A Framework for Quick Rejection of Dissimilar Binary Images

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***Abstract*—Given two binary images, how can we determine if the images are different? The most common technique used in the image processing arena is to calculate the correlation between the two images or simply subtract the two images. Both of these methods require some type of operation to be applied to the whole image. This implies that as the image size increases, more time will be required. In this paper, we show that –regardless of image size– only a limited number of points need to be checked to arrive at a required confidence level that the images are different. In fact, for completely different binary images only 8 points need to be checked to arrive at a 99% confidence level, 11 points need to be checked to arrive at a 99.9% confidence level and only 15 points need to be checked to arrive at a 99.99% confidence level. As a result, this method is magnitudes faster than traditional methods. Tests with real images are presented to show the validity of our technique.**

***Index Terms*—image matching, image mapping, image registration, template matching and image retrieval**

I. Introduction

A misconception about binary images is that they are simple; in reality they might be simpler to analyze than multi-bit images, but they are not simple. Their complexity increases exponentially with increasing size. The number of possible image variations for a binary image of size $M \times N$ is,

$$\aleph_1(M,N) = 2^{M \cdot N} \tag{1}$$

For example, a small 2×2 binary image has $\aleph_1 = 16$ possible image variations, as shown in Fig. 1. A 3×3 binary image has $\aleph_1 = 512$ possible image variations as shown in Fig. 2. A 4×4 image has $\aleph_1 = 65{,}536$ different image variations! In image analysis, where most image sizes are usually much larger than 128×128 pixels, a 128×128 binary image has more than $\aleph_1 = 10^{4931}$ different image variations!

When two binary images are to be compared, then the number of possible image pair variations is,

$$\aleph_1^2(M,N) = 2^{2 \cdot M \cdot N} \tag{2}$$

The numbers produced by this equation are extremely large, even for small image sizes; for 4×4 binary images, the number of possible image pair variations is $\aleph_1^2 = 4.295 \times 10^9$ different image pairs. While the complexity of images increases with image size, fortunately the number of possible image mappings does not. In fact, for binary images there are only 15 different image mappings.

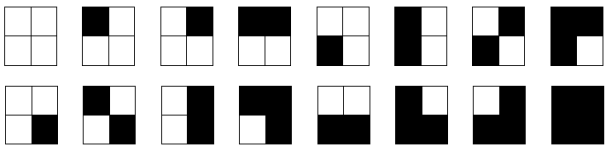

Figure 1.The 16 image variations for a 2×2 binary image.

Figure 2.The 512 image variations of a 3×3 binary image.

In this paper, we show that by exploiting image mappings, we can simplify the matching process drastically. We will show that only a few number of points need to be checked to arrive at a required confidence that the images are different. For example, only 8 points need to be checked to arrive at a 90% confidence, 11 points need to be checked to arrive at a 99% confidence and only 15 points need to be checked to arrive at a 99.9% confidence that the images are different. As a result, this method is magnitudes faster than traditional methods which are image size dependent. Tests with real images are conducted to show the validity of our technique.

This paper is organized as follows: section II points out related literature, section III presents the main theme of this paper and explains image mappings and how they can simplify matching. Section IV discuses the results of applying our method to synthetic and real binary images. This paper concludes in section V and states where our future work is headed.

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II. Related Literature

A vast amount of research has been done on image matching and can be found in the literature. Binary image matching is usually accomplished by calculating the correlation between the images [1] or simply by subtracting the two images [2]. We presented another technique that accomplishes matching by minimization the image intensity combinations between the two images with excellent results [3]. Lewis [4] showed that for template matching the unnormalized cross correlation can be efficiently normalized using pre-computed tables containing the integral of the image and the image search window. More recently, Sleit *et. al.* [5] presented a sub image matching algorithm for binary images that is an enhancement to the Chain Code based exact match algorithm. Tang and Tao [6] presented an approach to accelerate multi-scale template matching by representing the template as a linear combination of a small number of Haar-like binary features that can easily adapt to template scale changes with negligible extra computation cost.

III. Image Mapping

Initially let us define some terms that will be used in this paper. Let u and v be two independent random variables that represent the image intensity values of the 1st image and 2nd image, respectively.

