A New Precoding Scheme for the Codebook Based MISO Beamforming

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***Abstract*—Feedback of channel information can be used in MISO precoding to increase link reliability. To reduce the overhead in feedback signaling channel, codebook based precoding scheme is adopted in practical systems including LTE. However, the size-limited codebook has obvious quantization error which will incur some performance loss compared to the optimal precoding. In this paper, we propose a new precoding scheme for the codebook based MISO system. The proposed scheme utilized the PMI report from UE to modify the precoding vector towards the optimal beamforming vector on per slot basis. Simulation results indicate that the proposed scheme has obvious SNR gain over the conventional precoding scheme under same feedback overhead.**

***Index Terms*—precoding, codebook, MISO**

I. Introduction

Transmit beamforming, also known as transmit precoding, has attracted a great deal of research interest in recent years [1]─[6]. For the scenarios where the base station (eNodeB) has multiple antennas and user equipment (UE) has one antenna, an adequate beamforming vector can concentrate the UE desired signal towards the direction of UE and hence achieve both array gain and diversity gain [5].

The implementation of perfect precoding requires the channel state information (CSI) to be known at the transmitter. In frequency-division duplex (FDD) systems, CSI is usually estimated by the receiver and then fed back to the transmitter through a reliable signaling channel. For a MISO (multi-input single-output) system with N_t transmit antennas, the CSI consists of N_t complex channel coefficients. It is practically difficult (if not impossible) to accurately feedback all those coefficients through a signaling channel with limited channel capacity. For this reason, practical systems such as LTE (Long Term Evolution) [1] has adopted the codebook based precoding.

In a codebook based precoding system, the precoding vector space $\pounds^{N_t}$ has been quantized into a subset (codebook $C \subset \pounds^{N_t}$ with finite elements (codes or precoding vectors). The UE, based on the channel estimation selects the best code in codebook C and informs eNodeB of the index of the selected code [7]─[9]. In LTE terminology, this index is referred to as Precoding Matrix Indicator (PMI).

In the codebook-based precoding schemes, most of the codebook designs [10], [11] are optimized with the one dimensional statistical distribution of the channel. The precoding scheme works independently among time slots. For each slot, the precoding vector is completely specified by the PMI report intended for that slot and the codebook is fixed for all slots. In this paper, we propose a new precoding scheme where the precoding codebook for current slot is a modification of the codebook used in previous slot and the modification is specified by the PMI feedback from UE. Without increasing the processing complexity at both transmitter and receiver, without increasing the feedback overhead, the proposed scheme can improve the precoding vector and increase the received signal to-noise power ratio (SNR).

The rest of the paper is organized as follows. Section II is the system model. Section III gives the details of the proposed scheme. Section IV is the simulation results and finally we conclude the paper in Section V.

II. System Model

Consider the system model shown in Fig.1. At n-th time slot, the data symbol s_n is multiplied by the unitary precoding vector $\boldsymbol{w}_n$ and then passed through a MISO channel with N_t transmits antennas and 1 receive antenna. The received signal at n-th slot is given by

$$y_n = \boldsymbol{h}_n \boldsymbol{w}_n s_n + z_n \tag{1}$$

where z_n is the additive white Gaussian noise (AWGN) which is modeled as a circularly symmetric complex Gaussian random variable $\mathcal{CN}(0,\sigma^2)$;

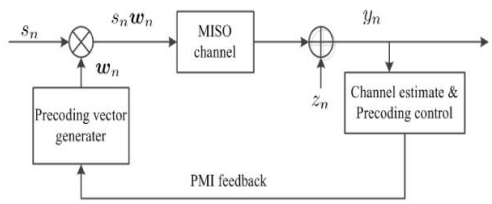

Figure 1. System model.

