The linear link model between the transmitter and receiver antenna can be represented as

$$y = Hx + n \quad (1)$$

where x is $(M \times 1)$ transmit signal vector, y is $(N \times 1)$ received signal vector, n is $(N \times 1)$ additive white Gaussian noise vector and H is $(M \times N)$ channel matrix which describes the wireless channel between transmitter and receiver and is given by

$$H = \begin{bmatrix} H_{11} & \dots & H_{1M} \\ \vdots & \ddots & \vdots \\ H_{N1} & \dots & H_{NM} \end{bmatrix}$$

The general entry (ij^{th} entry) of matrix H denoted by H_{ij} represents the path gain between the i^{th} antenna and j^{th} receive antenna. In rich scattering environment the channel gains $|h_{ij}|$ are usually Rayleigh Distributed. Also the column of H are assumed to be independent [2].

A. MIMO Channel Capacity

According to Shannon's law the capacity in terms of signal to noise ratio and bandwidth is given by

$$C = BW \log_2(1 + SNR) \quad (2)$$

A basic approximation of MIMO channel capacity is a function of number of spatial streams, bandwidth and signal to noise ratio and is given as

$$C = nBW \log_2(1 + SNR) \quad (3)$$

where 'n' is number of spatial streams.

Channel capacity with CSI unknown at transmitter: For, narrow band MIMO system, when the transmitter has no knowledge about the channel, it is optimal to spread the energy equally among all transmitter antennas in the case channel capacity is given by

$$C = \log_2 \left[\det \left(I + \frac{SNR}{M} HH^H \right) \right] \\ = \sum_{i=1}^{\min(M,N)} \log \left(1 + \frac{SNR}{M} \lambda_i \right) \quad (4)$$

where, λ_i is non zero eigen values of HH^H and HH^H is complex conjugate transpose of H [3]. Here we have assumed that MIMO channels are deterministic where the channel gain remains constant and the CSI is perfectly known to receiver but unknown to the transmitter side [4].

In general case, MIMO channels change randomly and hence H is a random matrix which means that its channel capacity is also randomly time varying and follows an ergodic process in practice. The capacity of such random channels is known as ergodic channel capacity. Ergodic capacity defines the maximum average rate under an adaptive transmission strategy averaged over all channel states. The ergodic channel capacity for the MIMO system without CSI at the transmitter side is given by [5]

$$C = E \left\{ \sum_{i=1}^{\min(M,N)} \log \left(1 + \frac{SNR}{M} \lambda_i \right) \right\} \quad (5)$$

Channel capacity with CSI known at the transmitter: When the channel is constant and known perfectly at the transmitter and the receiver, the MIMO channel can be converted to parallel, non-interfering single input single output (SISO) channels through a singular value decomposition of the channel matrix [6]. The model decomposition can be performed as shown in the Fig. 3, in which a transmitted signal is preprocessed with V in the transmitter and then, a received signal is post processed with U^H in the receiver.

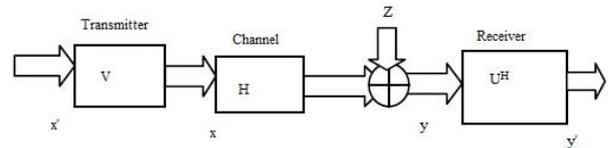


Figure 3. System model when channel knowledge is available at the transmitter side

The system model when channel knowledge is available at transmitter, is given by

$$y' = U^H (HVx' + n) \\ y' = U^H HVx' + U^H n \quad (6)$$

Single value decomposition of H yields $H = U \Sigma V^H$, putting this in (6) we get,

$$y' = \Sigma x' + n \quad (7)$$

Now, If the transmit power for the i^{th} transmit antenna is given by P_i , then the capacity of the i^{th} channel is

$$C_i(P_i) = \log_2 \left(1 + \frac{SNR P_i}{M} \lambda_i \right) \quad (8)$$

where $I=1, 2, \dots, r$ and $r=\min(M, N)$. The capacity in (8) can be maximized by solving the following allocation problem:

$$C = \sum_{i=1}^r \log_2 \left(1 + \frac{SNR P_i}{M} \lambda_i \right) \quad (9)$$

subjected to $\sum_{i=1}^r P_i = M$.

It is assumed that total power available at the transmitter side is 'M'.

The capacity can be increased using water filling algorithm by assigning various levels of transmitted power to various transmitting antennas. This power is assigned on the basis that higher the gain in the channel, the more power it gets and vice versa. This is an optimal energy allocation algorithm [7].

