Elementary NGD IIR/FIR Systems

Blaise Ravelo

IRSEEM, EA 4353/ESIGELEC, Av. Galilée, BP 10024, 76801 St Etienne du Rouvray Cedex, France

Email: blaise.ravelo@yahoo.fr

Abstract—This paper deals with the synthesis and design of impulse response digital systems exhibiting negative group delay (NGD). It is demonstrated that under certain condition, certain first order FIR and IIR systems can provide the NGD function. Synthesis methods of NGD FIR/IIR transfer functions are introduced. Then, the basic properties specific to the NGD digital functions are established. To validate the theory proposed, NGD FIR and IIR systems were synthesized and implemented with Matlab. As expected, group delay frequency responses with NGD in baseband frequency well-correlated to the theoretic hypothesis were found. Then, thanks to the NGD effect, arbitrary form discrete transient signals with limited bandwidth propagating in time advance were demonstrated in time-domain. It is proved that this counterintuitive function does not forbid the causality principle and the NGD numerical system respects the functioning condition of the classical digital system. The NGD principle presented is potentially useful for the group delay correction processes and the signal pure delay reduction.

Index Terms—digital system, negative group delay (NGD), synthesis method, NGD system design, delay equalizer

I. INTRODUCTION

Since the middle 1990s, theoretical and experimental investigations have demonstrated that certain electronic circuits are capable to operate with negative group delay (NGD) [1]-[4]. Similar to the classical electronic functions (filter, amplification, phase shifter…), it was found that the NGD circuits respect the criteria for the correct functioning of electronic systems. Under specific conditions between the NGD transfer function parameters and considered electrical signal bandwidth, it was shown several times that through the NGD circuit, the electrical signal output can propagate in time-advance of its input [5]-[9]. In this case, it was pointed out that this counterintuitive phenomenon does not forbid the causality principle [3], [4].

At the beginning, the NGD electronic circuit was designed with low frequency components as the operational amplifier associated with passive networks [1]-[4]. Therefore, its applications were limited to very low frequency systems only up to some MHz. Then, NGD passive devices operating at microwave wavelengths were proposed [10]-[14]. But analytical theories and experimentations confirmed that with such passive devices, the apparition of NGD effect is systematically accompanied with significant losses [1], [10]-[13].

To overcome this limitation, NGD microwave active circuit topologies were developed recently [15]-[17]. It was stated that these NGD active topologies were susceptible to operate up to several GHz, offering then possibilities of loss compensations. So, several types of NGD circuit applications, in particular, in telecommunication area were introduced [15]-[17]. Currently, one of technological issues which bottleneck the progress of current digital electronic systems is caused by the delay effects. As examples, it was reported that designers of microwave devices [18], [19] and electronic boards and circuits [20], [21] must take into account the different kinds of delays induced by the electrical links and interconnect busses. To cope with such aspects, various applications of NGD circuits based on the delay and phase compensation techniques were introduced recently. First, in the microwave area, low group delay broadband phase shifter [22] operating independently with the frequency were synthesized and implemented. In addition, enhanced structure of feed-forward amplifier was proposed [23]. Then, further application based on the group delay equalization dedicated to the analogue-numerical or mixed microelectronic systems was also established [24]-[27]. It consists in the recovery of the mixed signal integrities degraded by the interconnect circuitries between the logic gates or electronic chips with possibilities of rise-/fall-time and propagation delay reductions [25].

In this paper, innovative principle of the digital systems exhibiting the outstanding NGD function is developed. Similarly to the aforementioned analogue delay compensation approaches, the NGD effects can be used for the diminution of the constraints caused by the fractional or pure delays in the digital filters and control systems [28]-[35]. In addition, by transposing in numerical area the techniques introduced in [24], [25], the NGD digital systems could be employed for the equalization of the group delay based on the digital signal processing (DSP).

For the better comprehension, this paper is structured in three main sections. Section II focus is on the analysis and synthesis of IIR and FIR digital systems capable to exhibit the NGD effect. Then, specific properties and characteristics of the NGD function will be also established. Section III is depicted with the verification and validation of the analytic concept established. Design examples and numerical experimentations run with Matlab illustrating the relevance of the theory will be
analyzed and discussed. Then, frequency-domain and transient analyses with different types of test signals will be made. Then, the last section is the conclusion of the paper.