Manuscript received May3, 2013; revised August 11, 2013

doi: 10.12720/ijsps.1.1.98-102

$\boldsymbol{h}_n = (h_{1,n}, h_{2,n}, \mathrm{K}, h_{N_t,n})$ is a row vector representing the MISO channel coefficient vector at n-th slot. Assume that $\boldsymbol{h}_n$ can be modeled as a circularly symmetric complex Gaussian random vector $\mathcal{CN}(0, I_{N_t})$; The average transmit power is $\mathbb{E}[|s_n|^2]$; The precoding vector $\boldsymbol{w}_n$ is a unitary column vector satisfying $\|\boldsymbol{w}_n\|^2 = 1$. We also assume that $\boldsymbol{h}$ can be perfectly estimated at receiver (UE).

Under the assumptions above, the signal-to-noise power ratio (SNR) at n-th time slot is given by

$$\gamma_n = |\boldsymbol{h}_n \boldsymbol{w}_n|^2 \cdot \frac{P}{\sigma^2}. \tag{2}$$

Obviously, the optimal precoding vector $\boldsymbol{w}_n^*$, which can maximize γ, is the vector that matches to the direction of the channel coefficient vector, i.e.

$$\boldsymbol{w}_n^* = \frac{\boldsymbol{h}_n^H}{\|\boldsymbol{h}_n\|} \tag{3}$$

where the superscript $(\cdot)^H$ denotes the Hermitian transpose. The SNR under the optimal precoding is given by

$$\gamma_n^* = \|\boldsymbol{h}_n\|^2 \cdot \frac{P}{\sigma^2}. \tag{4}$$

In a FDD system, the channel direction information (CDI), i.e. $\boldsymbol{h}_n / \|\boldsymbol{h}_n\|$, is only known to UE and it is impossible for eNodeB to realize a precoding vector exactly specified by (3) unless the feedback signaling channel has infinite capacity.

For this reason, codebook based precoding schemes have been adopted in many systems including LTE [1]. In these precoding schemes, the space of all possible $\boldsymbol{w}_n^*$, i.e. $\pounds^{N_t}$ has been quantized into a set of M elements: $\boldsymbol{C} = \{\mathrm{c}_0, \mathrm{c}_1, \mathrm{K}\, \mathrm{c}_{M-1}\}$ where $\mathrm{c}_i \in \pounds^{N_t}$ and the quantization set $\boldsymbol{C}$ is referred to as *codebook*. For example, the codebook designed by LTE for 2×1 scenario is $\boldsymbol{C} = \{\mathrm{c}_0, \mathrm{c}_1, \mathrm{c}_2, \mathrm{c}_3\}$ with elements defined by [1]

$$\begin{cases} \mathrm{c}_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \\ \mathrm{c}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \\ \mathrm{c}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\\mathrm{j}\end{pmatrix} \\ \mathrm{c}_3 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-\mathrm{j}\end{pmatrix} \end{cases} \tag{5}$$

Instead of feedback the whole CDI $\boldsymbol{h}_n / \|\boldsymbol{h}_n\|$ to eNodeB, UE selects a code in $\boldsymbol{C}$ which will yield the largest SNR, and the index is informed to eNodeB. This index is termed as Precoding Matrix Indicator (PMI) in 3GPP specifications. When the index k arrived at UE via feedback signaling channel, eNodeB uses the code $\mathrm{c}_k \in \boldsymbol{C}$ as the precoding vector for the n-th time slot.

$$k = \underset{i=0,1,\mathrm{L},M-1}{\operatorname{argmin}} |\boldsymbol{h}_n \mathrm{c}_i|^2 \tag{6}$$

The SNR under the precoding vector c_k is given by

$$\gamma_n = |\boldsymbol{h}_n \mathrm{c}_k|^2 \cdot \frac{P}{\sigma^2} \tag{7}$$

which is upper bounded by γ_n^* given by (4).