A. Pixel Mapping

Pixel mapping (P_n) between two n-bit images refers to how a pixel value in the first image maps to the corresponding pixel value in the second image, i.e. how the intensity values of the two images map to each other in a specific direction,

$$P_n = \{u \rightarrow v,\ \forall\ u = 0 \ldots 2^n \text{ and } v = 0 \ldots 2^n\} \tag{3}$$

The '→' symbol is used to denote pixel mapping. Hence, $u \rightarrow v$ implies pixel value u in the first image maps to pixel value v in the second image. For binary images, pixel values are either 0 or 1 and hence there are four possible pixel mappings between any two binary images,

$$P_1 = \{0 \rightarrow 0, 0 \rightarrow 1, 1 \rightarrow 0, 1 \rightarrow 1\} \tag{4}$$

Note that the mapping order is important, i.e. 0→1 is not the same as 1→0. For ease of illustration we will use the labels (A-D) shown in Table I as shorthand for these four mappings. Hence,

$$P_1 = \{A, B, C, D\} \tag{5}$$

The number of possible different pixel mappings for two n-bit images (NP_n) is given by,

$$NP_n = 2^{2n} \tag{6}$$

B. Image Mapping Variations and Mapping Tuples

Image Mapping between two images refers to which pixel mappings are present when mapping two images. Let V_n refer to the possible image mapping variations (or simply referred to as mapping variations) between two n-bit images. Since the number of pixel mappings, NP, is finite, the number of possible image mapping variations between any two images (NV_n) –regardless of image size– is also finite. For binary images (n = 1), the following NV_1=15 image mapping variations are possible,

$$\begin{aligned} V_1 = \{&(A), (B), (C), (D), (A, B), (C, D), (A, C), \\ &(B, D), (A, D), (B, C), (A, B, C), (A, C, D), \\ &(A, B, D), (B, C, D), (A, B, C, D)\} \end{aligned} \tag{7}$$

TABLE I. Pixel Mapping Labels for Binary Images

Label	Pixel Mapping
A	$0 \rightarrow 0$
B	$0 \rightarrow 1$
C	$1 \rightarrow 0$
D	$1 \rightarrow 1$

These 15 image mapping variations will be labeled μ_1 to μ_{15}.

$$V_1 = \{\mu_1, \mu_2, \ldots, \mu_{15}\} \tag{8}$$

Instances of V_1 are shown in Fig. 3. As an example, μ_{12} has mappings (A, C, D) consisting of the 3 pixel mapping types: 1) A:0→0, 2) B:1→0, and 3) C:1→1.

Mapping two binary images result in at least one type of pixel mapping present and up to a maximum of all NP_1 = 4 pixel mappings present. If only one pixel mapping is present, we call the resulting image mapping a 1-tuple mapping. If we have m pixel mappings present, we call the resulting image mapping an m-tuple mapping. Let $T_{n,m}$ denote the image mapping set consisting of m-tuples for n-bit images. Hence, for binary images we have,

$$\begin{aligned} T_{1,1} &= \{\mu_1, \mu_2, \mu_3, \mu_4\} \\ T_{1,2} &= \{\mu_5, \mu_6, \mu_7, \mu_8, \mu_9, \mu_{10}\} \\ T_{1,3} &= \{\mu_{11}, \mu_{12}, \mu_{13}, \mu_{14}\} \\ T_{1,4} &= \{\mu_{15}\} \end{aligned} \tag{9}$$

which are the 15 image mapping variations divided up into groups based on mapping tuple size.

C. From Image Mapping to Deduction

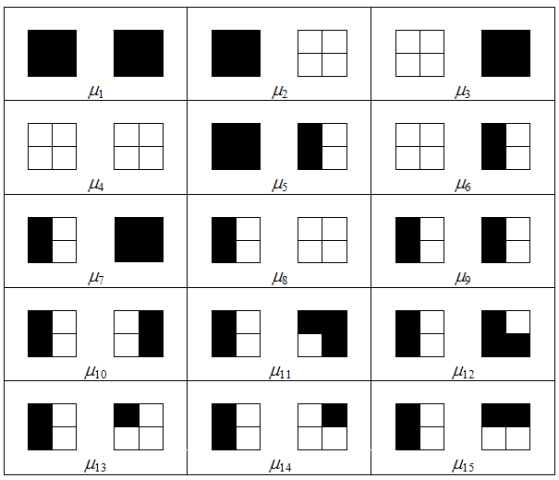

Figure 3. Instances of the 15 image mappings for binary images.

