Application of Windowed-sinc Band-pass Filter to Accurate and Fast Calculation of Impedance for Digital Distance Relaying

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***Abstract*— Extraction of fundamental components of both voltage and current is essential and is a challenge in digital distance protection. This paper introduces an effective filtering technique to fulfil this purpose. The technique proposed here is to use a windowed-sinc band-pass filter, which presents an outstanding immunity to both DC and harmonic components' influence when used to extract fundamental components of voltage and current. Then, the computed impedance is an accurate indication of the fault point along a transmission line and experiences the least amount of fluctuation. Compared to existing algorithms, it has a faster determination of the fault impedance of the line under protection and it takes slightly more than half a cycle of 50Hz system to reach the accurate impedance value. An experiment was also conducted to verify the validity of the designed filter to extract 50Hz fundamental component.**

***Index Terms*—digital relaying, impedance relay, windowed-sinc bandpass filter**

I. INTRODUCTION

In digital relaying, one fundamental issue is to determine the magnitudes and angles of voltage, current and impedance accurately and quickly, especially for high voltage transmission line protection. The whole delay in such protection includes detecting time in the relay and interrupting time of breakers. There is no way to remove the delay in the breaker. Yet one can try to shorten the detection time of the relay. By doing so, the overall clearing time can be reduced.

The factors that should be considered when developing an effective digital relaying algorithm are 1) DC offset; 2) harmonics; 3) Response time and stability of the designed algorithm etc [1]-[4]. There has been some research on extracting magnitudes and angles of fundamental components of voltage and current by considering one or two of these three factors. How to consider all the three factors and search for a faster algorithm is still left being improved. In [1]-[5], the authors proposed different effective methods to detect the fundamental component. Nevertheless the methods developed still take more than one cycle to work out the fundamental component.

In this paper, we introduce a narrow windowed-sinc band-pass filter to solve the problem of detecting speed and accuracy. It is found that the designed filter works very effectively to achieve the following: 1) effective DC component removal; 2) Faster and more stable response; 3) Immunity to harmonics. Such an approach is easier to implement and suitable for power system digital distance protection. The algorithm has been examined in the Matlab platform. It can be implemented in other platforms as well.

The paper is organized as follows: Section II introduces the basic methods for detecting magnitudes and angles of voltage, current and impedance; Section III discuss the design of the proposed narrow windowed-sinc band-pass filter and its experimental examination of the validity of the designed filter. Section IV introduces the overall power system under study and the implementation of the designed filter to extract the fundamental components of voltage and current; Section V concludes this paper.

II. REVIEW ON COMPUTATION OF PHASORS OF VOLTAGE, CURRENT AND IMPEDANCE

There are several basic methods [5]-[9] used in digital relays for computing magnitude and angle of voltage, current and impedance, one of which is first-order and second-order derivative method as reviewed below.

The magnitude of the impedance by this method is

$$|Z| = \left\{ \left[v'(t)^2 + \left(v''(t)/\omega_0 \right)^2 \right] \Big/ \left[i'(t)^2 + \left(i''(t)/\omega_0 \right)^2 \right] \right\}^{1/2} \quad (1)$$

where $v'(t)$ and $i'(t)$ are the first-order derivatives of voltage and current; $v''(t)$ and $i''(t)$ are the second-order derivatives of voltage and current.

The argument of the measured impedance is

$$\theta_Z = \tan^{-1}\left[i''/(\omega_0 i') \right] - \tan^{-1}\left[v''/(\omega_0 v') \right] \quad (2)$$

For this method, the following expressions are used to compute first- and second-order derivatives:

$$f_k' \approx (f_{k+1} - f_{k-1})/(2\Delta t) \quad (3)$$

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$$f_k'' \approx (f_{k+1} - 2f_k + f_{k-1}) / (\Delta t)^2 \quad (4)$$

where f could be i or v, Δt is the sampling interval, and k+1, k and k-1 are subscripts referring to a consecutive samples.