II. THEORETICAL APPROACH ON THE ELEMENTARY NGD DIGITAL IMPULSE RESPONSES

First and foremost, let us consider the first order IIR system excited by the discrete input signal and generating samples of output respectively, denoted \( x[n] \) and \( y[n] \) ( \( n \)-th samples) at the discretized time \( t_n = nT_s \) ( \( T_s \) is the considered sampling time) defined by the difference equation:

\[
y[a+1] = H_0 \{ x[a+1] + b \cdot x[n] \} - a \cdot y[n]
\]

(1)

with \( H_0 > 0 \), \( a \) and \( b \) \((a \neq b)\) are real constant. The transfer function canonical form of this system is written as:

\[
H(z) = Y(z)/X(z) = H_0 (1 + b \cdot z^{-1})/(1 + a \cdot z^{-1})
\]

(2)

Fig. 1 represents the block diagram of this IIR system under consideration.

![Diagram of 1st order IIR system](image)

To preserve the stability of the system under study, along this paper, \( |a| \) is supposed lower than 1.

A. Analysis of Elementary NGD IIR Filter

It is well-known that the harmonic responses of the system expressed in Eq. (2) is realized replacing \( z \) by \( e^{j\omega} \) \((\omega = 2\pi f \cdot T_s \) is the angular frequency and \( j = \sqrt{-1} \)). So, the magnitude and phase responses are respectively expressed as:

\[
H(\omega) = H_0 \sqrt[2]{ |1 + b \cos(\omega)|^2 + b^2 \sin^2(\omega)} / \sqrt[2]{ |1 + a \cos(\omega)|^2 + a^2 \sin^2(\omega)}
\]

(3)

\[
\varphi(\omega) = \arctan \left( \frac{b \sin^2(\omega)}{1 + b \cos(\omega)} \right) - \arctan \left( \frac{a^2 \sin^2(\omega)}{1 + a \cos(\omega)} \right)
\]

(4)

Therefore, the group delay frequency response defined by \( \tau(\omega) = -\dot{\varphi}(\omega)/\dot{\omega} \), is given by:

\[
\tau(\omega) = \frac{a[\cos(\omega)] - a[\cos(\omega)]}{1 + 2a \cos(\omega) + a^2} \cdot T_s
\]

(5)

**Synthesis Method:** At very low frequencies \((\omega \approx 0)\), the magnitude of \( H(\omega) \) becomes:

\[
H(0) = H_0 \left| \frac{1+b}{1+a} \right|
\]

(6)

The same, the group delay frequency responses will be expressed as:

\[
\tau(0) = T_s (b-a)/(1+a)(1+b)
\]

(7)

One finds that this quantity is negative under the condition:

\[
\frac{b-a}{(1+a)(1+b)} < 0 \Leftrightarrow b - a < 0, (1+a)(1+b) > 0
\]

(8)

In this case, an innovative type of IIR system with negative group delay (NGD) is realized. By exploiting the previous frequency analysis, a synthesis method of NGD IIR system can be established according the desired values of negative group delay \( \tau_0 \) and gain \( a \) at very low frequencies \((\omega=0)\). Based-on the canonical form expressed in Eq. (2), the design of the NGD IIR system depends on the determination of parameters \( H_0 \), and \( b \) by fixing \( a \) (or inversely, \( b \) by fixing \( a \)). For example, by fixing the parameter \( a \), the term \( b \) can be synthesized by inverting equation \( \tau(0) = \tau_0 \). Therefore, the following synthesis formula is yielded:

\[
b = \frac{a \cdot T_s + (1+a) \tau_0}{(1+a)T_s}
\]

(9)

It is interesting to note that in order to design an NGD IIR system, \( \tau_0 \) must be negative. The parameter \( H_0 \) can be determined similarly, by solving the equation \( H(0) = a \). So, one gets the following expression:

\[
H_0 = \frac{a(T_s - (1+a)\tau_0)}{(1+a)T_s}
\]

(10)

Knowing these synthesis formulae, it can be wondered what are the specific characteristics and properties of the IIR systems with the NGD function.