A lot of information can be extracted about the images being mapped if the mapping between them is known.

We use the term *blank* image for images that have no content (i.e. image entropy = 0) and hence consist of a single intensity value. When (at least) one of the images is *blank* we refer to the mapping as *trivial*. Table II lists the 15 mappings μ_1 to μ_{15} along with their interpretation:

TABLE II. IMAGE MAPPING AND CONTENT INFORMATION

μ_i	Pixel mappings	Tuple size	Image intensities 1st	Image intensities 2nd	Image contents
μ_1	*A*	1	1	1	same
μ_2	*B*	1	1	1	inverse
μ_3	*C*	1	1	1	inverse
μ_4	*D*	1	1	1	same
μ_5	*A,B*	2	1	2	different
μ_6	*A,C*	2	2	1	different
μ_7	*A,D*	2	2	2	same
μ_8	*B,C*	2	2	2	inverse
μ_9	*B,D*	2	2	1	different
μ_{10}	*C,D*	2	1	2	different
μ_{11}	*A,B,C*	3	2	2	different
μ_{12}	*A,B,D*	3	2	2	different
μ_{13}	*A,C,D*	3	2	2	different
μ_{14}	*B,C,D*	3	2	2	different
μ_{15}	*A,B,C,D*	4	2	2	different

a) **$T_{1,1}$: 1-tuple mappings (μ_1, μ_2, μ_3, μ_4)**: When the mapping between images result in 1-tuple mappings then we have the trivial case where both images are *blank*.

b) **$T_{1,2}$: 2-tuple mappings (μ_5, μ_6, μ_7, μ_8, μ_9, μ_{10}):** Here at least one of the images consists of two intensity values. Furthermore, we can divide them into three groups :

- **μ_5 and μ_{10}:** The images are different, but the 1st image is a blank image (trivial case).
- **μ_6 and μ_9:** The images are different, but the 2nd image is a blank image (trivial case).
- **μ_7:** The pixel mapping between the images is (*A*,*D*). The images are exactly the same and both images are non-blank images with two intensity values.
- **μ_8:** The pixel mapping between the images is (*B*,*C*). The images are exactly the inverse of each other and both images are non-blank images with two intensity values.

c) **$T_{1,3}$: 3-tuple mappings (μ_{11}, μ_{12}, μ_{13}, μ_{14}):** Here the images are different and both consist of two intensity values.

d) **$T_{1,4}$: 4-tuple mappings (μ_{15}):** Here we have different images that are completely different.

Two important notes:

- 8 of the 15 mappings (μ_1, μ_2, μ_3, μ_4, μ_5, μ_6, μ_9, μ_{10}) –i.e. more than half the mappings– represent the trivial case. This leaves 7 mappings that are non-trivial: μ_7, μ_8, μ_{11}, μ_{12}, μ_{13}, μ_{14} and μ_{15},. These 7 mappings are the mappings produced when matching real world non-blank images.
- 3-tuple and 4-tuple mapping can only occur if the images are different. Consequently, when investigating the dissimilarity between two images, once the mapping between the two images reaches a 3-tuple mapping, no further mapping is necessary as the images have been proven to be different.

D. Probability of Occurrence of a M-Tuple

Let *Pr*(*m*,*p*) represent the probability of occurrence of an *m*-tuple on the pth pixel mapped. When the first pixel is mapped from the 1st image to the 2nd image four possible outcomes are possible: *A*,*B*,*C* or *D*; Since they are all 1-tuple mapping we have: Pr(1,1) = 1 and Pr(2,1)= Pr(3,1) = Pr(4,1) = 0.

when *p* = 2, the sample space increases to 16 possibilities: *AA*, *AB*, *AC*, *AD*, *BA*, …, *DD*, which are all 2-tuple mappings. However, since duplicate mappings (e.g. *AA*) are in essence 1-tuple mappings, 4 of the outcomes reduce to 1-tuple mapping. This results in: Pr(1,2) = 4/16 = 0.25, Pr(2,2) =12/16=0.75 and Pr(3,2) = Pr(4,2) = 0.