The main advantage of this method is that DC component has no effect on the signal extraction, since it is cancelled out from all difference expressions. The disadvantage associated with this algorithm is that higher-frequency components, due in particular to fault-induced travelling waves, can produce significant errors in the first-order and second-order difference equations, thereby causing large errors in successive impedance estimates. This will be illustrated in Section IV.

III. Proposed Windowed-Sinc Band-Pass Filter and Its Experimental Examination

A. Design of Windowed-Sinc Band-Pass Filter

When a fault occurs in a power system, unavoidably the fault current contains both AC and DC components. Furthermore the high-frequency traveling wave also exists in the fault current. The DC component poses a difficulty for accurately extracting fundamental AC components. To solve the problem brought by both DC and high-order harmonic components, one can use filters, including different types of finite-impulse response (FIR) and infinite-impulse response (IIR) filters. In the following sections, a very effective windowed-sinc band-pass filter will be introduced to remove both DC and harmonic components.

The kernel of the proposed windowed-sinc filter used in our filter design is given below [8], [10], [11]

$$h(i) = K \cdot \sin\left[2\pi f_c (i - M/2)\right] / (i - M/2) \cdot \left[0.42 - 0.5\cos(2\pi i / M) + 0.08\cos(4\pi i / M)\right] \quad (5)$$

where f_c is the cutoff frequency, expressed as a fraction of the sampling rate, a value between 0 and 0.5. The sample number of the kernel is determined by M, which must be an even integer. The sample number i is an integer that runs from 0 to M, resulting in M+1 total points in the filter kernel. The constant K is chosen to provide unity gain at zero frequency. To avoid a divide-by-zero error, for $i=M/2$, use $h(i)=2\pi f_c K$.

The procedure for designing a windowed-sinc band-pass filter with a center frequency of 50Hz system is as follows:

Step 1: Design a windowed-sinc low-pass filter with a cut-off frequency of 30Hz; Normalize its kernel H1 for unity gain at DC.

Step 2: Design a windowed-sinc low-pass filter with a cut-off frequency of 70Hz; Normalize its kernel H2 for unity gain at DC.

Step 3: Change the low-pass filter in step 2 into a high-pass filter kernel H3 using spectral inversion.

Step 4: Add the normalized low-pass filter kernel H1 in Step 1 to the normalized high-pass filter kernel H3 in Step 3 to form a band-reject filter kernel H4.

Step 5: Change the band-reject filter kernel H4 in Step 4 into a band-pass filter kernel H by using spectral inversion.

In Section IV, we will show that by using such a filter, the extracted magnitude and angle of impedance for a distance relay are very accurate. It has the features of high immunity to the influence of both DC and harmonic components.

B. Experimental Examination of the Effectiveness of the Designed Band-Pass Filter

To examine the effectiveness of the designed windowed-sinc band-pass filter, a dSPACE 1104 based hardware was setup, which contains other equipment including signal generator and oscilloscope. The signal generator is used to produce sinusoidal and triangular signals of 50Hz. ADC1 of dSPACE 1104 was used to acquire the signal with a sample rate of 20kHz. Then the acquired signal was processed and filtered by the C program running at the platform of ControlDesk Developer. The filtered signal was output to oscilloscope via DAC1 of dSPACE [10].

Fig. 1 shows the input and filtered signals of 50Hz sinusoidal waveform observed from oscilloscope Lecroy 6050. The upper half contains both the input and filtered signals of 50Hz. We observed that the filtered signal perfectly overlaps the input sinusoidal signal. For visibility purpose, the scales used for upper half two waveforms in Fig. 1 are different. The lower half in Fig. 1 shows the Fourier transform results of filtered signal, from which one can see that it only contains 50Hz signal and other frequency components are almost zero.