**Properties of NGD IIR Systems:** In practice, the group delay of first order linear systems can be negative but its frequency bandwidth must be limited. The NGD cut-off frequencies can be determined from equation \( \tau(2\pi f \cdot T_s) = 0 \). By using Eq. (5), it results the following formula:

\[
f_c(k) = \frac{\arccos \left(-a+b\right) \left(1+a \cdot b\right) + 2k\pi}{2\pi T_s}
\]

(11)

where \( k \) is a constant integer. The targeted application of the NGD concept established in this paper concerns the field of base band signal processing. So, the operating signal frequency spectrum must start from zero to certain frequency limit (in order of GHz). For this reason, the first cut-off frequency can be assumed as the lowest of frequencies defined in Eq. (11). \( f_c = f_c(0) \). Moreover, substituting Eq. (9) into Eq. (11), the following relation
between the desired value of NGD \( \tau_0 \) and the cut-off angular frequency is extracted:

\[
f_c = \frac{1}{2\pi f_s} \arccos \left[ \frac{2a \cdot T_s + (1-a^2)\tau_0}{(1-a^2)\tau_0 - (1+a^2)T_s} \right]
\] (12)

This expression states that for the first order NGD IIR systems, the cut-off angular frequency \( f_c \) is inversely proportional to \( [\tau_0] \). In order to achieve a uniform propagation of each spectral component of the signal exciting the NGD systems expressed in Eq. (2), the cut-off frequency \( f_c \) must be higher or equal to the signal bandwidth for the simplification here denoted \( f_c \). During the synthesis process, the maximal value of the desired NGD must be equal to:

\[
\tau_{0\text{max}} = T_s \left[ \frac{2a + (1 + a^2) \cos(2\pi f_s T_s)}{(a^2 - 1)[1 - \cos(2\pi f_s T_s)]} \right]
\] (13)

It is noteworthy that the performance of NGD systems depend on the surface \( A(\tau < 0) \) delimited by the group delay \( \tau(\omega) \), \( x \)- and \( y \)-axis as will be seen in the next section about the numerical experimentation. This surface is mathematically defined as:

\[
e^{-2\pi \tau_j} \int_{\omega = 0}^\omega \tau(\omega)d\omega = \varphi(0) - \varphi(2\pi f_c \cdot T_j)
\Rightarrow A(\tau < 0) = \arctan \left[ \frac{(a - b) \sqrt{(a^2 - 1)(b^2 - 1)}}{b^2 \sin^2(\omega)} \right]
\] (14)

B. Analysis of Elementary NGD FIR Filter

One can realize a FIR system from the IIR transfer function expressed in Eq. (2) by taking \( a = 0 \). The canonical form becomes:

\[
H(z) = H_0(1 + b \cdot z^{-1})
\] (15)

Frequency Analysis: The gain and phase harmonic responses are respectively expressed as:

\[
H(\omega) = H_0 \sqrt{[1 + b \cdot \cos(\omega)]^2 + b^2 \sin^2(\omega)}, \quad \varphi(\omega) = \arctan \left[ \frac{b^2 \sin^2(\omega)}{1 + b \cdot \cos(\omega)} \right]
\] (16)

Through the derivative calculation with respect to \( \omega \), yields the group delay of the FIR system understudy:

\[
\tau(\omega) = \frac{b[b + \cos(\omega)]}{1 + 2b \cos(\omega) + b^2} \cdot T_j
\] (18)

Synthesis Method: One can see that at very low frequencies, the magnitude and the group delay responses are written as respectively:

\[
H(0) = H_0 \cdot \|1 + b\|, \quad \tau(0) = T_jb/(1 + b)
\] (19)

(20)

By considering \( |b| < 1 \), this last expression can be negative under the condition:

\[
b/(1 + b) < 0 \iff (b < 0, 1 + b > 0)
\] (21)

Similar to the previous case of the IIR systems, the NGD cut-off angular frequencies can be determined with equation:

\[
\cos[2\pi f_c kT_s] = -b \Rightarrow f_c = \frac{\arccos(-b)}{2\pi f_s}
\] (22)

By inverting equation Eq. (20) and taking \( \tau(0) = \tau_0 \), one finds the following synthesis relation:

\[
b = \tau_0 / (T_s - \tau_0)
\] (23)

As \( \tau_0 \) must be chosen as to be negative, the absolute value \( |b| \) must be lower than 1. Similarly, by solving equation \( H(0) = H \), the second parameter of the NGD FIR proposed is established:

\[
H_0 = H(1 - \tau_0)
\] (24)

C. Analysis of High Order NGD Digital Function

When cascading two first-order NGD FIR \( H_1(j\omega) \) and \( H_2(j\omega) \), a second order FIR system with the following transmittance is realized:

\[
H(j\omega) = H_1(j\omega) \cdot H_2(j\omega)
\] (25)

So that, the total group delay is equal to:

\[
\tau(\omega) = \tau_1(\omega) + \tau_2(\omega)
\] (26)

Meanwhile, the group delay value is proportional to the number of cells in cascade. Contrarily to the classical systems with positive delay, in the present case NGD systems, the negative delay increases with the number of cells. In time-domain, it corresponds to the increase of the signal time-advance instead of signal delay. This finding is important for the optimization of high order systems with significant value of NGD surface area in wide frequency band. By cascading \( k \) identical NGD digital systems, the total group delay must be equal to \( \tau_k(\omega) = k \cdot \tau(\omega) \) and the surface delimited by expressed in Eq. (14) the NGD curve must be \( A_k(\tau < 0) = k \cdot A(\tau < 0) \).