III. Proposed Scheme

Consider a UE which has been scheduled for transmission at 0-th time slot and will continue to transmit in the following slots. At the first slot ($n = 0$), based on the estimated channel coefficients vector $\boldsymbol{h}_0$, UE determines which code in $\boldsymbol{C}_0 = \boldsymbol{C}$ should be used by eNodeB and the index will be fed back by UE. Then the eNodeB uses c_k as $\boldsymbol{w}_0$, the precoding vector for 0-th slot.

$$k = \underset{i=0,1,\mathrm{L},M-1}{\operatorname{argmax}} |\boldsymbol{h}_0 \mathrm{c}_i|^2 \tag{8}$$

In the conventional precoding scheme, the same procedure as above will be repeated at all subsequent slots. We now propose a new scheme which can improve the system performance without additional feedback load and processing complexity.

At ($n+1$)-th slot, instead of using the fixed codebook $\boldsymbol{C}_0 = \boldsymbol{C}$, a new codebook

$$\boldsymbol{C}_{n+1} = \{\mathrm{c}_{0,n+1}, \mathrm{c}_{1,n+1}, \mathrm{K}\, \mathrm{c}_{M-1,n+1}\},\ n \ge 0 \tag{9}$$

is used as the codebook for $\boldsymbol{w}_{n+1}$, the precoding vector for the ($n+1$)-th time slot. The codes in $\boldsymbol{C}_{n+1}$ is given by

$$\mathrm{c}_{i,n+1} = \frac{\boldsymbol{w}_n + \delta \cdot \hat{\mathrm{c}}_i}{\|\boldsymbol{w}_n + \delta \cdot \hat{\mathrm{c}}_i\|}, i = 0,1,\mathrm{L}\ ,M-1 \tag{10}$$

where $\boldsymbol{w}_n \in \boldsymbol{C}_n$ is the precoding vector used by eNodeB at n-th time slot, $\boldsymbol{C}_n$ is the codebook for n-th time slot, $\delta > 0$ is a predefined step size, $\hat{\mathrm{c}}_i$ is defined as

$$\hat{\mathrm{c}}_i = \begin{cases} \mathrm{c}_{i,n}, & \boldsymbol{w}_n \ne \mathrm{c}_{i,n} \\ \alpha\left(\mathrm{c}_{i,n} - \sum_{m=0}^{M-1} \mathrm{c}_{m,n}\right)\|\hat{\mathrm{c}}_i\|, & else \end{cases} \tag{11}$$

where α is a scaling factor which normalize $\hat{c}_i$ to a unitary vector. In other words, the set $\{\hat{c}_0, \hat{c}_1, \ldots, \hat{c}_{M-1}\}$ is the same set as $\boldsymbol{C}_n = \{c_{0,n}, c_{1,n}, \ldots c_{M-1,n}\}$ except that the code being used in n-the slot, say $\boldsymbol{w}_n = c_{k,n}$, has been replaced by $-\alpha \sum_{m \neq k} c_{m,n}$.

The rest of the operation is the same as the conventional scheme. UE selects a code from $\boldsymbol{C}_{n+1}$ and feeds back the index

$$k = \underset{i=0,1,\ldots,M-1}{\operatorname{argmax}} \left| \boldsymbol{h}_{n+1} c_{i,n+1} \right|^2 \tag{12}$$

Upon the received index k, eNodeB constructs the precoding vector as

$$\boldsymbol{w}_{n+1} = c_{k,n+1} = \frac{\boldsymbol{w}_n + \delta \cdot \hat{c}_k}{\left\| \boldsymbol{w}_n + \delta \cdot \hat{c}_k \right\|} \tag{13}$$

where $\hat{c}_k$ is constructed from (11).

To illustrate the principle of the proposed scheme, consider a simple example where the channel is static: $\boldsymbol{h}_0 = \boldsymbol{h}_1 = \ldots = (0.1, 1)$. The optimal precoding vector for this $\boldsymbol{h}$ is

$$\boldsymbol{w}^* = \frac{1}{\sqrt{1.01}} \begin{pmatrix} 0.1 \\ 1 \end{pmatrix}$$

and the corresponding SNR is $\gamma^* = 1.01\bar{\gamma}$ where $\bar{\gamma} = P/\sigma^2$.

