MUSIC Based on Uniform Circular Array and Its Direction Finding Efficiency

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***Abstract*—The mathematical model of uniform circular array was established and some simulation experiment systems were constructed. The direction finding efficiency of MUSIC based on uniform circular array was studied by simulation experiments. Then MUSIC based on uniform circular array was compared with MUSIC based on uniform linear array. The conclusions are as following. Two kinds of MUSIC can find the directions of independent signals and their DF accuracy is close. They all cannot find the directions of coherent signals. The advantage of MUSIC based on UCA: the array size is smaller; it is easier to increase antenna to improve the DF accuracy; the angle distinguishing ability of signals is higher; there is neither image vagueness nor aperture vagueness. The weakness of MUSIC based on UCA: it cannot eliminate the coherence of coherent signals; the ability to find the direction of weak signal is poorer.**

***Index Terms*—uniform circular array, uniform linear array, MUSIC, simulation, direction finding**

I. Introduction to MUSIC Based on UCA

Ref. [1] discussed the MUSIC for direction finding (DF) of independent waves, Ref. [2] discussed the Smoothing algorithm for direction finding of coherent signals and Ref. [3] discussed matrix decomposing algorithm for pre-estimation of number and coherent state of signals.

Above three algorithms are all based on uniform linear array (ULA). Theoretical analyses and simulation experiment results indicated that ULA was able to complete the task of multiple emitter direction finding and pre-estimation of number and coherent state of signals. The structure of ULA is simple and mathematically modeling based on ULA is easy. Because of these advantages, researchers will choose ULA at first when they design and layout antenna array. However, ULA is not perfect. The weakness is mainly manifested in the following two aspects.

- The array is too long. Suppose a linear array is composed of N antennas, the distance of two adjacent antennas is d, then the array length is $(N-1)d$. The array for DF is usually constituted with more than 8 antennas, and the distance of two adjacent antennas is generally half a wave-length. The wave-length of short-wave signal used in the wireless communication is usually in meter grade or ten-meter grade. Therefore, the length of ULA for DF of short-wave signal approaches tens of meters or even hundreds of meters. Sometimes, the number and the distance of antennas should be increased to find the directions of signals more accurately. This will increase the array length further. For so long array, it is difficult to remain the array uniform linear in battlefield. Even in strategy DF, the unequal lengths of feedback lines of antennas will make channel inconsistent or unbalanced and thus arouse DF error although it is possible to choice a neat and spacious place.

On the other hand, since the electromagnetic waves traveling along the ground fade rapidly, researchers hope to lift the transmitting or receiving antennas, i.e. to use airborne or space borne electronic support system. Let the emitting power of radio wave is 10W, the sensitivity of electronic support system is 1~5 μV/m, then radio signal traveling along the ground can be detected by ground-based electronic support system in 10km. Whereas air to air interception range for this signal is more than 150km. Furthermore, airborne electronic support system can be close to or even deep into the enemy battlefield. So airborne electronic support system can greatly improve SNR, and then improve DF accuracy. However, it is unthinkable to layout ULA more than tens of meters or even hundreds of meters in an aircraft (especially small UAV) or in a satellite.

- MUSIC based on ULA can not find 2-D directions of signals. Radio communication signals can arrive at receiver through ionosphere reflection in short-wave band, so the DF must jointly measure the azimuth and elevation of a signal. Therefore, the array structure can't be straight line, but should be plane array or three-dimensional array.

In view of the weakness of linear antenna array, it is necessary to consider some other array structures, such as circular array. Compared with linear array, the size of circular array can be fixed. For a circumference with the fixed radius, the number of antennas which are arranged in it is flexible. Furthermore, to increase the number of antennas does not increase the size of the array.

Circular array with 8 antennas is commonly applied to DF. The mathematical model of circular array with 8 antennas is shown in Fig. 1. Eight antennas are arranged

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in a horizontal circle with the radius of d uniformly, the north antenna is numbered as 0, the else antennas are numbered as 1~7 clockwise.

At first, we consider the inducted voltage of one signal at each antenna. For simple, we just find the azimuth of signal, i.e. suppose the elevation of signal to be 90°.