III. VALIDATION WITH NUMERICAL EXPERIMENTS

The effectiveness of the previous theoretic concepts is intended to be verified in this section. Design examples of NGD IIR and FIR systems are realized and tested via frequency- and time-domain numerical analyses. Different types of transient digital signals are considered. It is noteworthy that the numerical results presented in this section were computed with Matlab.

A. Design and Synthesis of NGD FIR System

As application of the previous concept, the desired values of magnitude and group delay are arbitrarily chosen respectively, \( H_{\text{des}} = -0.57 \text{ dB} \) and \( \tau_0 = -2 \text{ ns} \). By
applying synthesis relations Eq. (23) and Eq. (24), the NGD FIR filter having the following difference equation is designed:

$$y(n+1) = 14.976x[n+1] - 0.937x[n]$$  \hspace{1cm} (27)$$

By implementing this NGD FIR function into Matlab program, the group delay plotted in Fig. 2 was obtained. As expected in theory, baseband NGD with minimal value equal to $-2$ ns with cut-off frequency of about $424$ MHz for the sampling time $T_s=0.13$ ns. This NGD frequency response is delimited by the shaded surface area shown in Fig. 2. So, an excellent agreement between the theory and the numerical experimentation is realized.

To illustrate the functioning of this NGD FIR, normalized discrete Gaussian input signals denoted $x_1$ and $x_2$ sampled in the time interval $[0, T_{max}=40]$ ns with 301 sampling points with half-width at half maximal heights $\Delta T_1 = 5.89$ ns and $\Delta T_2 = 1.47$ ns are considered.

As displayed in the bottom of Fig. 2, the frequency spectra of $x_1$ and $x_2$ present frequency bandwidths respectively of about $f_{s1}=429$ MHz and $f_{s2}=1.71$ GHz. Therefore, as can be seen in Fig. 3, the output signal $y_1$ practically similar to $x_1$ in time-advance of about $-2$ ns is obtained. However, as $f_{s2}$ is widely higher than the cut off frequency of the NGD FIR understudy, the output signal $y_2$ (see Fig. 3) corresponding to the input $x_2$ is completely distorted. It is worth underlining that this time advance does not forbid the causality principle. As illustrated in Fig. 4, at the starting point of the signal, the output $y_1[0]$ appear at the same time as the input $x_1[0]$. Then, as sketched in Fig. 4, thanks to the NGD function, this output behaves as an anticipated form of the input.

Furthermore, as explained by the experimentation of truncated signal plotted in Fig. 5, the output value $y_1[kT_s]$ is generated from $x_1[kT_s], x_1[(k-1)T_s]$ but not from the future value $x_1[kT_s+\tau_0]$. Meanwhile, the future value of the input does not affect the present value of the output.

The NGD FIR does not anticipate the signal discontinuity because this singularity point contains harmonic components outside the NGD bandwidth. It is important to note that the NGD function can operate also with different forms of smoothed input signals. As
demonstrated in Fig. 6, once again, output signals in advance compared to the input is occurred when injecting a signal with arbitrary form.

Figure 6. Demonstration of time-advance with arbitrary signals.

B. Design of NGD IIR System

Similar numerical test as in previous subsection was made with an NGD IIR system but in this case the desired values of gain and group delay are arbitrarily chosen equal to $H_{0dB} = 0$ dB and $\tau_0 = -1.5$ ns. By considering a pulse signal and 30ns periodical signal sampled in the time interval $[0, T_{max}=60\text{nsec}]$ with 501 sampling points. By applying synthesis relations Eq. (9) and Eq. (10), the NGD FIR system with difference equation was designed:

$$y[n+1] = 13.775[x[n+1] - 0.9274x[n]] - 0.25y[n]$$

(28)

Figure 7. Time-domain responses of NGD IIR filter expressed in (27).