Fig. 2 shows the input and filtered signals of 50Hz triangular waveform observed from oscilloscope. The upper half contains both the input and filtered signals of 50Hz, which cross zero and peak simultaneously. For visibility purpose, the scales used for upper half two waveforms in Fig. 2 are different. The lower half shows the Fourier transform results of filtered signal, from which one can see that it only contains 50Hz signal and other frequency components are almost zero.

Both cases above show the capability of 50Hz signal extraction by the designed windowed-sinc band-pass filter.

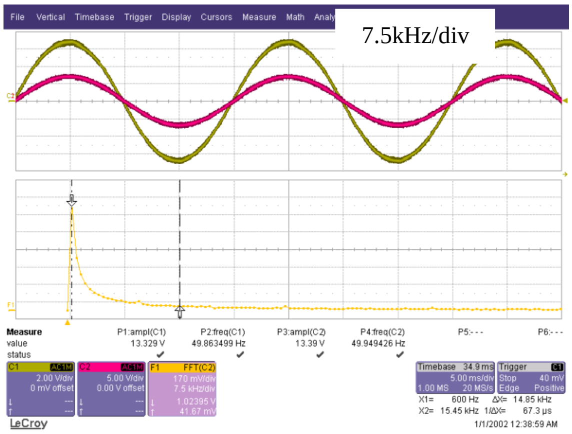

Figure 1. Input signal of 50Hz sinusoidal wave, its filtered signal and FFT result of filtered signal observed from oscilloscope Lecroy 6050

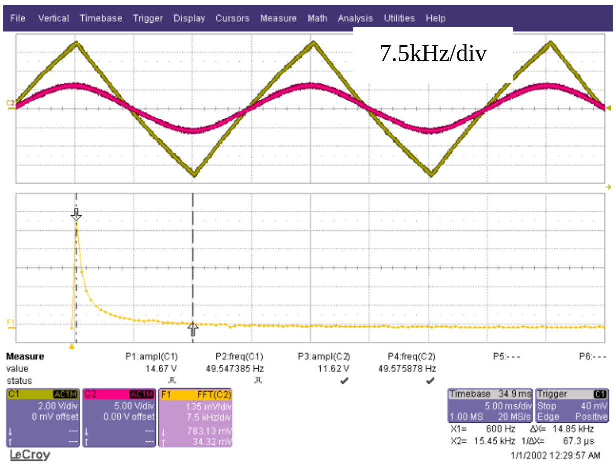

Figure 2. Input signal of 50Hz triangular wave, its filtered signal and FFT result of filtered signal observed from oscilloscope Lecroy 6050

IV. Implementation of Digital Filtering for Extracting the Phasors of Voltage, Current and Impedance

A three-phase power system under study is shown in Fig. 3, where three-phase source voltages are

$$v_a(t) = 2694\sin(2\pi 50t) \ \ \text{(V)}$$
$$v_b(t) = 2694\sin(2\pi 50t - 2\pi/3) \ \ \text{(V)}$$
$$v_c(t) \quad 2694\sin(2\pi 50t \quad 2\pi/3) \ \ \text{(V)}$$

Source impedance is R_s=0.1Ω, L_s=20mH; line impedance is R_{Line}=1Ω, L_{Line}=20mH; load: R_L=80Ω. Three-phase distance protection is installed at bus 1 to protect the system. Assume that voltage transformers and current transformers installed at bus 1 have a voltage transformer ratio (VTR) of 50 and a current transformer ratio (CTR) of 50.