A general formula for the occurrence of the probabilities of an *m*-tuple can be shown to be [7],

$$\Pr(m,p) = \frac{m!}{4^p}\binom{4}{m} S_m^p \qquad m = 1\ldots 4 \tag{10}$$

where,

$$\binom{n}{m} = \frac{n!}{m!(n-m)!} \tag{11}$$

is the combination function and *S* is the *Stirling numbers of the second kind* function defined by,

$$S_m^p = \frac{1}{m!}\sum_{j=0}^{m}(-1)^{m-j}\binom{m}{j} j^p \qquad 0 \le m \le p \tag{12}$$

The probability of occurrence for each of the four tuple sizes can be individually simplified from Eq. (10) to,

$$\Pr(1,p) = \frac{1}{4^{p-1}} \qquad p \ge 1 \tag{13}$$

$$\Pr(2,p) = \frac{3}{4^{p-1}}(2^{p-1}-1) \qquad p \ge 2 \tag{14}$$

$$\Pr(3,p) = \frac{3}{4^{p-1}}(3^{p-1}-2^p+1) \qquad p \ge 3 \tag{15}$$

$$\Pr(4,p) = \frac{1}{4^{p-1}}(4^{p-1}-3^p+3\cdot 2^{p-1}-1) \qquad p \ge 4 \tag{16}$$

Fig. 4 shows plots of these four equations. From the plots we observe:

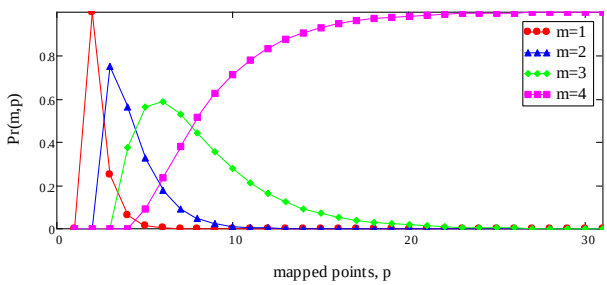

Figure 4. Plots of Pr(*m*, *p*) for *m* = 1…4.

- The m = 1 and m = 2 probability curves quickly diminish after their initial peaks and are practically non-existent for $p > 10$.
- The m = 3 probability curve reaches a maximum value of 0.586 at p = 6 and slowly starts to diminish beyond $p > 6$ and becomes negligible for $p > 22$.
- The m = 4 case represents maximum randomness between two images. The quick dominance of this probability curve over the other probability curves is evident as its value quickly approaches unity with increasing p (e.g. Pr(4,7) > 0.5, Pr(4,13) > 0.9 and Pr(4,29) > 0.999). This implies that given two completely random images there is a 50% chance that the mapping between the two images will have 4-tuple mapping by the 7th pixel being matched, a 90% chance by the 13th pixel being matched, and a 99.9% chance by the 29th pixel being matched, which agrees with our intuition that as we map more and more pixels between two different images, random pixel mapping will dominate.

E. Probability of Occurrence of a Mapping Variation

The probability of occurrence of a mapping variation (Pv) can be calculated from the mapping occurrence of the m-tuples,

- **1-tuple mapping variations ($\mu_1 - \mu_4$)**: Since these four variations all have equal likelihood of occurrence, they all have the same probability of occurrence,

$$\mathrm{Pv}(1, p) = \mathrm{Pv}(2, p) = \mathrm{Pv}(3, p) = \mathrm{Pv}(4, p) \quad (17)$$

which is equal to ¼ the probability of occurrence of a 1-tuple mapping,

$$\mathrm{Pv}(v, p) = \frac{1}{4^p} \quad p \geq 1,\ v = 1 \ldots 4 \quad (18)$$

- **2-tuple mapping variations ($\mu_5 - \mu_{10}$)**: Since these six variations all have equal likelihood of occurrence, they all have the same probability of occurrence,

$$\mathrm{Pv}(5, p) = \mathrm{Pv}(6, p) = \mathrm{Pv}(7, p) = \mathrm{Pv}(8, p) = \mathrm{Pv}(9, p) = \mathrm{Pv}(10, p) \quad (19)$$

which is equal to 1/6 the probability of occurrence of a 2-tuple mapping,

$$\mathrm{Pv}(v, p) = \frac{1}{2} \cdot \frac{1}{4^{p-1}} (2^{p-1} - 1) \quad p \geq 2,\ v = 5 \ldots 10 \quad (20)$$