For the conventional codebook based precoding scheme with codebook defined by (5), the PMI fed back by UE will be $k = 0$ since $\left|\boldsymbol{h} c_0\right| \geq \left|\boldsymbol{h} c_i\right|$ for all i. Then the precoding vector to be used by eNodeB will be $\boldsymbol{w}_0 = c_0$. Since the channel is assumed as static, the PMI fed back by UE and the precoding vector applied at eNodeB will be the same for all time slots. Since $\left|\boldsymbol{h} c_0\right|^2 = 0.605$, the received SNR will be 2.22dB less compared to the optimal precoding.

In the proposed scheme, the operation in the first slot ($n = 0$) is same as the conventional scheme. The difference lies in the remaining slots ($n > 0$). At the slot $n = 1$, form (11), we have

$$\begin{cases} \hat{c}_0 = -\alpha(c_1 + c_2 + c_3) = \dfrac{1}{\sqrt{10}} \begin{pmatrix} -3 \\ 1 \end{pmatrix} \\ \hat{c}_1 = c_1 \\ \hat{c}_2 = c_2 \\ \hat{c}_3 = c_3 \end{cases}. \tag{14}$$

Assuming $\delta = 0.3$, the codebook defined for slot 1, $\boldsymbol{C}_1$, can be constructed from (10) and the results are listed as follows

$$\begin{cases} c_{0,1} = (0.4661,\ 0.8847)^{\mathrm{T}} \\ c_{1,1} = (0.8805,\ 0.4741)^{\mathrm{T}} \\ c_{2,1} = (0.78,\ 0.6+0.18\mathrm{j})^{\mathrm{T}} \\ c_{3,1} = (0.78,\ 0.6-0.18\mathrm{j})^{\mathrm{T}} \end{cases}. \tag{15}$$

Comparing $\left|\boldsymbol{h} c_{i,1}\right|$ for all $i = 0,1,2,3$, UE will feed back PMI=0 and the precoding vector will be $\boldsymbol{w}_1 = c_{0,1}$. The SNR at slot 1 will be $\gamma_1 = 0.8674\bar{\gamma}$ which is 0.66dB less than the optimal precoding.

Similarly, the codebook for slot 2 is

$$\begin{cases} c_{0,2} = (0.2925,\ 0.9563)^{\mathrm{T}} \\ c_{1,2} = (0.5795,\ 0.8150)^{\mathrm{T}} \\ c_{2,2} = (0.5489,\ 0.8348+0.0423\mathrm{j})^{\mathrm{T}} \\ c_{3,2} = (0.5489,\ 0.8348-0.0423\mathrm{j})^{\mathrm{T}} \end{cases} \tag{16}$$

and the SNR at slot 2 is $\gamma_2 = 0.9712\bar{\gamma}$ which is 0.17dB less than the optimal precoding.

This procedure will continue in all the following slots and it can be shown that

$$\lim_{n \to \infty, \delta \to \infty} \boldsymbol{w}_n = \boldsymbol{h}^{\mathrm{H}} \tag{17}$$

implying that the proposed scheme can essentially approach to the optimal precoding vector. In the practical systems, there is a tradoff for the design of δ. A small δ can result in a better precoding vector but will cost longer convergence time.

IV. Simulation Results

In this section, we use simulation to compare the performance of the proposed scheme and the conventional one.

The comparison is based on the SNR gain. Given channel response $\boldsymbol{h}$ and the precoding vector $\boldsymbol{w}$, the received SNR at UE is proportional to $\left|\boldsymbol{h}\boldsymbol{w}\right|^2$. At the slot n, the SNR gain of the proposed scheme over the conventional scheme can be expressed as, in deci-Bell,

$$\text{Gain}=20\lg \frac{\left|\boldsymbol{h}_n \boldsymbol{w}_n^{\text{prop}}\right|}{\left|\boldsymbol{h}_n \boldsymbol{w}_n^{\text{conv}}\right|} \tag{18}$$

where $\boldsymbol{w}_n^{\text{prop}}$ and $\boldsymbol{w}_n^{\text{conv}}$ stands for the precoding vector in the proposed and conventional scheme, respectively.