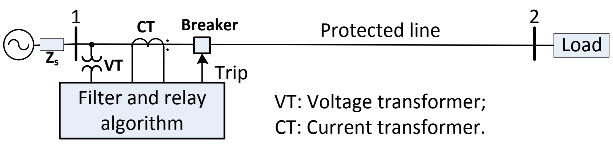

Figure 3. Power system under study

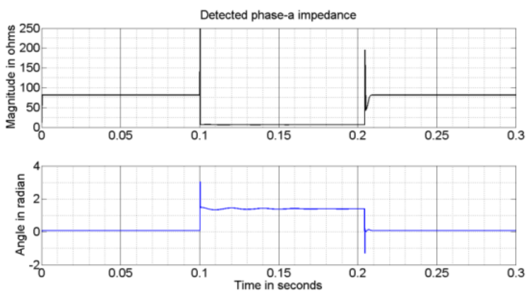

Figure 4. Magnitude and angle of phase-a impedance sensed by the digital relay

To examine the effectiveness of the method in Section II.A, the model for the power system in Fig. 3 was developed in Simulink. A bolted three-phase fault at bus 2 was assumed, as indicated in Fig. 3, which occurs at 0.1s and disappears at 0.205s. Modeling results are shown in Fig. 4, from which one can see that when there are no harmonics in the system, the algorithm based on the method in Section II.A can effectively work out the magnitude and angle of impedance. After the fault occurs, the computed magnitude and angle of impedance quickly dwells at $(1.0 + j6.283)\Omega$, which is the line impedance between bus1 and bus 2. So its capability of removing the influence of the DC component by this method is very good.

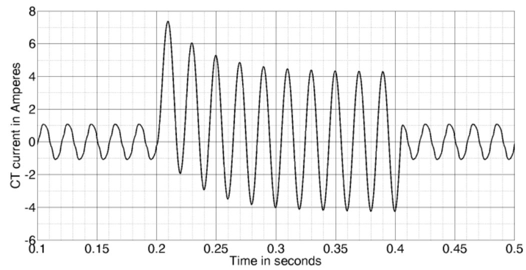

Figure 5. Phase-a current from CT

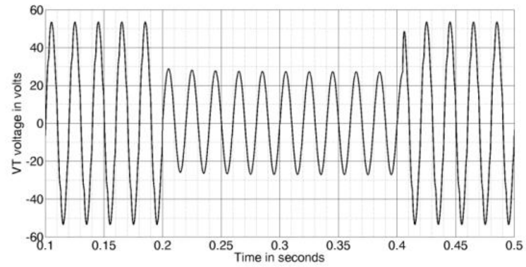

Figure 6. Phase-a voltage from VT

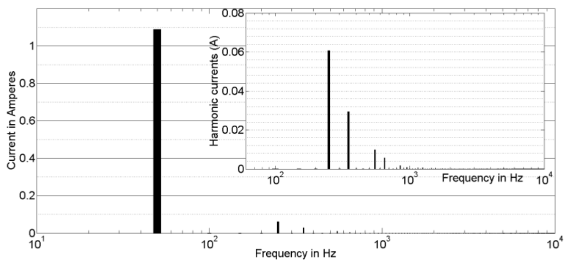

Figure 7. Fundamental and harmonics of current

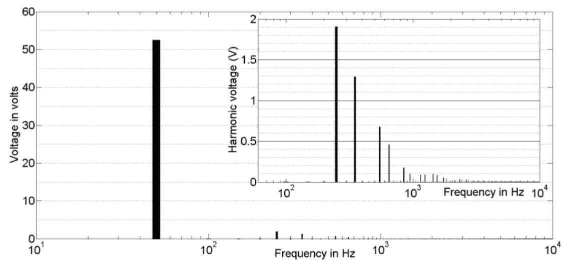

Figure 8. Fundamental and harmonics of voltage

Nevertheless in a real power system, there always exists harmonic components. Also the traveling wave, after the fault occurs, deteriorates the situation of high order harmonic components. These components may pose severe influences on the computation of voltage, current and impedance phasors. To study such an influence, another DC load which is powered through a three-phase naturally-commutating rectifier is connected in parallel with the load at bus 2 in Fig. 3. The same algorithm in Section II.A is used to study the influence of harmonics. The bolted three-phase fault is assumed to occur at 0.2s and disappear at 0.4s. From Fig. 5 and 6 one can see that voltage and current waveforms contain harmonic components and the distortion of the waveform is mild. The fundamental and harmonic components of voltage and current are shown in Figs.7-8. The total harmonic distortion (THD) of current and voltage under steady-state is the same and equal to 4.69%.

