Generalized Synchronization of Topologically-Nonequivalent Chaotic Signals via Active Control

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***Abstract*—Synchronization of the broadband-like information-maskable state trajectories of the Lorenz-63 and Burke-Shaw chaotic systems is presented in this paper. The two systems are three-dimensional dissipative chaotic systems which have similar algebraic structures but are topologically nonequivalent. Active controllers were designed and employed to track the exponentially divergent information-masking signals of their state trajectories into synchrony, based on the Lyapunov stability criteria. Numerical simulation results via MATLAB 7 software confirmed the global synchronization of the systems for all initial conditions and the asymptotic stabilization of the resulting synchronization error dynamics in the sense of Lyapunov. In addition, the robustness of the generalized synchronization scheme to parametric perturbation in the nonlinear hyperbolic structure of the Burke-Shaw slave system when interchanged between sinh and cosh holds possibility for online tuning of the coupled systems to vary the broadband spectrum density of the information-masking carrier when applied to modelling and design of chaos-based secure communication systems.**

***Index Terms*—lorenz-63 system, burke-shaw system, synchronization, active control, lyapunov stability**

I. Introduction

Research into the control and synchronization of chaotic systems has gained the attention of researchers cutting across various disciplines especially mathematics, physics and engineering. This is justified by the increasing applications of the principles of chaos control and synchronization to practical system designs in secure communication and other fields. Chaos is a behavioural dynamics which results from the extreme sensitivity or hypersensitivity to perturbations in the structural parameters and initial conditions of some deterministic dynamic systems and has been found to be prevalent in many man-contrived and natural systems such as the ecology [1], economics [2], power systems [3], neurology and medicine [4], electronics and communication systems [5]. Chaos synchronization occurs when under appropriate coupling configurations, the trajectories of two systems achieve synchrony in finite time. As novel chaotic systems continue to be evolved, the imperative of studying the characteristics of these systems to ascertain their applicability in real-life modelling and design becomes a fascinating endeavour. Essentially, when a chaotic system meets the conditions of controllability and synchronizability, it becomes a possible candidate for multi-faceted applications, especially in information theory and secure communications where the primary objective, among others, is to mask transmitted information so that it assumes a pseudo-noisy broadband spectrum in a transmitting channel. Consequent to this development, various methodologies have been studied and employed to synchronize identical or non-identical chaotic signals. Among them are the feedback control [6], active control [7], linear control [8], adaptive control [9], fuzzy control [10], sliding mode control [11], linear matrix inequality (LMI)-based fuzzy control [12]. Equally, several chaotic systems have received much attention in the literature due to their topological properties and structural adaptability such as the Lorenz-63 system [13], Lorenz-84 systems [14], Chen [15], Lu [16], Sprott [17], Chua [18]. However, in recent years, attention has also been focused on the usefulness of recently evolved systems in the context of synchronization. These systems include the Lu-Chen-Cheng [19], Chen-Lee [20], Lorenz-63-Pelivan [21], hyperchaotic Lu [22] and so on. In this paper, attention is focused on the coupling of the Lorenz-63 and Burke-Shaw system which has received less attention in the literature, but is nonetheless dynamically complex with distinctive properties.

II. The Lorenz-63 and Burke-Shaw Chaotic System

The Lorenz-63 system is a three-dimensional chaotic system that has two nonlinear terms in its system equations [13]. When the parameters of the system are appropriately adjusted, it evolves into the popular butterfly-shape attractor. The algebraic structure of the system is governed by the following equations:

$$\begin{aligned} \dot{g}_1 &= -\gamma g_1 + \gamma g_2 \\ \dot{g}_2 &= \mu g_1 - g_2 - g_1 g_3 \\ \dot{g}_3 &= g_1 g_2 - \varepsilon g_3 \end{aligned} \tag{1}$$

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where g_1, g_2, g_3 are states of the system, γ, μ, ε are constant parameters that determines the shape of the evolved chaotic attractor. For values of $\gamma=10$, $\mu=28$, $\varepsilon=8/3$, the system evolves the popular butterfly attractor. The information-maskable state trajectories of the Lorenz-63 system are depicted in Fig. 1(i)-Fig. 1(iii).