The principle of the waterfilling theorem (in Fig. 4) sees the division of total power in such a way that a greater portion goes to the channels with higher gain and less or even none to the channels with small gains. This distributes the available transmit power over the Eigen modes of the channel realization based on the eigenvalues of the channel.

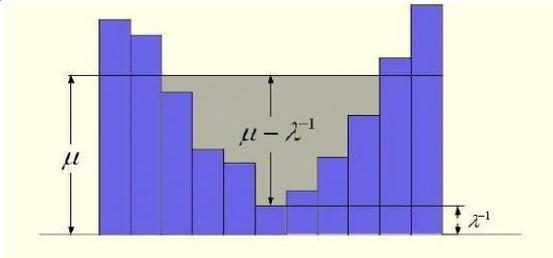


Figure 4. Illustration of water filling

Waterfilling the transmit power over the parallel MIMO channels whose gains are given by λ_i leads to power allocation

$$P_i = \sum_{i=1}^r (\mu - 1/\lambda_i)^+ \quad (10)$$

where, P_i is the power in the i^{th} Eigen mode of the channel, μ is chosen from waterfill algorithm and $(*)^+$ denotes take only those terms which are positive.

If P_i is positive then the power is allocated to the i^{th} sub channel otherwise, the sub channel is left unused. The resulting capacity is given by

$$C_{WF} = \sum_{i=1}^r \log_2 (\mu \lambda_i)^+ \quad (11)$$

III. MIMO PRECODING

By precoding we mean all methods applied at the transmitter that facilitate detection at the receiver. Precoding is multi stream beamforming. It is considered to be all spatial processing that occurs at the transmitter. The main reason for which the precoding techniques are so important in wireless MIMO systems is that if the precoding techniques are applied at the source signals before they are transmitted it reduces the performance loss caused by interference and channel fading. Thus, the

advantage of MIMO system will be enhanced. Precoding techniques are performed with the help of downlink Channel State Information (CSI). The assumption that full CSI is available at the transmit side is valid in Time Division Duplex (TDD) systems because the uplink and downlink share the same frequency band. For Frequency-Division Duplex (FDD) systems, however, the CSI needs to be estimated at the receiver and fed back to the transmitter. With precoding techniques employed at the transmit side, the required computational effort for each user's receiver can be reduced and eventually the receiver structure can be simplified [8], [9]. There are mainly two types of precoding techniques, linear and non-linear. Linear precoding is characterized by its simplicity since the data signal is linearly transformed at the transmitter. The nonlinear precoding is named from its nonlinear processing, and a superior performance is achieved compared to the linear precoding algorithms. The nonlinear Tomlinson-Harashima Precoding (THP) is another technique that can decompose the MU-MIMO channel into parallel channels by utilizing a successive pre-processing cancelation at the transmit side. Because of MUI is cancelled out successively, the performance of THP strongly depends on the ordering of the precoded symbols.

A. Tomlinson-Harashima Precoding for MIMO Channels

Tomlinson-Harashima precoding (THP) is a pre-equalization technique originally proposed for channels with inter symbol interference (ISI) [10], [11]. Based on the knowledge of CSI at the transmit side, the interference of the parallel streams of a MIMO system with spatial multiplexing can be subtracted from the current stream. This successive interference cancellation technique at the transmit side is known as THP.

For this exposition we will assume a real-valued transmission model of the type

$$\tilde{Y} = Bx + \tilde{n} \quad (12)$$

where B is the effective transmission matrix, i.e., a transformed version of H such that the interference that Tomlinson-Harashima precoding cannot take into account is minimized (i.e., the upper triangular part of the matrix B is minimized). We additionally assume that the effective transmission matrix is scaled such that it has unit diagonal, i.e., $b_{kk}=1; k=1, \dots, K$. Channel matrix H is QL (Q is orthogonal matrix and L being lower triangular matrix) decomposed as

$$H = F^T S \quad (13)$$

where, F is orthonormal, S is lower triangular. We can force the upper triangular part to be zero, obtaining $B = \text{diag}(\frac{1}{S_{11}}, \dots, \frac{1}{S_{kk}})S$, (then $\tilde{Y} = Fy = Bx + \tilde{n}$) [12].