Just due to these non-severe harmonics, the computed impedance is totally distorted as shown in Fig. 9, from which one can see that both the magnitude and angle are distorted as compared with those shown in Fig.4.

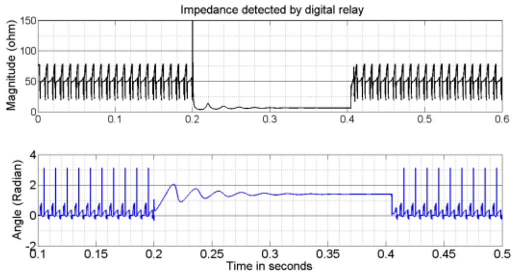

Figure 9. Phase-a impedance detected by the digital relay with no filter

In [1], [5], the developed algorithms were quite effective. Yet the detection time could be longer than one cycle. In some cases it could be longer than one and a half cycles. To have more effective and faster fault detection, a new algorithm must be introduced.

In the following study, we introduce the narrow windowed-sinc band-pass filter with a band-width of 30Hz through 70Hz. The sampling frequency adopted here is 50kHz. The corresponding kernel of the designed windowed-sinc band-pass filter is shown in Fig. 10, whose magnitude and step responses are shown Fig. 11. From Fig. 11, one can see that the designed filter has a super property to reject DC component. Its rejection of high-order harmonic components is also very good.

The fault cases under study is the same as the previous one: bolted three-phase fault occurs at bus 2 at 0.2s and disappears at 0.4s. Results by using the designed filter are shown in Figs. 12-14. Figs. 12 and 13 show that the fundamental component can be extracted accurately and very fast, as one can see that after the fault lasts some time, the DC component decays to zero, and the extracted AC component just overlaps with the fault current. Besides extracting the fundamental AC component, the DC component after the fault can also be extracted swiftly and accurately. Fig. 14 is a zoomed-in figure from Fig.11, from which one can see that immediately after the fault occurs at 0.2s, the extracted DC component contains part of the AC component and the extracted AC component is not very accurate. But this lasts for a very short time. By 0.215s, the extracted fundamental AC component is almost steady and so the DC component after this moment follows the correct decaying trace. More detailed DC component comparison is shown in Fig. 15, from which one can see that at around 0.215s, the extracted DC component is almost the same as the real DC component. The real DC component is obtained by exporting CT current data from Simulink into a workspace where the AC component is found from its steady state value at around 0.39s. Then the total current is subtracted by the AC component to reach the real DC component.

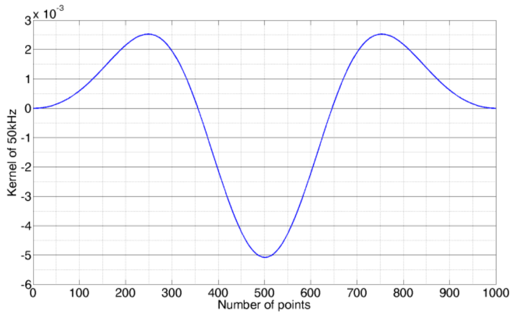

Figure 10. Kernel of designed windowed-sinc band-pass filter with a sampling rate of 50kHz