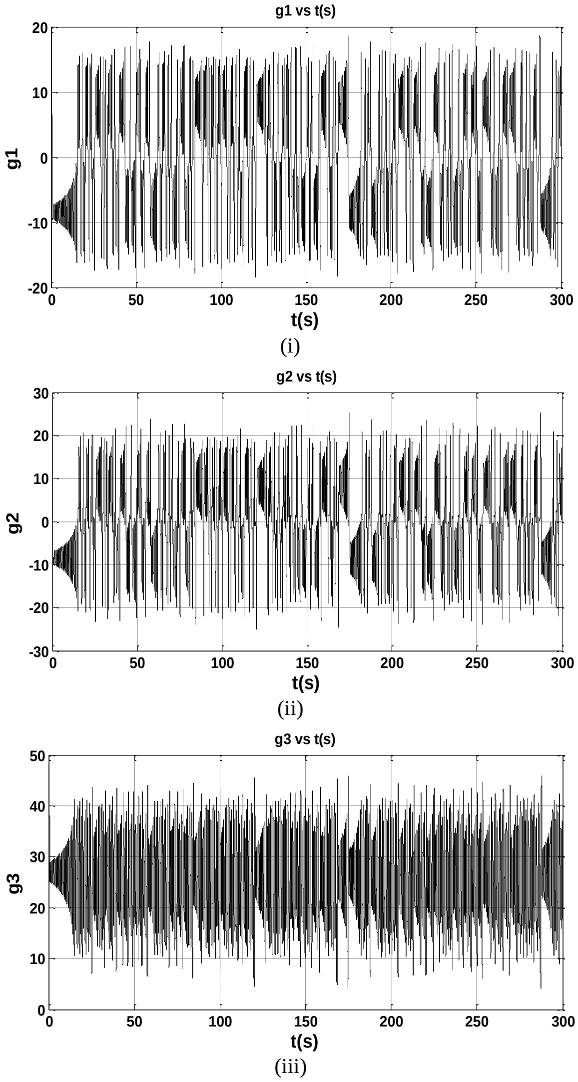


Figure 1. (i)-(iii) Information-Maskable state trajectories of the Lorenz-63 system

The Burke-Shaw (BS) chaotic system [23] is also three-dimensional system that possesses two quadratic nonlinear terms in its system equations. The BS system was evolved from the Lorenz-63 system, and therefore has similar algebraic structure to that of the Lorenz-63 system, but is topologically nonequivalent. Detailed structural and parametric analyses have been reported in [24]. The governing equation of the system is given by the following equations:

$$\begin{aligned} \dot{h}_1 &= -\eta h_1 - \eta h_2 \\ \dot{h}_2 &= h_2 - \eta h_1 h_3 \\ \dot{h}_3 &= \eta h_1 h_2 + \psi \end{aligned} \qquad (2)$$

where h_1, h_2, h_3 are state variables, η, $\psi > 0$ are positive constants. Eq. (2) unfolds different attractors as ψ is varied positively. For typical values of $\eta=10$, $\psi=4.272$, the information-maskable trajectories of the system are depicted in Fig. 2(i)-Fig. 2(iii).

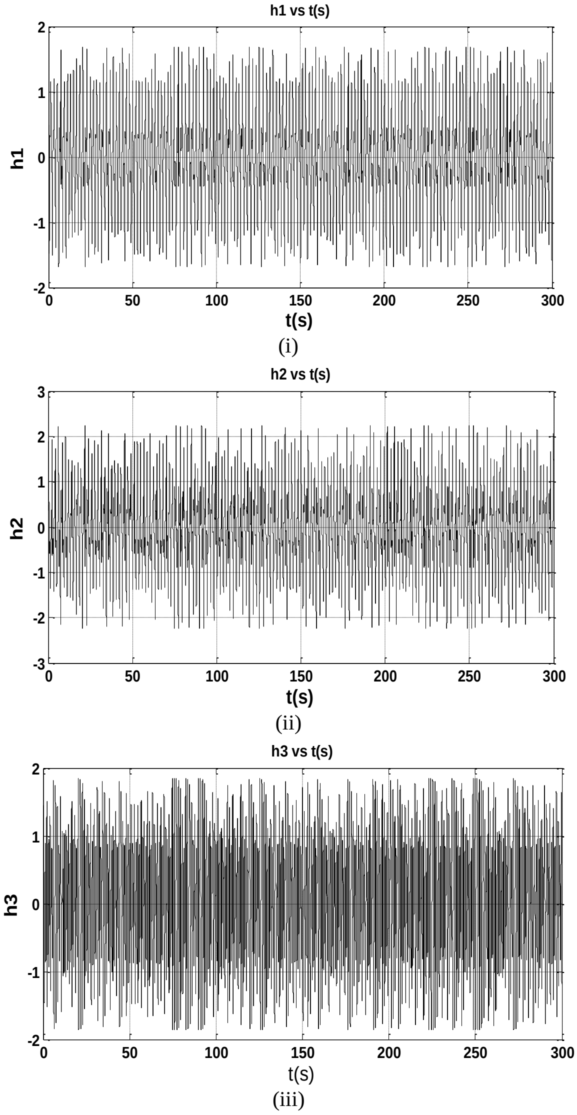


Figure 2. (i)-(iii) Information-Maskable state trajectories of the Burke-Shaw system