- **3-tuple mapping variations ($\mu_{11} - \mu_{14}$)**: Since these four variations all have equal likelihood of occurrence, they all have the same probability of occurrence,

$$\mathrm{Pv}(11, p) = \mathrm{Pv}(12, p) = \mathrm{Pv}(13, p) = \mathrm{Pv}(14, p) \quad (21)$$

which is equal to ¼ the probability of occurrence of a 3-tuple mapping,

$$\mathrm{Pv}(v, p) = \frac{3}{4^p} (3^{p-1} - 2^p + 1)$$

$$p \geq 3,\ v = 11 \ldots 14 \quad (22)$$

- **4-tuple mapping variations (μ_{15})**: There is only a single variation of tuple size 4. The probability of occurrence of this mapping variation is equal to the probability of occurrence of a 4-tuple mapping,

$$\mathrm{Pv}(15, p) = \frac{1}{4^{p-1}} (4^{p-1} - 3^p + 3 \cdot 2^{p-1} - 1) \quad p \geq 4 \quad (23)$$

Note that the probability of the mapping variations sum up to the mapping tuple for each size:

$$\mathrm{Pr}(1, p) = \sum_{v=1}^{4} \mathrm{Pv}(v, p), \quad \mathrm{Pr}(2, p) = \sum_{v=5}^{10} \mathrm{Pv}(v, p)$$
$$\mathrm{Pr}(3, p) = \sum_{v=11}^{14} \mathrm{Pv}(v, p), \quad \mathrm{Pr}(4, p) = \mathrm{Pv}(15, p) \quad (24)$$

Naturally, all probabilities add up to unity,

$$\sum_{v=1}^{15} \mathrm{Pv}(v, p) = \sum_{m=1}^{4} \mathrm{Pr}(m, p) = 1 \quad (25)$$

F. Similar and Dissimilar Images

Let us define some terms that will be used to indicate the closeness between two images based on a pixel to pixel comparison. The closeness will be categorized as either *similar* or *dissimilar*:

- Similar (S): The two images are considered to be the same and are of two types:
  1) *Exact* (E): The two images have the same intensity values at each pixel.
  2) *Inverse* (I): The two images have the complement intensity values at each pixel.
- Dissimilar (D): The two images are different.

Based on the above definitions the pixel intensity values must be the same (or inverted) at all corresponding locations in the two images for them to be *similar*. If they are not 100% different, and differ at a single location then the two images are considered to be *dissimilar*, even though a large resemblance exists between the two images. This condition can be relaxed by introducing a quality factor (QF) term that can account for such cases (not discussed in this paper).

Determining if two images are *similar* or *dissimilar* is easily accomplished by examining the mapping size between the two images, as summarized in Table III,

a) if the mapping tuple size is 1 then the images are *similar*. Further investigation into the mapping variation can reveal which type:

- if the mapping variation is μ_1 or μ_4 then the images are *exact.*
- if the mapping variation is μ_2 or μ_3 then the images are *inverse.*

TABLE III. IMAGE CLOSENESS BASED ON TUPLE MAPPINGS

Tuple mapping size	Image closeness	Further investigation
1	*Similar*	*exact*: μ_1, μ_4 *inverse*: μ_2, μ_3
2	undetermined and further investigation required	*similar*: μ_7 (*exact*), μ_8 (*inverse*) *dissimilar*: μ_5, μ_6, μ_9, μ_{10}
3 or 4	*Dissimilar*	

b) if the mapping tuple size is 2 then the closeness of the images cannot be determined by solely examining the tuple size and further investigation is required by examining the mapping variation:

- if the mapping variation is μ_7 or μ_8 then the images are *similar*; *exact* or *inverse*, respectively.
- if the mapping variation is μ_5, μ_6, μ_9 or μ_{10} then the images are *dissimilar*.

c) if the tuple mapping size is either 3 or 4 the images are *dissimilar*.

G. *Probability of Occurrence of Similar and Dissimilar Images*

In this section the equations that define the probability of occurrence of *similar* images is introduced; the probabilities of occurrence of both types of *similar* images, *exact* and *inverse*, are given. Finally, the probability of occurrence of *dissimilar* images is obtained.