Now we assume that the symbols a_k to be transmitted in x are taken from the real signalling set corresponding to Mary QAM constellation. The real "boundary" interval is consequently given as $[-A/2, A/2)$, where we define $A=\sqrt{M}$.

Transmitting these symbols over the channel described by the matrix B, the components of x have to be “coded” such that the interference from the first symbol into the output variable corresponding to the second, third, etc., symbol is eliminated, that from the second to the third, fourth, etc., and so on.

The way this is done in Tomlinson-Harashima precoding for this particular set of transmit symbols is to use the symmetric modulo operation $\text{mods}_A(x)$, which reduces its argument x to $-A/2 \leq x \leq A/2$, and select, with $x_1 = a_1$,

$$x_2 = \text{mods}_A(a_2 - b_{21}x_1) \quad (14)$$

or more generally for sub channel k,

$$x_k = \text{mods}_A(a_k - \sum_{k=1}^{k-1} b_{kk}x_k) \quad (15)$$

At the receiver, we obtain for the highest index subchannel

$$\tilde{Y}_k = b^k x + \tilde{n}_k \quad (16)$$

(b^k denoting the Kth row of B), which evaluates to

$$\begin{aligned} \tilde{Y}_k &= \sum_{k=1}^k b_{kk}x_k + \tilde{n}_k \\ \tilde{Y}_k &= x_k + \sum_{k=1}^{k-1} b_{kk}x_k + \tilde{n}_k \end{aligned} \quad (17)$$

where we have used $b_{kk}=1$, and from (15) we obtain

$$\begin{aligned} \text{mods}_A(\tilde{Y}_K) &= \text{mods}_A\left(a_K - \sum_{k=1}^{K-1} b_{Kk}x_k + \sum_{k=1}^{K-1} b_{Kk}x_k + \tilde{n}_K\right) \\ \text{mods}_A(\tilde{Y}_K) &= \text{mods}_A(a_K + \tilde{n}_K) \end{aligned} \quad (18)$$

For a sub channel $l < K$, the detection of a_l is affected by interference from sub channels $(l+1, \dots, K)$ as long as B has nonzero entries in the upper right half:

$$\begin{aligned} \text{mods}_A(\tilde{Y}_l) &= \text{mods}_A\left(\sum_{k=l+1}^{K-1} b_{lk}x_k + \tilde{n}_l\right) \\ &= \text{mods}_A\left(a_l - \sum_{k=1}^{l-1} b_{lk}x_k + \sum_{k=1, k \neq l}^k b_{lk}x_k + \tilde{n}_l\right) \\ &= \text{mods}_A\left(a_l + \sum_{k=l+1}^K b_{lk}x_k + \tilde{n}_l\right) \end{aligned} \quad (19)$$

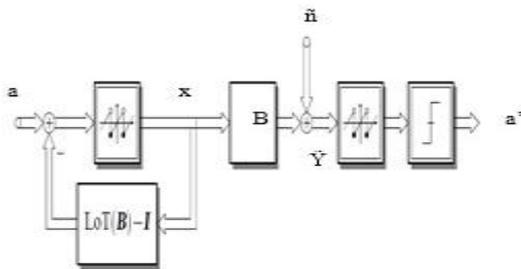


Figure 5. Tomlinson-Harashima precoder operating on effective channel B

Compared to the solution based on the decomposition given above, which forces the inter-channel interference

to 0, allowing some residual inter-channel interference can be favourable if the resulting sum disturbance, $\sum_{k=l+1}^K b_{lk}x_k + \tilde{n}_l$ has less power. A block diagram of Tomlinson-Harashima precoding is shown below in Fig. 5.

Here the transmitter side processing corresponding to (15) is performed by the feedback filter based on the lower-triangular part of B. The symmetric modulo operation is depicted as a saw tooth block. An equivalent formulation of the precoding operation is shown in Fig. 6

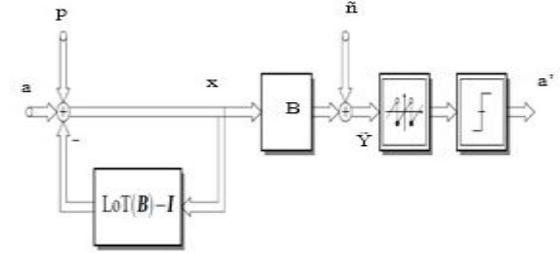


Figure 6. Linear description of Tomlinson-Harashima precoder operating on effective Channel B, using precoding vector p

In this figure the precoding vector p replaces the modulo operation.