III. Generalized Synchronization

Generalized synchronization, which is a special case of identical synchronization occurs in cases where two chaotic systems g and h are coupled such that in transient time, the trajectories of h are regulated by g. In a master-slave (transmitter-receiver) topology, if the information-maskable state variables of the master system is denoted as $g(t)=(g_1,\ g_2,\ ...,\ g_n)$ where n is the dimension of the system and $h(t)=(h_1,\ h_2,\ ...,\ h_n)$ denotes the state variables of the slave system, then global synchronization occurs if there exist a coupling function Π such that after time

$$t\big|_{g(t_0),h(t_0)} < t << \infty;\ g(t_0) = g_1(0),\ g_2(0),\ g_3(0);$$
$$h(t_0) = h_1(0),\ h_2(0),\ h_3(0)$$

The resulting coupled dynamics can be expressed as $\left(g_1,\ g_2,\ ...,\ g_n\right) = \prod\left(h_1,\ h_2,\ ...,\ h_n\right)$, which implies that $g(t)$ regulates the evolution of $h(t)$. Generally, the coupling function is a nonlinear control law.

IV. Control Objectives

Consider two chaotic systems of the form:

$$x' = Mx + N(x) \tag{3}$$

$$y' = Py + Q(y) + u_i \tag{4}$$

where $x,\ y$ are system states, $p,\ q$ are vector fields that models the chaotic systems and $u_i\,(i = 1,\ 2,\ ...,\ n)$ are the active controllers to be designed. If $M = P$, the two systems are identical, otherwise, they are non-identical. Let $e = (e_1,\ e_2,\ e_3,\ ...e_n) = y - x$ be the synchronization errors where $x = (x_1,\ x_2,\ ...\ x_n),\ y = (y_1,\ y_2\ ,...y_n)$ and

$$\begin{aligned} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3 \\ e_n &= y_n - x_n \end{aligned} \tag{5}$$

where n is the dimension of the chaotic systems. By subtracting (3) from (4), we can obtain the error dynamics expressed in the following form:

$$e' = Py - Mx + Q(y) - N(x) + u \tag{6}$$

The control objective is to design active controllers that will enable the information-maskable state dynamics of the master to regulate the evolution of the slave's state dynamics and also asymptotically stabilize the resulting error dynamics of the synchronized systems in the sense of Lyapunov. i.e.

$$\lim_{t\to\infty}\left\|e(t)\right\| = 0, \forall e_i(0) \tag{7}$$

V. Active Controller Design

Active control design strategy has been applied in several works in the literature [7], [21]. Let the Lorenz-63 system in (1) be taken as the master system. The slave and responding system in (2) can be rewritten with the control functions as:

$$\begin{aligned} h_1' &= -\eta h_1 - \eta h_2 + u_1 \\ h_2' &= h_2 - \eta h_1 h_3 + u_2 \\ h_3' &= \eta h_1 h_2 + \psi + u_3 \end{aligned} \tag{8}$$

Subtracting (1) from (8) and applying the synchronization error given in (5), we obtained the following error dynamics:

$$\begin{aligned} e_1' &= -\eta h_1 - \eta h_2 + \gamma g_1 - \gamma g_2 + u_1 \\ e_2' &= -h_2 - \eta h_1 h_3 - \mu g_1 + g_2 + g_1 g_3 + u_2 \\ e_3' &= \eta h_1 h_2 + \psi - g_1 g_2 + \varepsilon g_3 + u_3 \end{aligned} \tag{9}$$

Setting $\eta = 10,\ \gamma = 10,\ \varepsilon = 8/3,\ \psi = 4.272,\ \mu = 28$ and using the convention in (5), the active controllers are chosen as:

$$\begin{aligned} u_1 &= 9e_1 + 10e_2 + 20x_2 \\ u_2 &= 10h_1h_3 + 28g_1 - g_1g_3 \\ u_3 &= -10h_1h_2 + g_1g_2 - 4.272 - 8/3g_3 - e_3 \end{aligned} \tag{10}$$

Inserting (10) into (9), the error dynamics becomes:

$$\begin{aligned} e_1' &= -e_1 \\ e_2' &= -e_2 \\ e_3' &= -e_3 \end{aligned} \tag{11}$$

Theorem 1: If the active controllers governed by (10) are selected, then the state vector dynamics of Lorenz-63 system (1) will regulate the evolution of the state dynamics of the Burke-Shaw system (2).