1) *Probability of Occurrence of Exact and Inverse Images*

For the images to be *exact*, the mapping variation between them must be μ_1, μ_4 or μ_7. The probability of occurrence of an *exact* image (P(E, p)) can then be calculated by,

$$\mathrm{P}(E, p) = \mathrm{Pv}(1, p) + \mathrm{Pv}(4, p) + \mathrm{Pv}(7, p) \quad (26)$$

which results in,

$$\begin{aligned}\mathrm{P}(E,p) &= \frac{2}{4^p}\Phi(p-1) + \frac{1}{2}\cdot\frac{1}{4^{p-1}}(2^{p-1}-1)\Phi(p-2) \\ &= \frac{1}{2}\cdot\frac{1}{4^{p-1}}\left(\Phi(p-1) + (2^{p-1}-1)\Phi(p-2)\right)\end{aligned} \quad (27)$$

where $\Phi(x)$ is the *Heaviside* unit step function. This equation reduces to the simple equation,

$$\mathrm{P}(E,p) = \frac{1}{2^p} \qquad p \geq 1 \quad (28)$$

For the images to be *inverse*, the mapping variation between them must be μ_2, μ_3 or μ_8. The probability of occurrence of an *inverse* image (P(I, p)) can then be calculated by,

$$\mathrm{P}(I, p) = \mathrm{Pv}(2, p) + \mathrm{Pv}(3, p) + \mathrm{Pv}(8, p) \quad (29)$$

which results in,

$$\mathrm{P}(I,p) = \frac{1}{2}\cdot\frac{1}{4^{p-1}}\left(\Phi(p-1) + (2^{p-1}-1)\Phi(p-2)\right) \quad (30)$$

This is the same equation obtained for P(E, p). Hence, the P(E, p) and P(I, p) curves are identical and are defined by,

$$\mathrm{P}(E,p) = \mathrm{P}(I,p) = \frac{1}{2^p} = \left(\frac{1}{2}\right)^p \qquad p \geq 1 \quad (31)$$

This curve is a geometric progression (geometric sequence) starting at p = 1 with initial value ½ and common ratio ½. The progression is:

$$\mathrm{P}(E,p) = \mathrm{P}(I,p) = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \ldots \qquad p \geq 1 \quad (32)$$

A plot of P(E,p) is shown in Fig. 5 and the first 15 values are tabulated in Table IV. It can be seen that P(E,p) approaches zero with increasing value of p,

$$\lim_{p\to\infty} \mathrm{P}(E,p) = \lim_{p\to\infty}\left(\frac{1}{2^p}\right) = 0 \quad (33)$$

An interpretation of the probability values: On the 1st pixel mapping there is ½ chance that the images are *exact* (or *inverse*), on the 2nd pixel mapping there is ¼ chance that the images are *exact* (or *inverse*), on the 3rd pixel mapping there is 1/8 chance that the images are *exact* (or *inverse*), …, etc. The probability values quickly decrease and approach zero; e.g., by the 10th mapping, the value of P(E,10) = P(I, 10) < 10^{-3}.

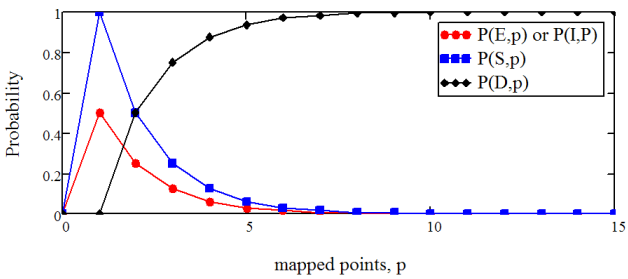

Figure 5. Plots of P(E, p), P(S, p) and P(D, p).