As explained in Fig. 7, a group delay response with NGD bandwidth of about 676MHz is realized with optimal value $\sim 1.5$ ns at very low frequencies. With Matlab numerical experimentations, once again, output signals in time-advance were confirmed by considering different types of inputs with arbitrary form as depicted in Fig. 8(a) and periodical form as displayed in Fig. 8(b). As aforementioned earlier, one of potential applications of the NGD digital systems lies on the principle of the delay compensation. Delay equalization process diagram is proposed in Fig. 9(a) by cascading a fractional delay having equation $(n_0=\text{int}(|\tau_0/T|))$ with the NGD IIR defined in Eq. (27) as experimentation of the proposed technique efficiency. The test fractional delay was set at $\text{Time-Delay}=\tau_0=1.5\text{ns}$ for verifying the delay compensation. As illustrated in Fig. 9(b), the delayed signal is given by $x_0(n,T_s) = x(n,T_s-Delay)$. After interaction with the NGD IIR system designed, as proved in Fig. 9(b), the NGD system output is approximately equal to the general input, $y[n] = x[n]$ for $n > n_0 = \text{int}(\text{Delay}/T_s)$.

Figure 8. Time-domain responses of NGD IIR filter expressed in (27): (a) arbitrary pulse signal – (b) 30ns periodical signal.

Figure 9. (a) Delay equalization process diagram with NGD IIR system - (b) demonstration of transient signal compensation.

IV. CONCLUDING REMARK AND FUTURE WORK

Several investigations on the NGD analogue devices based on the electronic analogue circuit were published in the literature [1]-[17]. For this reason, innovative synthesis and design of IIR and FIR digital systems susceptible to exhibit NGD effect were devoted in this paper. By considering simple first-order transfer function
digital systems, it was demonstrated that NGD IIR/FIR functions can be realized. Then, synthesis method based on the desired value of NGD was established. Then, properties and characteristics specific to the NGD digital systems notably the determination of the NGD bandwidth and the effects of the association of NGD digital cells in cascade were developed.

In order to substantiate the relevance of the concept established, NGD IIR and FIR systems are designed and implemented into Matlab program. So, as expected, group delay response presenting NGD in base band frequency were obtained. Then, by considering different types of pulse signals, it was proved that when the frequency band of the input signal belongs to the NGD bandwidth, the output signal can propagate in time-advance to the input. It was confirmed also that the NGD function does not forbid the causality and the stability. In addition, it was demonstrated that this NGD effects can be potentially employed to the equalization of pure delay especially, in the numerical or mixed control or/and telecommunication systems.

Due to the incessant increasing number of applications involving digital signal processing (DSP) and numerical filtering, the variety of requirements that have to be met by digital systems has increased considerably [36]-[39]. Digital filters are integral parts of many DSP systems, including control systems, systems for audio and video processing, communication systems and systems for medical applications. In these systems, group delay equalizers play an important role by connecting in cascade with digital filters. In the continuation of this work, by improving the equalization method as described in [40]-[42], the study of the present NGD concept integration in the DSP for the synchronization of numerical data and delay correction in the complex high-speed system structures is in progress.

REFERENCES


Blaise Ravelo is currently an Assistant Professor at the Graduate School of Engineering ESIGELEC, located in Rouen, France. He is a pioneer of the development of NGD circuits. His research interest focus is on the applications of negative group delay (NGD) circuits, modeling of PCB interconnect for the signal integrity and radiated transient electromagnetic compatibility/interference (EMC/EMI) on microwave and digital microelectronic systems. He offers lectures on different topics as electronic circuit theory, telecommunication science and technology, and microwave/digital engineering.

He received the Dipl.-Ing. degree in electronic engineering from the Graduate Polytechnic School of Engineering, University of Antsiranana, Madagascar, in 2000. From 2001 to 2002, he was a construction engineer at MADIMEX Inc. which is located in Majunga, Madagascar. He received the Professional/Research Sci. Master degrees, and the Ph.D. degrees from Brest University, Brest, France, in 2004, 2005, and 2008, respectively. He hold his dissertation to lead researches (“HDR: Habilitation à Diriger des Recherches”) from the Univ. of Rouen in 2012.

Dr. Blaise Ravelo is regularly involved to participate and to lead national and international research projects (FP7, INTERREG, ANR, FUI...) on the electronic and microwave engineering. He is the Coordinator of the international project titled TEC5 (Time-Domain Electromagnetic Characterization and Simulation for EMC applications) which is funded by the programme INTERREG IVA Channel France/UK 2007-2013. He is (co)author of more than 100 papers published in international journals, book (and book chapters) and conference proceedings.