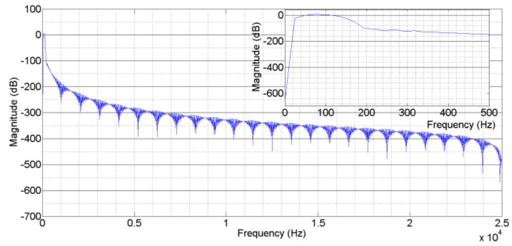

Figure 11. Properties of the filter: Magnitude against frequency

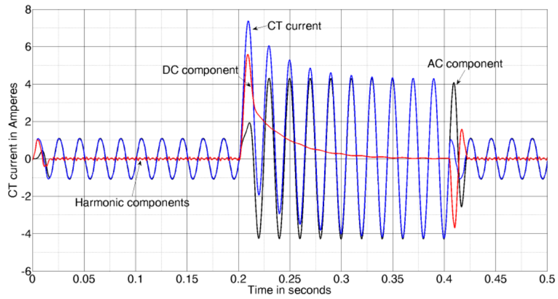

Figure 12. Phase-a current and its filtered fundamental component by the narrow bandpass filter

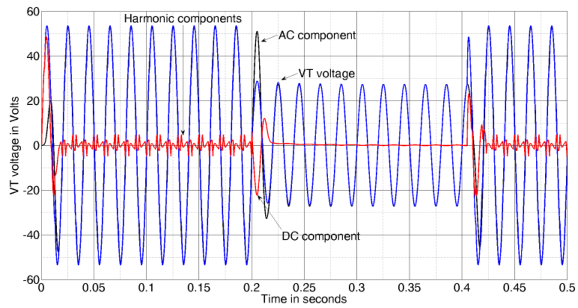

Figure 13. Phase-a voltage and its filtered fundamental component by the narrow bandpass filter

Fig. 16 shows the computed magnitude and angle of impedance. One can see that after the fault occurs, the impedance quickly dwells at an impedance of

$6.3659\angle 80.75^0 = (1.02 + j6.283)\Omega$, which is almost the same as the impedance of 1ohm resistor in series with a 20mH inductor or the total line impedance. Both angle and impedance are accurate. The detection time by the proposed filter is very short. By 0.212s, both the magnitude and angle of the impedance dwell correctly at their respective values. This shows that detection time is around 0.012s, slightly more than half cycle of 50Hz system.