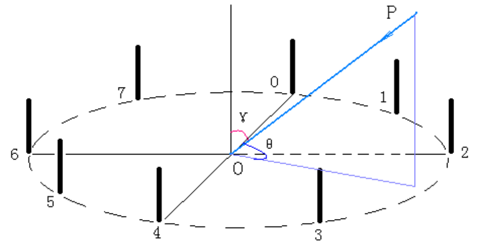

Figure 1. The uniform circular array with 8 antennas

Let the signal at the center O of the circle is $s(t)$. Connect the center O with the ith antenna and gain the line $Oi\ (i=0,1,2,\cdots,7)$. The angle from the wave vector $\vec{P}$ to $O0$ is θ. Mark θ^i as the angle from $\vec{P}$ to Oi, then $\theta^i=\theta-i\times45°$. The phase difference between the signal received by the ith antenna and $s(t)$ is $\varphi^i=(2\pi d/\lambda)\cos\theta^i\ (i=0,1,\cdots,7)$, so the signal received by the ith antenna is

$$x_i(t)=s(t)e^{j\varphi^i}=s(t)e^{j\frac{2\pi d}{\lambda}\cos\theta^i}\ (i=0,1,\cdots,7).$$

In general, suppose K signals are narrowband plane waves which are mutually independent, and mark

λ : the wavelength of K signals,

$\theta_1,\theta_2,\cdots,\theta_K$: the directions of K signals,

$\theta_k^i=\theta_k-i\times45°\ (k=1,2,\cdots,K;i=0,1,\cdots,7)$: the angle from Oi to the wave vector of the kth signal,

$\beta_k^i=(2\pi d/\lambda)\cos\theta_k^i$: the phase difference of the kth signal between the signal received by the ith antenna and $s_k(t)$,

$a(\theta_k)=(e^{j\beta_k^0},e^{j\beta_k^1},\cdots,e^{j\beta_k^7})^T$:the direc-tion vector of the kth signal,

$A(\theta)=\left(a(\theta_1),\cdots,\ a(\theta_K)\right)$: the direction matrix of K signals,

$S(t)=\left(s_1(t),\cdots,\ s_K(t)\right)^T$: the signal vector of K signals at the center O of the circle,

$n_i(t)\ (i=0,1,\cdots,7)$: the noise received by the ith antenna,

$N(t)=\left(n_0(t),n_1(t),\cdots,n_7(t)\right)^T$: the noise vector received by the array,

$X(t)=\left(x_0(t),x_1(t),\cdots,x_7(t)\right)^T$: the array induc-tion voltage vector received by the array,

then the array induction voltage vector can be expressed as following

$$\begin{aligned}X(t)&=a(\theta_1)s_1(t)+\cdots+a(\theta_K)s_K(t)+N(t)\\&=A(\theta)S(t)+N(t)\end{aligned}$$

The steps of MUSIC based on uniform circular array (UCA) is as following: at first, record the array induction voltage vector $X(t)$; secondly, calculate the array covariance matrix; thirdly, decompose the array covariance matrix based on eigenvalue; fourthly, construct the spatial spectrum function; and finally, search azimuth angles along the spatial spectrum and get the directions of signals.

These steps are exactly same as the MUSIC based on ULA, so we shall not relate them any more.

II. Introduction to Simulation System of MUSIC Based on UCA

The simulation system of MUSIC based on UCA consists of six modules, such as signal transmitter, channel, UCA, receiver, MUSIC and the results display. These modules form a complete simulation system of MUSIC based on UCA according to the logic sequence, as shown in Fig. 2.

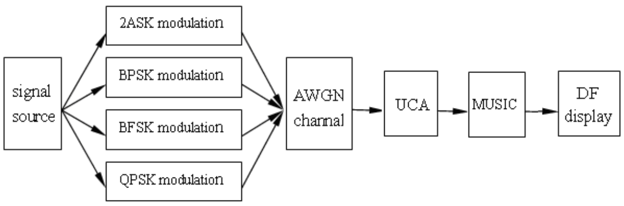

Figure 2. The simulation system of MUSIC based on UCA

III. Simulation Analyses of DF Efficiency of MUSIC Based on UCA

In following experiments, the radius of UCA is 1.0λ, the search step of MUSIC is 0.1° and the noise intensity is -12dB except for special declare.

A. DF Efficiency of Independent Signals

The simulation system interface of MUSIC for independent signals DF is shown in Fig.2. In the simulation system, it is possible to find directions of 1, 2, 3 or 4 signals by setting the intensity of some signals to be 0. In four experiments, the signals are independent for their modulation modes are different. The signal parameters and DF results of four experiments are shown in Table I.