A. Static Channel

Firstly we take account into the transmitting scenario in static channel. The simulation parameters are shown in Table I

TABLE I. SIMULATION ASSUMPTIONS FOR 2×1 STATIC CHANNEL

Parameter	Simulation Cases
Channel	Standard Rayleigh Channel
Antenna ports	2×1
Is channel irrelevant	Yes
Number of converging timeslots	20
Code book used	Table 6.3.4.2.3-1 in [1]

For different value of δ, the simulated results at slot $n=20$ are shown in Fig. 2 for a 2×1 MISO system. Since $\boldsymbol{h}$ is a random vector, the gain defined by (18) is a random variable.

The curves shown in Fig. 2 are the cumulative distribution function (CDF) of the gain corresponding to different values of δ.

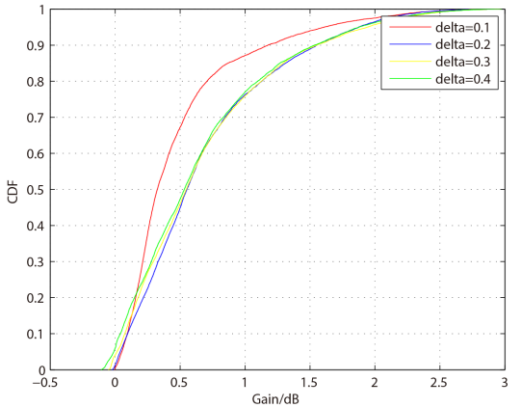

Figure 2. The CDF of the gains after 20 timeslots in 2 × 1 MISO system.

We can see from Fig. 2 that the gain is randomly distributed in the interval $[0,3]$ dB. If δ is appropriate (for example, 0.3), in the static channel, the new scheme will have 1dB gain in 20 percent cases in 2×1 MISO system. If δ is very small, the precoding vector constructed by system can be finally very close to the channel status vector as suggested by (17).

However, this accuracy is not worthy when considering the converging time it costs. The reason that the red line (corresponding to smaller δ) has less gain is due to the convergence speed — at this small step size, it needs more slots for $\boldsymbol{w}_n$ to converge. On the other hand, if δ is too large, for example 0.4, the constructed precoding vector may significantly deviate the optimal direction if the original precoding vector is very close to the channel status vector. That's why the left end of the green line in Fig. 2 is lower than 0dB. In most cases, $\delta=0.3$ is good enough. In fact, when δ is within the range 0.2 to 0.4, the performance of the proposed scheme is insensitive to the setting of δ.

The simulation result for 4×1 MISO system is shown in Fig. 3. The corresponding simulation parameters are listed in Table II.

TABLE II. SIMULATION ASSUMPTIONS FOR 4×1 STATIC CHANNEL

Parameter	Simulation Cases
Channel	Standard Rayleigh Channel
Antenna ports	4×1
Is channel irrelevant	Yes
Number of converging timeslots	20
Code book used	Table 6.3.4.2.3-2 in [1]

Fig. 3 indicates that if δ is appropriate (for example, 0.3), in the static channel, the new scheme will have 1dB gain in 80 percent cases and 2dB gain in 30 percent cases in 4×1 MISO system. Obviously the proposed scheme performs better when the number of antenna ports increases.

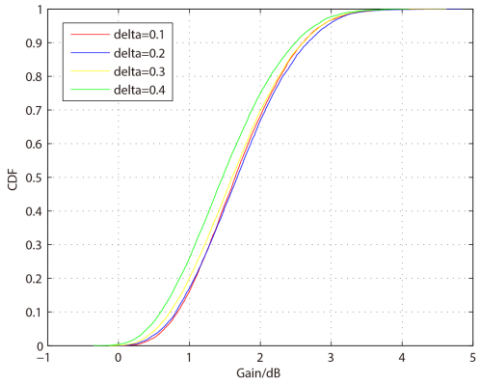

Figure 3. The CDF of the gains after 20 timeslots in 4 × 1 MISO system.