$$\begin{aligned} x_k &= \text{mods}_A\left(a_k - \sum_{k=1}^{k-1} b_{kk}x_k\right) \\ &= a_k - \sum_{k=1}^{k-1} b_{kk}x_k - A\left[\left(a_k - \sum_{k=1}^{k-1} b_{kk}x_k + A/2\right)/A\right] \\ &= a_k - \sum_{k=1}^{k-1} b_{kk}x_k + p_k \end{aligned} \quad (20)$$

where $p_k = -Ar_k \in A\mathbb{Z}$ and $p = [p_1, \dots, p_k]^T \in A\mathbb{Z}^K$. In the transmit signal we see for component k the 2^{k-1} different possible values of the interference, leading to a total of 2^k distinct values.

Using a matrix F on y at the receiver, interference from lower-number sub channels to higher-number sub channels is avoided and an effective transmission matrix is obtained. Also $H = F^T S$ with F orthonormal and S lower triangular, and introducing the scaling matrix $r = \text{diag}\left(\frac{1}{S_{11}}, \dots, \frac{1}{S_{kk}}\right)$, we obtain a unit-diagonal lower triangular matrix $B = rFH = rS$

The resulting transmission system including the pre- and post-processing (in contrast to Fig. 5 and Fig. 6) is depicted in Fig. 7

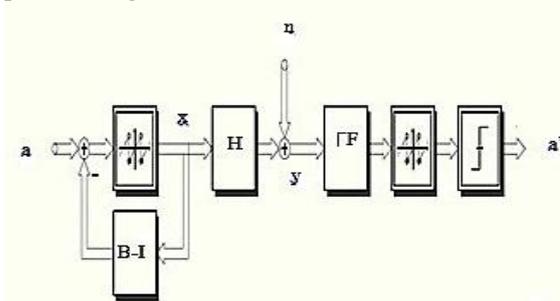


Figure 7. Tomlinson-Harashima precoding for zero-forcing interference suppression, centralized receiver

Since the equivalent parallel channels are characterized by the diagonal entries of S , an optimization of these factors by permutation of the sub channels is possible.

Instead of trying to suppress the residual interference that Tomlinson-Harashima precoding cannot take care of at the receiver side, it is also possible to do so at the transmitter side [13], [14].

Now, $H = SF^T$ is formed by decomposition of H^T . only "causal" interference (described by the lower triangular S) is present if the transmit signal is rotated by F prior to transmission. With the scaling matrix $r = \text{diag}(\frac{1}{s_{11}}, \dots, \frac{1}{s_{kk}})$, we now have the unit-diagonal lower triangular, $B = HF r$.

It turns out that for the multiple antenna MIMO point-to-point channel, both schemes, receiver-side and transmitter-side rotation, have similar performance. We should however emphasize that only the latter method, rotating the transmit signal such that only causal interference is present, is applicable to broadcast transmission systems [15], [16]. For SU MIMO systems, it can be advantageous to switch between the two methods according to the present realization of H , selecting the one that yields a higher sum capacity of the parallel channels.

IV. SIMULATION AND RESULTS

A. Simulations for Capacity of MIMO Channel

Fig. 8 shows the plot of ergodic capacity of the MIMO channel with respect to SNR as the number of transmit antenna (M) and receive antennas (N) is varied, when CSI is not known at the transmitter side.

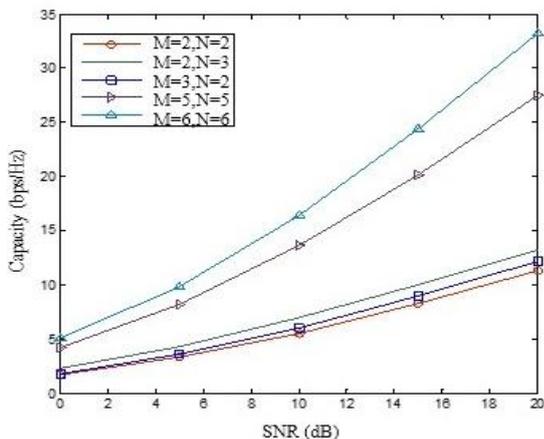


Figure 8. Ergodic MIMO channel capacity when CSI is not available at the transmitter

From the Fig. 8 we can interpret that increasing the number of antenna elements does increase the channel capacity, however at slow rate. Channel capacity of $M \times N$ MIMO system with $M > N$ is less than that of $M \times N$ MIMO system with $M < N$ i.e., the capacity of MIMO channel with $M=3$ and $N=2$ is less than the capacity of MIMO channel with $M=2$, $N=3$. Thus, keeping M constant capacity increases logarithmically with the increase in number of receive antenna N .