The experiment results indicate that MUSIC based on UCA can find the directions of 1, 2, 3 or 4 independent signals accurately, so MUSIC based on UCA has the function of finding the directions of independent signals.

Experiment 1 shows that MUSIC based on UCA can find the direction of one signal.

TABLE I. THE SIGNAL PARAMETERS AND SIMULATION RESULTS OF INDEPENDENT SIGNALS

Number	Signal parameters				DF results		DF errors
	Signal	Modulation mode	Amplitude	Angle(°)	Angle(°)	Peak value	
1	1	2ASK	5.0	-50.0	-50.000000	818.375305	0.0
2	1	2ASK	5.0	-50.0	-49.899998	573.329773	0.100002
	2	BPSK	5.0	48.0	48.000004	1058.373657	0.000004
3	1	2ASK	5.0	-50.0	-50.000000	270.664612	0.0
	2	BPSK	5.0	48.0	48.000004	285.595612	0.000004
	3	QPSK	4.0	170.0	169.900009	320.817078	0.099991
4	1	2ASK	5.0	-50.0	-50.000000	741.809570	0.0
	2	BPSK	5.0	48.0	48.000004	743.294617	0.000004
	3	QPSK	4.0	170.0	170.000000	395.218781	0.0
	4	BFSK	4.0	240.0	240.000000	196.823456	0.0

B. *Angle Distinguishing Ability*

Two simulation experiments are to investigate the angle distinguishing ability of two independent signals. The search step of MUSIC is 1.0°. The signal parameters and DF results of two experiments are shown in Table II.

Table II shows that the angle distinguishing ability of two independent signals of MUSIC based on UCA reaches to 2° at least and 1° at most when the search step is 1.0°. Whereas the simulation experiments showed that the angle distinguishing ability of two independent signals of MUSIC based on ULA reached to 5° at least and 4° at most. So the angle distinguishing ability of MUSIC based on UCA is higher than MUSIC based on ULA.

It must be pointed out that the angle distinguishing ability is related to the search step. The angle distinguishing ability will be higher when the search step is less. So we can improve the angle distinguishing ability of MUSIC based on UCA by decreasing the search step when necessary.

TABLE II. THE ANGLE DISTINGUISHING ABILITY OF MUSIC BASED ON UCA

Number	Signal parameters				DF results		DF errors
	Signal	Modulation mode	Amplitude	Angle(°)	Angle(°)	Peak value	
1	1	2ASK	5.0	48.0	48.000000	619.398987	0.0
	2	BPSK	5.0	50.0	50.000000	696.122375	0.0
2	1	2ASK	5.0	49.0	-	-	∞
	2	BPSK	5.0	50.0	50.000000	787.617554	0.0

C. *Mirror Image Vagueness*

MUSIC based on ULA has the problem of mirror image vagueness. In order to solve the problem, MUSIC based on orthotropic ULAs should be used[2]. For two antenna arrays, MUSIC searches the angle in (-90°, 90°) according to defined step respectively, and determines the directions of signals by seeking the same angles of two sets of angles found. The spectrum curve of MUSIC based on ULA in (-90°, 90°) is symmetrical with the curve in (90°, 270°), therefore, it is not necessary to search in (-90°, 270°), it is enough to search only in (-90°, 90°). By this way, the search range of MUSIC based on single ULA is reduced by half. During the simulation experiment, MUSIC based on two ULAs search the angle at the same time, then the time of simulation experiment can be reduced one time by parallel processing technology.

The simulation results show that MUSIC based on UCA does not have the problem of mirror image vagueness. During the simulation experiment, the search range of MUSIC based on UCA is not (-90°, 90°), but (-90°, 270°). Therefore, the search range is double to MUSIC based on ULA, so the search time is doubled. Under the same simulation environment and with the same electromagnetic parameters, the running time of MUSIC based on UCA is nearly double to MUSIC based on orthogonal ULAs.

D. *Aperture Vagueness*

MUSIC based on ULA has the problem of aperture vagueness. Aperture vagueness occurs when the distance of two adjacent antennas is large than 0.5λ. Here, we study whether MUSIC based on UCA has the problem of aperture vagueness by simulation experiment. The method is to observe the spectrum curve changing with the change of radius of UCA. The electromagnetic environments of six experiments are same. One signal is 2ASK modulation, direction angle is -38.0°, another signal is BPSK modulation, direction angle is 57.0°. The amplitudes of two signals are all 5.0. The experiment results are shown in Table III.