To verify the asymptotic stability of the error dynamics, we apply the Lyapunov stability criteria [25] and choose the Lyapunov function candidate

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^3) \tag{12}$$

Theorem 2: If the partial derivative of (12) along the trajectories of the error dynamics (11) is negative semi-definite or negative definite, then the error dynamics is asymptotically stable in the sense of Lyapunov for all initial conditions.

$$g_1(0),\ g_2(0),\ g_3(0) \neq h_1(0),\ h_2(0),\ h_3(0)$$

Proof:

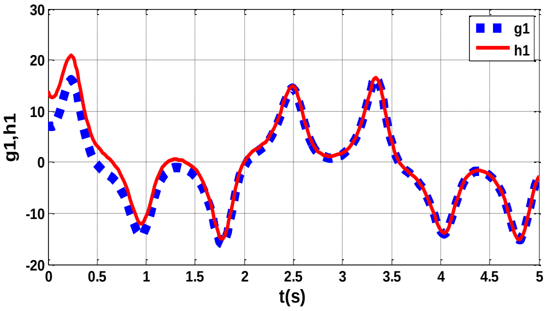
$$V'(e_1, e_2, e_3) = e_1e'_1 + e_2e'_2 + e_3e'_3 \tag{13}$$

By inserting (11) into (13), $V'(e_1,\ e_2,\ e_3) \leq 0$ which is negative definite function on $\Re^3$. Consequently, the trajectories of (9) will be locally and globally asymptotically stable in the sense of Lyapunov as $t \to 0$.

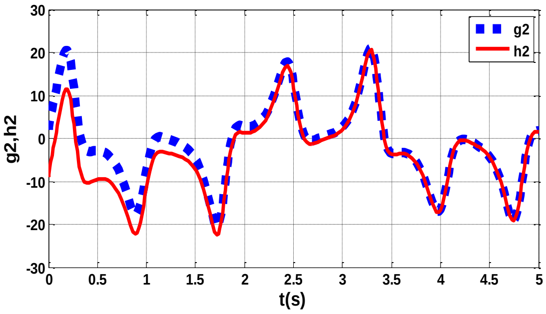
VI. Numerical Simulation Results

The synchronized systems were simulated with MATLAB software for the following initial conditions: Master system, $g(0)\big|_{g_1(0),g_2(0),g_3(0)} = [8,\ 2,\ 10]$ and the Slave system, $h\big|_{h_1(0),h_2(0),h_3(0)} = [14,\ -9,\ 6]$ which gives

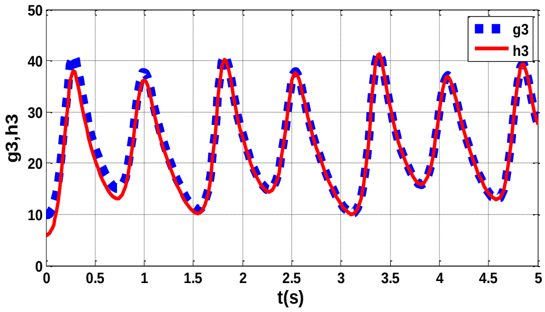
$e(0)\big|_{e_1(0),e_2(0),e_3(0)} = [22,\ -7,\ 16]$. Fig. 3, Fig. 4 and Fig. 5 are plots depicting the simulation results.



(a)



(b)



(c)

Figure 3. (a)-(c) Converged trajectories of the synchronized signals

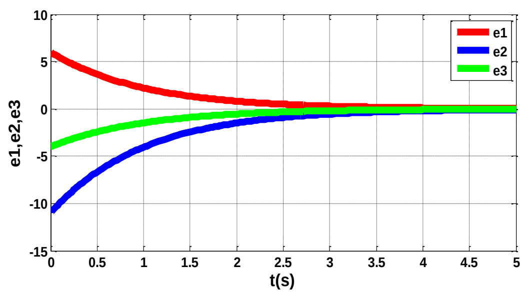


Figure 4. Time series of the synchronization error dynamics

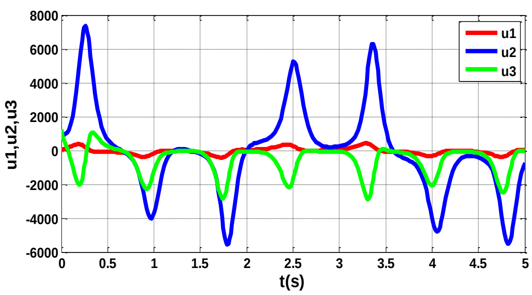


Figure 5. Dynamics of the active control signals

VII. Conclusion

Global synchronization of non-identical information-maskable state trajectories of the Lorenz-63 and Burke-Shaw chaotic systems has been demonstrated via numerical simulations. The designed active controllers seamlessly synchronized the two systems and asymptotically stabilized the error dynamics that resulted from the coupled trajectories. The designed controller was also robust in cases where the positive constant ψ was varied between 4.272 and 13. The relevance of this synchronization lies in their usefulness in chaos-based secure communication scheme in which the broadband information-maskable signals could serve as encryption keys on vulnerable transmitting channels that are susceptible to third party information pilfering. The variability of the responding signals indicates the possibility of online tuning of the spectrum density in the transmission channel to enhance information security.

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