B. Time-Varing Channel

In this subsection we observe the performance of the proposed scheme in the time-varying channel. The channel is simulated by Jakes' Model. The system parameters are shown in Table III and Table IV for the 2×1 and 4×1 MISO system, respectively.

In LTE, the basic scheduler time unit is called subframe which lasts 1 ms, so we make the timeslot equal to 1ms. There are 4 time-invariant reference signals in one timeslot (1 ms), so the sampling rate is taken as 4000Hz([12], [13]). We compare the average gain which is defined as

$$\text{Gain}=10\lg\frac{\mathbb{E}\left[\left|\boldsymbol{h}_n\boldsymbol{w}_n^{\text{prop}}\right|^2\right]}{\mathbb{E}\left[\left|\boldsymbol{h}_n\boldsymbol{w}_n^{\text{conv}}\right|^2\right]} \qquad [19]$$

where $\mathbb{E}[\cdot]$ is the mathematical expectation.

The comparisons shown in Fig.4 and Fig.5 are under the same Doppler frequency shift which is specified by

TABLE III. SIMULATION ASSUMPTIONS FOR 2×1 TIME-VARYING CHANNEL

Parameter	Simulation Cases
Channel model	Jakes' Model
Sampling rate	4000Hz
Central carrier frequency	2G Hz
Antenna ports	2×1
Is channel irrelevant	Yes
Timeslot unit	1 ms
Codebook used	Table 6.3.4.2.3-1 in [1]
δ	0.4

UE's moving speed (in km/h). In the simulation, we have also taken into consideration the feedback delay. The step size is fixed to $\delta = 0.4$ for both 2×1 and 4×1 systems.

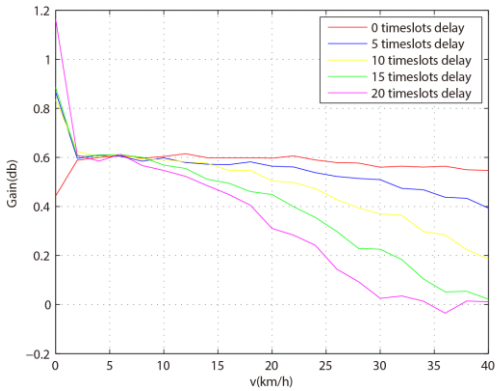

Figure 4. Gains with different timeslots delays in the 2×1 MISO system.

Both Fig. 4 and Fig. 5 indicate that the proposed scheme can keep a positive gain even in a time varying channel. The gain is more significant for slower varying channel, less feedback delay and more antenna ports.

TABLE IV. SIMULATION ASSUMPTIONS FOR 4×1 TIME-VARYING CHANNEL

Parameter	Simulation Cases
Channel model	Jakes' Model
Sampling rate	4000Hz
Central carrier frequency	2G Hz
Antenna ports	4×1
Is channel irrelevant	Yes
Timeslot unit	1 ms
Codebook used	Table 6.3.4.2.3-2 in [1]
δ	0.4

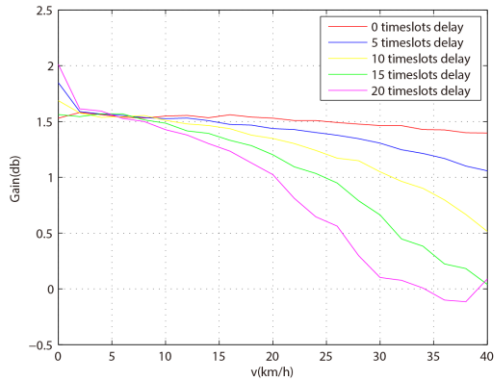

Figure 5. Gains with different timeslots delays in the 4×1 MISO system.

V. CONCLUSION

A new precoding scheme has been proposed. In this scheme, we use the PMI report to dynamically change the codebook and the codebook constructed can asymptotically approach to the optimal precoding vector in static channels. Simulation results show that, under Rayleigh fading channels, the proposed scheme can bring more than 1dB benefit for a majority of users.

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