TABLE IV. VALUES OF P(E, p), P(S, p) AND P(D, p)

	P(E, p)*	P(S, p)	P(D, p)
1	0.50000	1.00000	0.00000
2	0.25000	0.50000	0.50000
3	0.12500	0.25000	0.75000
4	0.06250	0.12500	0.87500
5	0.03125	0.06250	0.93750
6	0.01563	0.03125	0.96875
7	0.00781	0.01563	0.98438
8	0.00391	0.00781	0.99219
9	0.00195	0.00391	0.99609
10	0.00098	0.00195	0.99805
11	0.00049	0.00098	0.99902
12	0.00024	0.00049	0.99951
13	0.00012	0.00024	0.99976
14	0.00006	0.00012	0.99988
15	0.00003	0.00006	0.99994

* Values of P(I, p) are equal to P(E, p)

2) *Probability of Occurrence of Similar Images*

The probability of occurrence of a *similar* image (P(S,p)) is the sum of the probabilities of occurrence of an *exact* image and an *inverse* image,

$$\mathrm{P}(S, p) = \mathrm{P}(E, p) + \mathrm{P}(I, p) = 2 \cdot \mathrm{P}(E, p) \quad (34)$$

which results in,

$$\mathrm{P}(S, p) = \frac{1}{2^{p-1}} = \left(\frac{1}{2}\right)^{p-1} \qquad p \geq 1 \quad (35)$$

This curve is also a geometric progression starting at p = 1, but with initial value 1 and common ratio ½. The progression is:

$$\mathrm{P}(S, p) = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \cdots p \geq 1 \quad (36)$$

A plot of P(S, p) is shown Fig. 5 and the first 15 values are tabulated in Table IV. It can be seen that P(S, p) approaches zero with increasing value of p,

$$\lim_{p\to\infty} \mathrm{P}(S, p) = \lim_{p\to\infty}\left(\frac{1}{2^{p-1}}\right) = 0 \quad (37)$$

An interpretation of the probability values: The probability values for the P(S, p) are twice as large as the probability values for P(E, p) (or P(I, p)); hence they decreases half as quickly as for P(E, p) as can be seen from the plot. On the 1st pixel mapping there is 100% chance that the images are *similar* (i.e either *exact* or *inverse*), on the 2nd pixel mapping there is ½ chance that the images are *similar*, on the 3rd pixel mapping there is ¼ chance that the images are *similar*, …, etc. The probability values quickly decrease and approach zero.

3) Probability of Occurrence of Dissimilar Images

The probability of occurrence of a *dissimilar* image (P(D, p)) is the sum of the probability of occurrence of the mapping variations: μ_5, μ_6, μ_9, μ_{10}, μ_{11}, μ_{12}, μ_{13}, μ_{14} and μ_{15}. This can be obtained directly from,

$$\mathrm{P}(D, p) = 1 - \mathrm{P}(S, p) \quad (38)$$

which results in,

$$\mathrm{P}(D, p) = 1 - \frac{1}{2^{p-1}} \qquad p \geq 1 \quad (39)$$

In difference to the previous progressions, this progression is not a geometric progression,

$$\mathrm{P}(D, p) = 0, \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}, \frac{63}{64}, \frac{127}{128}, \cdots \qquad p \geq 1 \quad (40)$$

A plot of P(D, p) is shown in Fig. 5 and the first 15 values are tabulated in Table IV. The progression here is quite different from P(E, p) and P(S, p), as it can be seen that P(D, p) approaches unity with increasing value of p,

$$\lim_{p\to\infty} \mathrm{P}(D, p) = \lim_{p\to\infty}\left(1 - \frac{1}{2^{p-1}}\right) = 1 \quad (41)$$

An interpretation of the probability values: On the 1st pixel mapping there is no possibility that the images are *dissimilar*, since they are *similar* (in agreement with the interpretation of P(S, p) above). On the 2nd pixel mapping there is a ½ a chance that the images are *dissimilar*. On the 3rd pixel mapping there is ¾ a chance that the images are *dissimilar*, …, etc. The probability values quickly increase and approach unity; e.g., P(D, 5) > 0.93, P(D, 8) > 0.99, , P(D, 11) > 0.999, P(D, 15) > 0.9999.

IV. Discussion

Based on the above probabilities we see that as the number of pixels mapped increases the probability that the images are different increases. This is expected for random variables reflecting different (random) images. As predicted by (39) we see that the probability of the images being different (i.e. P(D, p)) increases quickly with increased mapping as was shown in Fig. 5. Hence, we arrive at the following conclusion when mapping images:

- There is a 75% chance that the images are different by the 3rd mapping.
- There is a 93.75% chance that the images are different by the 5th mapping.
- There is a 99.22% chance that the images are different by the 8th mapping.
- There is a 99.90% chance that the images are different by the 11th mapping.
- There is a 99.99% chance that the images are different by the 15th mapping.