Table III shows that the accuracy of DF improves gradually with the radius of UCA increasing, and there is no vagueness angle. Therefore, MUSIC based on UCA does not have the problem of aperture vagueness.

TABLE III. THE EXPERIMENT RESULTS OF MUSIC BASED ON UCA WITH DIFFERENT RADIUSES

Number	Radius	Angles of DF(°)		Errors of DF(°)		Average error(°)
		-38.0	57.0	-38.0	57.0	
1	0.1λ	-38.000000	57.100002	0.0	0.100002	0.050001
2	0.5λ	-37.899998	57.000004	0.100002	0.000004	0.050003
3	0.8λ	-38.000000	56.900002	0.0	0.099998	0.049999
4	1.0λ	-38.000000	57.000004	0.0	0.000004	0.000002
5	1.5λ	-38.000000	57.000004	0.0	0.000004	0.000002
6	2.0λ	-38.000000	57.000004	0.0	0.000004	0.000002

E. Influence of SNR on DF Accuracy

Above simulation experiments are carried out at -12dB noise intensity. Next, we do the simulation experiment of MUSIC based on UCA at different SNR. The electro-magnetic environments of six experiments are same. One signal is 2ASK modulation, direction angle is -38.0°; another signal is BPSK modulation, direction angle is 57.0°. The experiment results are shown in Table IV. The average errors of MUSIC based on ULA are listed at the last column to compare MUSIC based on UCA with MUSIC based on ULA.

It is easy to find from Table IV that, the DF accuracy of MUSIC based on UCA decreases with the decrease of SNR. When SNR is lower than -3dB, the DF error is more than 1° for some signals. Compare the last two columns of TAB. IV and it can be found that, at the same SNR, there is not much difference between the average errors of two kinds of MUSIC, their DF accuracies are similar.

TABLE IV. THE EXPERIMENT RESULTS OF MUSIC BASED ON UCA AT DIFFERENT SNR

Number	SNR(dB)	Angles of DF(°)		Errors of DF(°)		Average error(°)	Average error(°) of MUSIC based on ULA
		-38.0	57.0	-38.0	57.0		
1	9	-37.899998	57.200001	0.100002	0.200001	0.150002	0.185001
2	6	-37.799999	57.300003	0.200001	0.300003	0.250002	0.210001
3	3	-37.799999	57.600002	0.200001	0.600002	0.400002	0.255003
4	0	-37.599998	58.000004	0.400002	1.000004	0.700003	0.295002
5	-3	-37.399998	58.800003	0.600002	1.800003	1.200003	0.725001
6	-6	-37.099998	60.400002	0.900002	3.400002	2.150002	2.704998

F. Influence of Signal Intensity on Peak Value of Spectrum Curve

Signal intensity influences the peak height of spectrum curve. When signal enhances, the peak height increases; otherwise, the peak height decreases. In the simulation experiments of MUSIC based on ULA, the signal amplitudes were set as 1.0. When SNR (dB) ≥ 0, the peak height corresponding to the signal was in 10^4 orders, the peak height of spectrum curve corresponding to the signal was rather high.

In above simulation experiments of MUSIC based on UCA, the signal amplitudes are enhanced to 5.0. The reason is that, for MUSIC based on UCA, the peak height of spectrum curve corresponding to the signal is usually low. In order to raise the peak height, it is necessary to enhance the signal amplitudes. Simulation results show that, under the same parameters, the peak value of MUSIC based on UCA is smaller two orders than MUSIC based on ULA.

Why is the peak value of MUSIC based on UCA smaller than MUSIC based on ULA? It can be explained by the array structure. For ULA, let the induction potential at the first antenna is $s(t)$, then the induction potential at the *i*th antenna is $s(t)e^{j\frac{2\pi(i-1)d}{\lambda}\sin\theta}$, namely the induced potential at each antenna is the induced potential at the former antenna multiplied by the factor $e^{j\frac{2\pi d}{\lambda}\sin\theta}$. This regularity strengthens the signal in the array. For UCA, the induction potential at each antenna does not own such regularity. The irregular variation of phase makes the signal of each antenna not be strengthened, maybe be offset each other. The total effect is that the signal reduces. The results reflecting to spectrum curve is that the peak height of spectrum curve is relatively low.