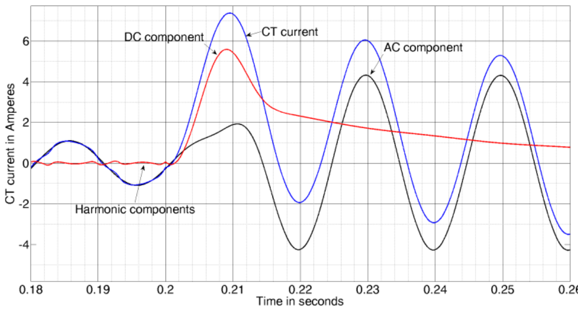

Figure 14. Zoomed-in current waveforms

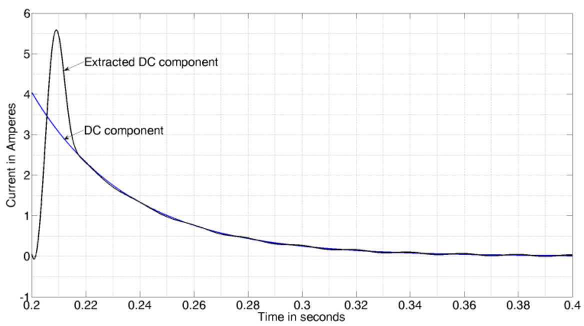

Figure 15. Extracted DC component and real DC component in the fault current

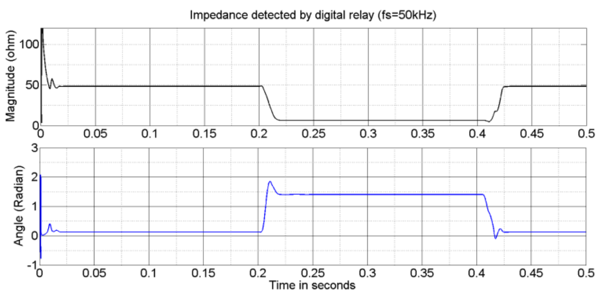

Figure 16. Computed impedance with a sampling rate of 50kHz

The total line impedance is (1+j6.283)Ω so we can set impedance of a conventional mho relay by amplitude comparison at $8\angle 80.75^0$ Ω. The impedance trajectory after the fault occurs and disappears is shown in Fig. 17. One can see that at t=0.212s the trajectory enters the mho relay circle. This can make the mho relay send out a trip signal to circuit breakers. The fault is set to self-clear at 0.4s. One can see that at t=0.41s the trajectory leaves the mho relay circle. From this, one can see that the algorithm can quickly determine circuit conditions.

The above results are based on 50kHz sampling rate. The same results can be obtained if one uses 5kHz sampling rate. The filter kernel of 5kHz is shown in Fig. 18 and the obtained impedance is shown in Fig. 19. One can see that the results in Figs. 16 and 19 are almost the same.

The overall system is tested in a Matlab platform [11] and it can be transferred to a digital relay readily.

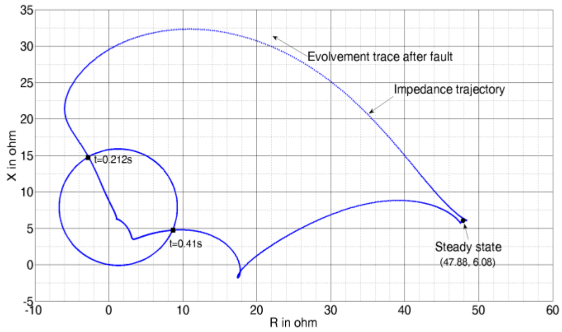

Figure 17. Phase-a current and its filtered fundamental component by windowed-sinc low pass filter

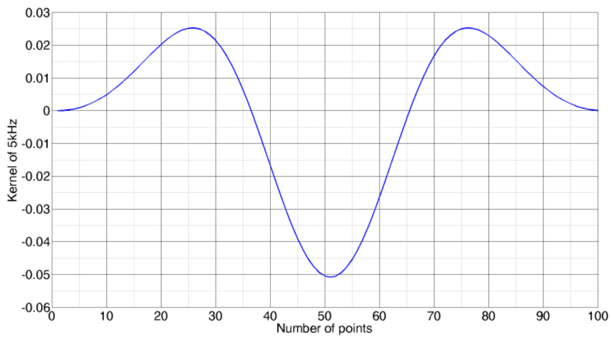

Figure 18. Kernel of designed windowed-sinc band-pass filter with a sampling rate of 5kHz

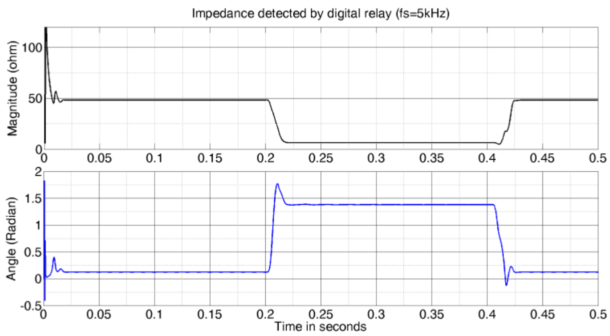

Figure 19. Computed impedance with a sampling rate of 5kHz

V. Conclusion

This paper presents an effective windowed-sinc narrow band-pass filter to extract the fundamental components of voltage and current waveforms with high immunity to DC and harmonic components. It is found that such a filter can effectively and simultaneously remove the influence of DC and harmonic components and quickly and accurately extract both magnitude and angle of voltage and current. Correspondingly, the computed impedance experiences the least amount of fluctuation and is an accurate indication of fault points along a transmission line. The detection time is around 0.012s, slightly longer than half a cycle of a 50Hz system. This helps distance relay make a correct and fast decision upon a fault occurrence.

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