These values are summarized in Table V. Therefore, depending on the confidence level required the maximum number of points to be mapped is predetermined. As an example, if a confidence level of 99% is required then a maximum of only 8 points need to be mapped. If after mapping 8 points the images haven't been found to be different, then the images have a 99.22% possibility of being similar.

It is important to note that the values appearing in Table V are for the ideal case when the images are ideally different. If the images are not ideally different, which is the usual case when dealing with real images, then more points need to be mapped as explained below.

A. Random Synthetic Images

Initially, tests were conducted on perfect different images, i.e. 100% random images, to verify the equations obtained. Fig. 6 shows two random binary images that were generated by a random number generator. Fig. 7 shows sample plots of the tuple size vs. the number of pixels mapped for the two random images. Here we see that the mapping tuples reach the maximum size of 4 as early as the 4th mapping and by the 17th mapping in the worst case. This was repeated 1000 times and the cumulative distribution function (cdf) of the histogram was constructed as shown in Fig. 8. From the cdf, the 50th percentile is at the 7th mapped value, the 90th percentile is at the 14th mapped value and the 99th percentile is at the 21st mapped value which exactly agrees with (16); the theoretical derived equation for a 4-tuple mapping.

TABLE V. NUMBER OF MAPPINGS AND THE ASSOCIATED CONFIDENCE LEVELS

Confidence Level	No. of mapped pixels
75.00%	3
93.75%	5
99.22%	8
99.90%	11
99.99%	15

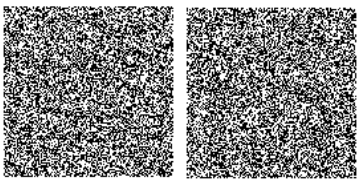

Figure 6.Two 128×128 random binary images.

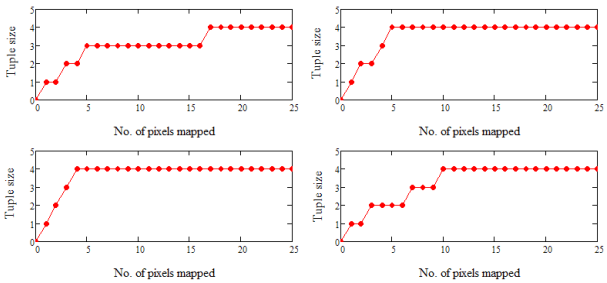

Figure 7.Sample plots of the tuple size vs. the number of pixels mapped for the two *random* images.

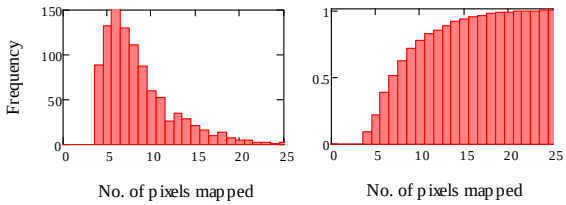

Figure 8.Mapping results for finding the number of pixels mapped to reach the maximum tuple size of 4 for the binary random images (1000 trials); (left) histogram and (right) cdf of the histogram.

Figure 9.Examples of real images used for testing.

B. *Real Images*

When tested with real images the number of mappings required to reach a certain confidence level increased depending on the images. For example for the 128×128 real images shown in Fig. 9, the corresponding number of mappings and confidence levels found were 50 and 95 for 90% and 99%, respectively. These values are more than 10 times the values tabulated in Table V. Nevertheless, even at 50 or 95 points, the technique is magnitudes faster than traditional methods.

V. CONCLUSION

In this paper we have presented a fast method for rejecting different binary images. Only a few points need to be mapped and checked to arrive at the confidence level for rejecting images as being different. Only 8 points need to be checked to arrive at a 90% confidence, 11 points need to be checked to arrive at a 99% confidence and only 15 points need to be checked to arrive at a 99.9% confidence that the images are different or the same. Compared to traditional image matching methods that are image size dependent, this method is magnitudes faster. The method does not require any preprocessing to the images.

Initial results on real image sets have shown that as the differences between images become large, the results obtained agree with the equations derived. As the differences between images become less, higher numbers of mappings are required to arrive at the required confidence level. Tests on real image sets are not yet complete and will be reported on shortly.

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