The peak value of MUSIC based on UCA is small. This fact means that the ability of MUSIC based on UCA to find the direction of weak signal is poor, because the peak value corresponding to the weak signal is very small, it is filtered out easily.

G. DF Ability of Coherent Signals

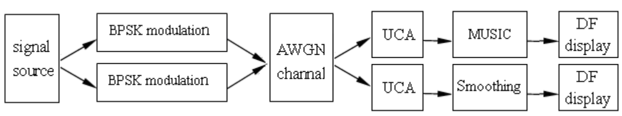

Figure 3. The simulation system of MUSIC and Smoothing algorithm based on UCL

The simulation system of MUSIC and Smoothing algorithm based on UCA is shown as in Fig.3.

The modulation mode of two signals is BPSK, and the amplitude of two signals is 5.0. The direction of one signal is 45° and the direction of another signal is 70°. Since the frequency and modulation mode of two signals are same, two signals are coherent signals.

Run simulation program, the simulation result of MUSIC is as shown in Fig. 4 and the simulation result of Smoothing algorithm is as shown in Fig. 5. The experiment results indicate that MUSIC and Smoothing algorithm based on UCA aren't able to find the directions of coherent signals.

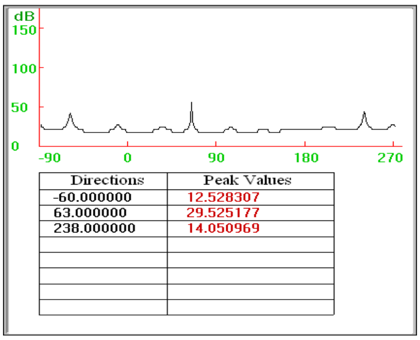

Figure 4. The simulation result of MUSIC based on UCA

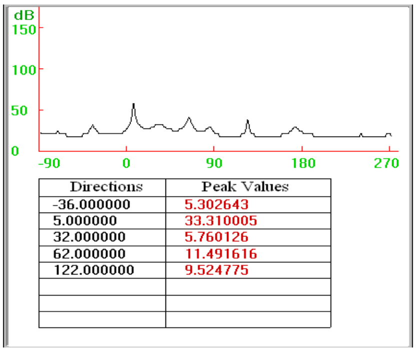

Figure 5. The simulation result of Smoothing algorithm based on UCA

Ref. [4] pointed out that we could find the directions of coherent signals based on UCA and the method was as following: carry on Smoothing to the mode space of UCA and gain the revised matrix; apply MUSIC to the revised matrix and estimate the directions of coherent signals. Ref. [5] put forward another method to find the directions of coherent signals. For UCA, when the antenna number is large enough, do discrete Fourier transform to the mode space and turn it into the form of ULA, then apply Smoothing algorithm based on ULA to distinguish coherent signals. However, these methods increased DF complications, therefore, UCA is worse than ULA for distinguishing coherent signals.

IV. Conclusions

The DF simulation system of MUSIC based on UCA is constructed. The efficiency of MUSIC based on UCA and MUSIC based on ULA are summarized as follows according to the simulation experiment results.

Two kinds of MUSIC are in common: they all have the DF function of single signal; they all have the DF function of multiple independent signals; the DF accuracy is related to many factors such as SNR, the search step length, the distance of two adjacent antennas, the number of antennas; the DF accuracy of two kinds of MUSIC is close in the same electromagnetic field; they all can not find the directions of coherent signals.

The advantage of MUSIC based on UCA: the array size is smaller; it is easy to increase antenna to improve the DF accuracy; the angle distinguishing ability of signals is higher than MUSIC based on ULA; there is no image vagueness; there is no aperture vagueness.

The weakness of MUSIC based on UCA: ULA can eliminate the coherence of coherent signals by spatial smoothing technique, but UCA can not directly eliminate the coherence of coherent signals, even can not eliminate the coherence of coherent signals by spatial smoothing technique. This is the most serious defect of UCA. The ability of MUSIC based on UCA to find the direction of weak signal is poor compared with MUSIC based on ULA.

In summary, we recommend MUSIC based on UCA in airborne DF system for the following reasons. At first, it is easier to arrange the circular array on an aircraft or a satellite. Secondly, the signals with same frequency are independent for aerial reflectors around the antenna are fewer, this avoid the defect of MUSIC based on UCA. Finally, the high SNR of airborne antennas can heighten the spectrum peak, and increase probability of signal capture.

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