But keeping N , constant capacity does not increase at all with increase in number of transmit antenna. Also, MIMO channel capacity grows linearly with $\min(M, N)$ for a given fixed transmitted SNR. Capacity for 5×5 MIMO channel is greater than capacity for 6×6 MIMO channel. Fig. 9 shows the ergodic channel capacity with CSI known at transmitter.

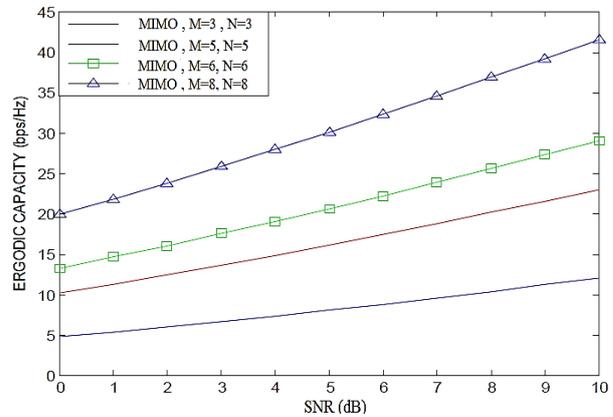


Figure 9. Ergodic capacity of MIMO channel with CSI known at transmitter

At a particular SNR value the ergodic capacity of $M \times N$ MIMO system is greater when channel state information is known at transmitter side than the case when CSI is not known at transmitter side. When SNR is 9dB and CSI is not known at transmitter side, the capacity of 5×5 MIMO system is 12.5bps/Hz but when CSI is known to transmitter the capacity of 5×5 MIMO system with same SNR(9dB) as previous is 40bps/Hz.

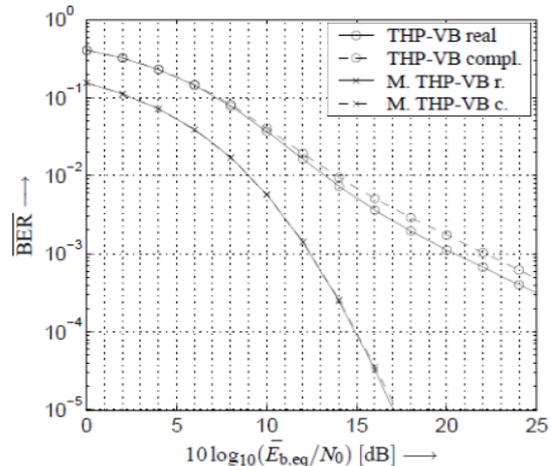


Figure 10. Average bit error rates achieved using Tomlinson-Harashima precoding with the V-BLAST algorithm, for real- and complex-valued matrices as well as zero forcing and MMSE solutions, $K_c = 4$, 4-QAM

B. THP Simulation Results

Fig. 10 shows error rate curves for zero-forcing and MMSE THP based on the real-valued equivalent model (THP-VB real/M. THP-VB r.) and those if the complex-valued model is used (THP-VB complex./M. THP-VB c.). We see that while in the zero-forcing case similar gains as in the detection setup are possible, for MMSE THP (labelled M. THP-VB) only a tiny improvement is visible

at already very low bit error rates ($<10^{-4}$). This is due to the good average performance of all sub channels of MMSE THP in this SNR range.

THP and MMSE THP based on an unsorted QR decomposition are labelled as THP (QR) and M. THP (QR), respectively. In all schemes depicted, equal rates were used in all sub channels.

V. CONCLUSION

In this paper we have described the capacity calculation of MIMO system. We have presented simulation results comparing channel capacities for different configuration of MIMO system. The simulation results shows that increasing the number of antenna elements does increase the MIMO channel capacity and capacity grows linearly with $\min(M, N)$ for a given fixed transmitted SNR. Also MIMO system with CSI available at the transmitter can greatly improve spectral efficiency over MIMO system without CSI at transmitter. The MIMO channel capacity depends on channel knowledge, SNR, number of transmit and receive antenna elements and on correlation between them. Also the performance of multiuser MIMO systems are better than that of single user MIMO system which can be further improved using THP precoding.

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